Classical Doppler Effect

(short recapitulation of the material from Ph 212):

Suppose that there is a source emitting a wave (e.g., a sound wave) of frequency f, and an observer – both at fixed positions.

<u>Comment</u>: in the K. Krane's text, the frequency is denoted by a Greek symbol ν ("nu"). However, in handwritten notes ν is too similar to v, u, used as velocity symbols – to avoid confusion, here we will use f instead

~~Let's plot~~ The wave propagates with speed c, and the wavelength (the distance between two adjacent "crests") is λ

The relation between f, c and λ is:

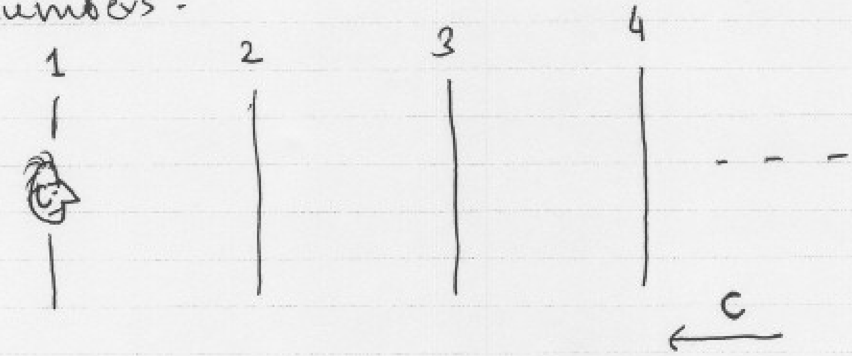
$$\lambda = \frac{c}{f}$$

We can also talk about the wave time period, T, which is related to the frequency as $T = \frac{1}{f}$

So, we can also write $\boxed{\lambda = cT}$

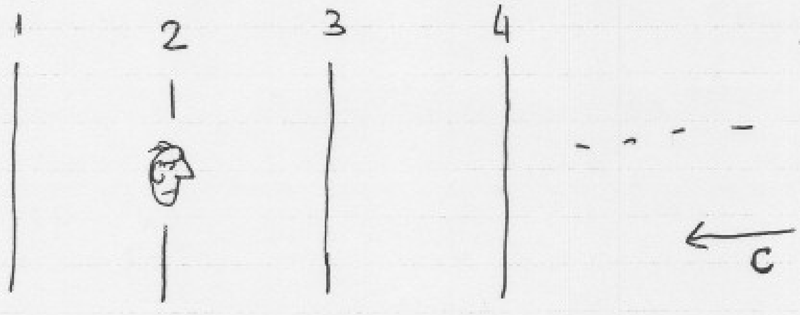
λ is the distance the wave travels in time T

Now, let's draw the situation in a blown-up scale, and let's number the "crests" with consecutive numbers:



