# THE WAVELIKE PROPERTIES OF PARTICLES

"Flashes of insight" as Ken Krane calls them.
All started from the idea of a young French aristocrate, Prince Louis de Broglie.
His thinking was in a large part driven by AESTETICISM (world is essentially beautiful, the laws of nature also should be).

The text below is copied from the book "Understanding quantum physics" by Michael A. Morrison. He writes good textbooks, and has a great sense od humor.

### *2.6 IT ALL STARTED WITH DE BROGLIE*

The experiments discussed in § 2.3–2.5 raise a host of questions. If electromagnetic radiation consists of photons—localized clumps of energy—how can we explain phenomena such as diffraction and interference? If not, then why did Compton have to use classical collision theory to explain the scattering of *x rays* by metals? On the other hand, if electrons are particles, why do they produce an interference pattern at the detector in the double-slit experiment? The behavior of electrons and photons in these experiments seems provocatively similar—crazy, to be sure, but crazy in the same way. Are electrons and photons in some sense the same?

Einstein was deeply puzzled by this question until he noticed a possible answer in the doctoral thesis of a young French physicist. In 1924, Einstein wrote in a letter to his Dutch colleague Hendrik Lorentz (1853–1928) that the research of Prince Louis de Broglie (1892–1975) "...is the first feeble ray of light to illuminate this, the worst of our physical riddles." De Broglie's achievement was to synthesize the wave-like and particle-like aspects of microscopic matter. Although de Broglie seems to have only dimly understood the nature of quantum particles, and his rather nebulous physical models of quanta have since been superseded, the importance of his contribution has not diminished. It initiated the development of modern quantum mechanics.

[20]For details, see J. S. Marsh, *Amer. Jour. Phys.*, **43**, 97 (1975).

### *Sometimes a Great Notion*

In 1910 de Broglie began studying history at the University of Paris; soon, however, he switched to physics. His studies were interrupted in 1913 by a six-year stint in the French army, during which he and his brother Maurice worked on wireless telegraphy. Then in 1919 he returned to Paris for his doctoral research.

From work on the x-ray spectra of heavy elements, de Broglie knew of photons and the Bohr model of atomic structure. And he was particularly intrigued by "Planck's mysterious quanta." So he set himself the task of "[uniting] the corpuscular and undulatory points of view and thus [penetrating] a bit into the real nature of quanta."

In 1923, lightning struck. As de Broglie tells it:

> As in my conversations with my brother we always arrived at the conclusion that in the case of x-rays one had [both] waves and corpuscles, thus suddenly—I cannot give the exact date when it happened, but it was certainly in the course of summer 1923—I got the idea that one had to extend this duality to the material particles, especially to electrons.

Thus did de Broglie come up with the idea of *matter waves.* This idea led him to the important notion that *all microscopic material particles are characterized by a wavelength and a frequency, just like photons.*

Aesthetic considerations seem to have influenced de Broglie's thinking towards the idea of matter waves. He evidently felt that nature should be symmetrical, so if *particles of light* (photons) were to be associated with electromagnetic radiation, then so should *waves of matter* be associated with electrons. Simply stated, his hypothesis is this: *There is associated with the motion of every material particle a "fictitious wave" that somehow guides the motion of its quantum of energy.*

In spite of its rather vague character, this idea was remarkably successful. For example, using the methods of classical optics (such as Fermat's principle) to describe the propagation of quanta, de Broglie was able to explain how photons (and, for that matter, electrons) diffract and interfere: It is not the particles themselves but rather their "guide waves" that diffract and interfere. In de Broglie's words, "the fundamental bond which unites the two great principles of geometrical optics and of dynamics is thus fully brought to light."

De Broglie proffered these ideas in his Ph.D. dissertation, which he wrote at age 31. His thesis did not fully convince his examiners, who were impressed but skeptical of the physical reality of de Broglie's matter waves. One examiner later wrote, "at the time of the defense of the thesis, I did not believe in the physical reality of the waves associated with the particles of matter. Rather, I regarded them as very interesting objects of imagination."[21] Nevertheless, de Broglie passed.

### *Beautiful Equations*

De Broglie's equations for the wavelength and frequency of his matter waves are elegant and simple. Even their derivations are not complicated.[22] In his seminal paper of 1923,

[21] It is not clear that de Broglie himself knew what he meant by a matter wave. In his thesis he notes that "the definitions of the phase wave and of the periodic phenomenon were purposely left somewhat vague." In subsequent papers, de Broglie tried several different interpretations of his elusive waves. [For a thorough discussion of these interpretations, see Chap. V of *The Historical Development of Quantum Theory*, Volume I, Part 2 by J. Mehra and H. Rechenberg (New York: Springer-Verlag 1982).]

