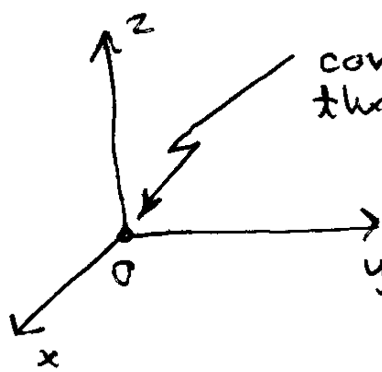
RELATIVITY

CLASSICAL RELATIVITY

Basic concepts: <u>observer</u> - it may be a human, but also anything that can "see" objects, e.g., a TV camera, or some other recording apparatus.

A <u>Frame of reference</u> - a system in which the observer is located.

To describe the position of objects in a frame of reference, we introduce a <u>coordinate system</u> - most often, a Cartesian system.



conventionally, we assume that the observer is located in the origin O.

Quotation from our textbook (Krane):
"A theory of relativity is in effect a way for observers in different frames or relativity to compare the resuts of their observation

~~inertial frames~~: <u>Transformation</u>: the mathematical basis for comparing the physical situation as seen from different frames of reference.

Inertial frame:

Suppose that an observer in her reference frame has the capability of testing the Newton's Laws. If she finds that the First Law holds, such a frame is called an inertial frame!

Question: can you tell me what the I Newton Law says?

Question: If the observer finds that the I Law holds, is it necessary to test the other laws?

Is it easy to find a real inertial frame? Well, not. A spacecraft in the interstellar space, nonrotating and with all rocket engines shut down...

Is our Mother Earth an internal frame? No! Because it's rotating and orbiting the sun. However, the deviations from the Newton Laws caused by these motions are really small, so in the first approximation we can consider Earth as a "nearly inertial" frame.

Questions: there are, however, some visible manifestations of the fact that Earth is not an inertial frame. Could you give some examples.

Hint: you go to the Oregon Coast, you stay there for several hours, and what you see?

Question: Give some examples of reference frames that are obviously non-inertial - you can tell that right away, without looking for subtle effects.

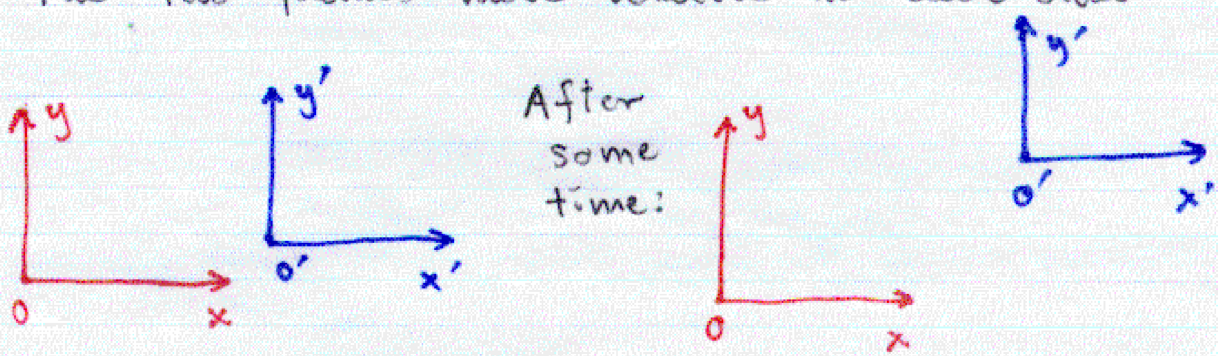
Galilean Transformation:

Named after Galileo – can you tell me something about that gentleman!

Two coordinate frames with axes x, y, z (O) and x', y', z' (O').

For simplicity, let's draw only two axes, x & y.

The two frames move relative to each other:



Q: Time – can you give me a (good) definition of time?