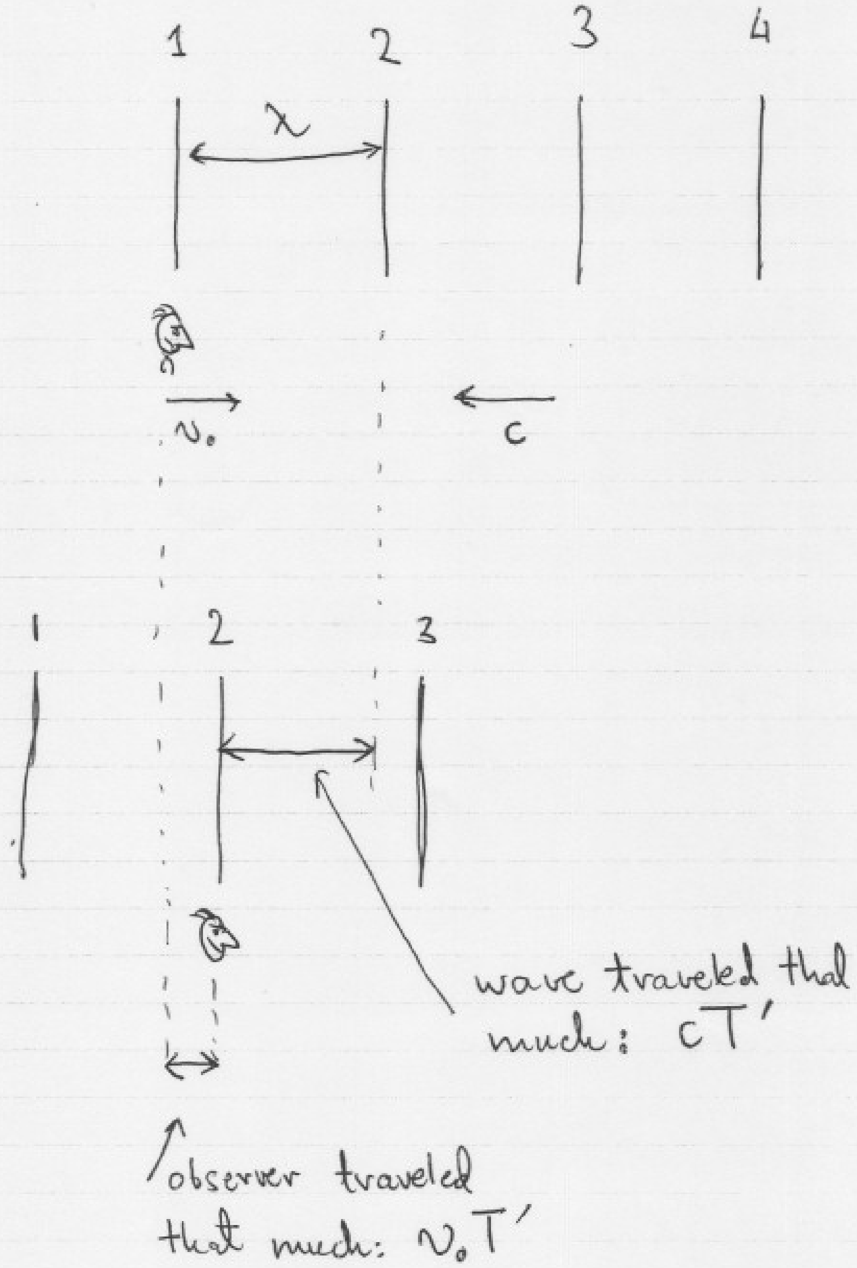
Take the time instant the crest #1 reaches the observer as $t = 0$

After time T elapses:



crest #2 reaches the observer at $t = T$

Now, suppose that the observer is in motion:



Take this moment as $t=0$

crest # 2 reaches the moving observer at $t=T'$

From the picture it is clear that

$$v_0 T' + cT' = \lambda$$

But $\lambda = cT$

So. $$T' = T\frac{c}{c+v_0}$$

T' is the wave period as seen by the moving observer in terms of frequencies: $f = \frac{1}{T}$ - the frequency heard by stationary observer

and $f' = \frac{1}{T'}$ - the frequency heard by the moving observer

we get:

$$f' = f \cdot \frac{c \pm v_o}{c}$$

+ sign - obs. moving toward S

− sign: obs. moving away from S

Also, the source may start moving with speed v_S. It is easy to show, in a similar way, that for stationary observer and moving source

$$f' = f \frac{c}{c \mp v_S}$$

"−" - source toward the observer

"+" - source away from the obser.

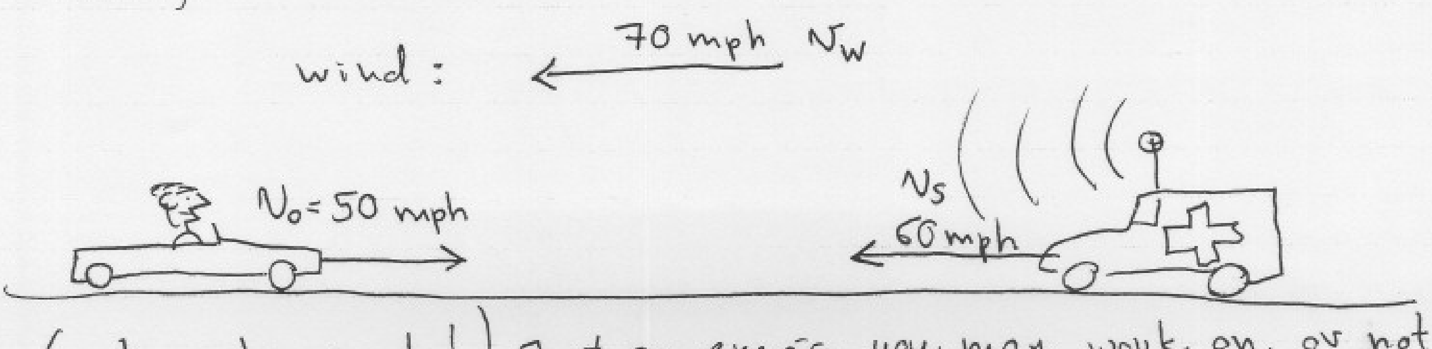
For both the observer and source moving:

$$f' = f \frac{c \pm v_o}{c \mp v_S}$$

In all cases we assume that the medium in which the wave propagates is at rest. So both speeds, v_o and v_s, are measured relative to the medium

What shall we do if the speed v_o and v_s are measured relative to the ground, and the medium is in motion?