[22] See Cropper, op. cit., pp. 57–63.

Read only if you really want to know the details of de Broigle reasoning:

de Broglie began with *light quanta*—photons—so I'll first recap the derivation of the equation relating the wavelength and momentum of a photon and then press on to material particles.

The photon is a relativistic particle of rest mass $m_0 = 0$. Hence the momentum $p$ of a photon is related to its total energy $E$ through the speed of light $c$ as

$$p = \frac{E}{c}. \qquad [\text{for } m_0 = 0] \tag{2.10}$$

To introduce the frequency $\nu$ of the photon, we use Einstein's equation for the photon energy

$$E = h\nu \tag{2.11}$$

to write Eq. (2.10) as

$$p = \frac{h\nu}{c}. \tag{2.12}$$

For a wave in free space, the wavelength is $\lambda = c/\nu$, so Eq. (2.12) becomes

$$p = \frac{h}{\lambda}. \tag{2.13}$$

Now, in contrast to a photon, a material particle such as an electron has a non-zero rest mass $m_0$. Therefore the relationship between the energy and momentum of such a particle moving at relativistic velocities (in a region of zero potential energy) is not Eq. (2.10), but rather

$$E^2 = p^2c^2 + m_0^2c^4. \tag{2.14}$$

If the velocity of the particle is non-relativistic ($v \ll c$), then its kinetic energy is simply[23]

$$T = \frac{p^2}{2m_0}, \tag{2.15a}$$

where $T$ is the kinetic energy,[24]

[23]The notational conventions of physics dictate a confusing ambiguity between Eqs. (2.14) and (2.15), because the rest mass energy $m_0c^2$ is included in $E$ in (2.14) but not in (2.15). The relativistic kinetic energy, which is sometimes denoted by the symbol $K$, is defined as

$$K = \sqrt{p^2c^2 + m_0^2c^4} - m_0c^2.$$

It is this quantity which, in the non-relativistic limit, reduces to Eq. (2.15*a*). [See *Classical Mechanics*, 2nd ed. by Herbert Goldstein (Reading, Mass.: Addison-Wesley, 1983, pp. 307–309).] I'll adhere to standard practice by using $E$ for the energy in the relativistic and non-relativistic cases.

[24]This equation is usually written with the symbol $E$ standing for the kinetic energy, as

$$E = \frac{p^2}{2m_0}.$$

I'll follow this convention in this book whenever dealing with non-relativistic particles (*i.e.*, most of the time).

$$T = E - m_0c^2. \qquad (2.15b)$$

In either case, the derivation of Eq. (2.13) cannot be applied to a material particle.

Nonetheless, de Broglie proposed that Eqs. (2.11) and (2.13) be used for material particles as well as photons.[25] Thus, *for electrons, atoms, photons and all other quantum particles, the energy and momentum are related to the frequency and wavelength* by

$$\boxed{\begin{aligned} p &= h/\lambda \\ E &= h\nu \end{aligned}} \qquad \text{de Broglie-Einstein equations} \qquad (2.16)$$

Notice that the de Broglie equation $\lambda = h/p$ implies an *inverse* relationship between the total energy $E$ of a particle and its wavelength, *viz.*,

$$\lambda = \frac{hc/E}{\sqrt{1 - \left(\frac{m_0c^2}{E}\right)^2}}. \qquad (2.17)$$

If applied to a photon (by setting the rest mass to zero), this equation reduces to Eq. (2.10). Hence the larger the energy of a particle, the smaller is its wavelength, and *vice versa.*

**Question 2–1**

**Derive** a relationship between the wavelength of an electron and its *kinetic energy T*. **Prove** that in the non-relativistic limit, your result reduces, as it should, to

$$\lambda = \frac{h}{p} = \frac{h}{m_0v}.$$

Only the equations:

$$\left.\begin{aligned} p &= \frac{\lambda}{h} \\ \text{and} \quad E &= h\nu \end{aligned}\right\} \text{de Broglie-Einstein equations}$$

are really what you have to remember.

For particles, the second equation is not often used, but the first is extremely important and it is usually written in the form:

$$\boxed{\lambda = \frac{p}{h}}$$