We can think of time as of a fourth coordinate. (a coordinate of different kind than x, y, z, but also a coordinate).

We assume that time is the same in both reference frames (a postulate of classical physics):

$$t = t'$$

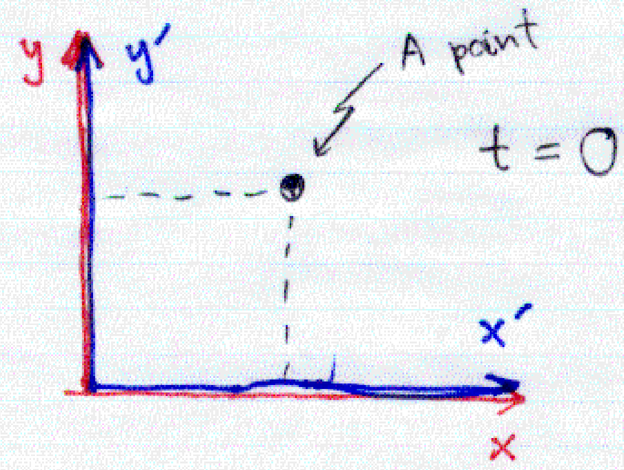
Now, consider that the two frames are moving with respect to each other with a constant velocity

Example: suppose there is a "well-behaved" river in which the current speed is u.
One observer is sitting on the river bank, (frame O), and the other (O') on a raft floating freely in the river water.

As far as the choice of the x, y, z is concerned. We have a total freedom of chosing their orientation!

So, for convenience, let's choose the orientation of the x and x' axes along the direction of the relative motion of the frames.

Also, let's choose the time coordinate such that at $t = 0$ the origins of the two frames are at the same point:



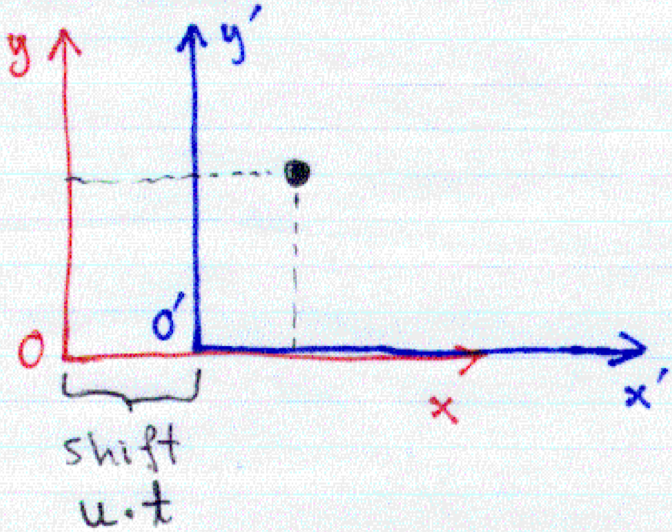
At $t=0$, of course, the coordinates of a certain point are the same in both systems.

$x' = x$

$y' = y$

$z' = z$

The situation, after some time t elapses:



So, now:

$$x' = x - ut$$

$$y' = y$$

$$z' = z$$

$$\text{and} \quad t' = t$$

We can also ask: what are the velocities of the point? (the point may also be moving):

It's very easy to find the velocities, simply by taking derivatives, since:

$$\vec{v} \equiv \frac{d\vec{r}}{dt}$$

Accordingly:

$$v_x = \frac{dx}{dt} \; ; \quad v_y = \frac{dy}{dt} \; ; \quad v_z = \frac{dz}{dt}$$

So:
$$v_x' = \frac{dx'}{dt'} = \frac{d}{dt}(x - ut)$$

$$= \frac{dx}{dt} - \frac{d(ut)}{dt} = v_x - u$$

$$v_y' = \frac{dy'}{dt'} = \frac{dy}{dt} = v_y$$

$$v_z' = \frac{dz'}{dt'} = \frac{dz}{dt} = v_z$$

$$\boxed{\begin{aligned} v_x' &= v_x - u \\ v_y' &= v_y \\ v_z' &= v_z \end{aligned}}$$

<u>And accelerations:</u> $\vec{a} \equiv \frac{d^2\vec{r}}{dt^2} = \frac{d\vec{v}}{dt}$

So:
$$a_x' = \frac{dv_x'}{dt'} = \frac{d}{dt}(v_x - u)$$

$$= \frac{dv_x}{dt} - \underbrace{\frac{du}{dt}}_{= 0 \text{ because } u = \text{const}} = \frac{dv_x}{dt} = a_x$$

$$a'_y = \frac{dv'_y}{dt'} = \frac{dv_y}{dt} = a_y$$

$$a'_z = \frac{dv'_z}{dt'} = \frac{dv_z}{dt} = a_z$$

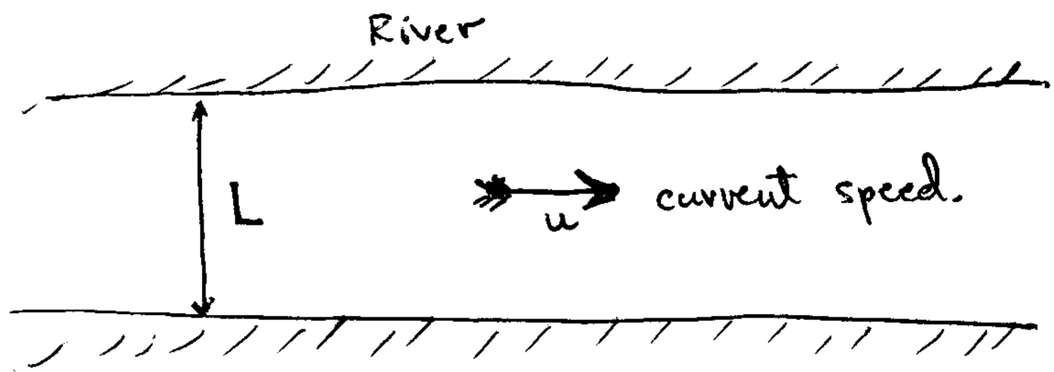
Altogether;

$$\boxed{\begin{array}{lll} x' = x - ut & ; \; v'_x = v_x - u & ; \; a'_x = a_x \\ y' = y & ; \; v'_y = v_y & ; \; a'_y = a_y \\ z' = z & ; \; v'_z = v_z & ; \; a'_z = a_z \\ & \quad t' = t & \end{array}}$$

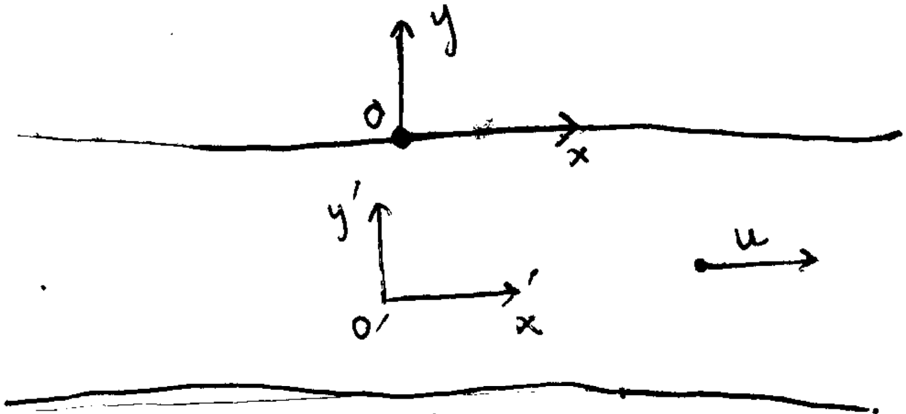
This is the full set of equations that express the <u>Galilean Transformation</u>

Difficult? Of course not!!!!

Now, a <u>practical example</u>, which is important, because we will use the results later.



A swimmer has to swim forth and back across the river, and then swim the same distance (as seen from the river bank) upstream and then downstream to the point she started.



The frame O is a fixed point at the river bank

The frame O' is an object floating freely in the water, (e.g., a person sitting on a raft with no engine or sail).

Our task is to find the time in each case.

Upstream - downstream first (easier case!).

The swimmer always swims with the same speed c. This is her speed relative to the river water!

In other words – the speed in the frame O'!

So, when she swims upstream, her $v'_x = -c$

But, according to the Galilean Transformation:

$$v_x' = v_x - u \Rightarrow v_x = v_x' + u$$

$$v_x = -c + u$$

It's her speed as seen from the bank

To swim the distance L (in the frame O) she needs the time:

$$t_{up} = \frac{-L}{v_x} = \frac{-L}{-c+u}$$ (the distance when swiming upstream is <u>negative</u> in the O frame)

When she turns downstream, her speed relative to river water is +C. So, $v_x' = C$

Again, using the same equation, we obtain:

$$v_x = v_x' + u = c + u$$

and:

$$t_{down} = \frac{L}{v_x} = \frac{L}{c+u}$$

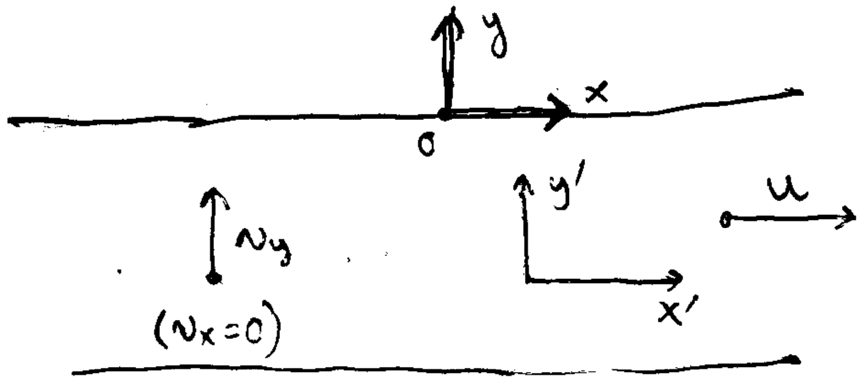
Total time needed:

$$t_{up-down} = t_{up} + t_{down} = \frac{-L}{u-c} + \frac{L}{u+c}$$

$$= L\left[\frac{-u-c+u-c}{(u-c)(u+c)}\right] = L \cdot \frac{-2c}{u^2-c^2}$$

$$= \frac{2L}{c} \frac{1}{1-u^2/c^2}$$

Finding the time when swimming across is more tricky. In order to swim directly across, the swimmer has to "compensate" for the current speed.



In the frame O only the velocity component v_y is non-zero, while $v_x = 0$.

From the Galilean Transf. we find the velocity component v_x' in the O' frame:

$$v_x' = v_x - u = -u \quad (\text{because } v_x = 0).$$

But the swimmer speed in O' is c. And

$$c = \sqrt{v_x'^2 + v_y'^2} \Rightarrow v_y'^2 = c^2 - v_x'^2 = c^2 - u^2$$

But, again using the transformation, we have

$$v_y = v_y' \Rightarrow v_y = \sqrt{c^2 - u^2}$$

This is what we need to find the ~~speed~~ time one-way:

$$t = \frac{L}{v_y} = \frac{L}{\sqrt{c^2-u^2}}$$

It is easy to show that the time needed for swimming back is the same, so

$$t_{tot} = \frac{2L}{\sqrt{c^2-u^2}} = \frac{2L}{c}\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$

Question: ~~Which time is longer?~~

Which swim takes more time?
Swimming across, or upstream-downstream?