For instance, during a windstorm, a person falls and brakes his ankle. His neighbor see that and she calls 911. An ambulance is dispatched and rushes to the accident site, with the siren on all the time. On the highway, you drive in the opposite direction. What frequency do you hear, if the siren in normal conditions (nothing moves) emitts a sound with $f = 500$ Hz?



(not a homework!) Just an exercise you may work on, or not.

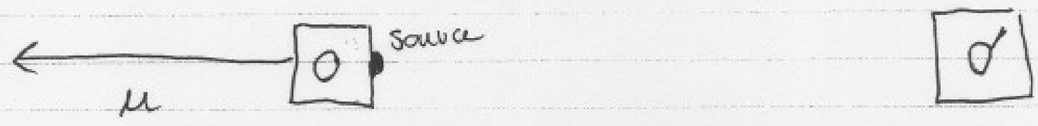
Relativistic Doppler Effect: applies to light waves

It's different than the classical one because:

(a) there is no medium

(b) time dilation effects play an important role

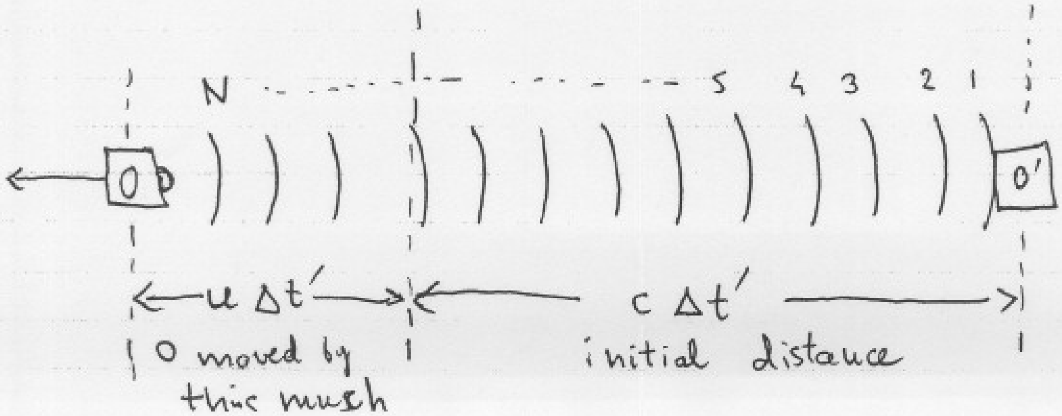
Consider the following situation: there are two frames, O and O′, and an observer in each of them. In frame O, there is light source.



O moves away from O′ with speed u.

At some instant the source in O starts emitting light:

After a time $\Delta t'$ passed (in O′), the first crest reaches O′



Total of N wave "crests" have been emitted.

For the observer in O': the total "light signal" consisting of N "crests" is stretched over a distance $\boxed{u\,\Delta t' + c\,\Delta t' = \Delta t'(u+c)}$

So, the wavelength λ' seen by the observer in O' is: $\lambda' = \dfrac{\Delta t'(u+c)}{N}$

Or, converting wavelength to frequency $\left(f = \frac{c}{\lambda}\right)$

The observer in O' sees the incoming wave of a frequency f':

$$f' = \frac{c}{\lambda'} = \frac{cN}{\Delta t'(u+c)} = \frac{N}{\Delta t'} \cdot \frac{1}{1+\frac{u}{c}}$$

Now, for the observer in O the emission of the total signal, N crests, took a time Δt_o.

So, the oscillation period of the wave this observer "sees" is $T = \dfrac{\Delta t_o}{N}$

Or, the frequency of the signal (in O) is

$$f = \frac{1}{T} = \frac{N}{\Delta t_o}$$

So that $N = f \cdot \Delta t_o$

Combining with the equation for f', we get:

$$f' = f \frac{\Delta t_0}{\Delta t'} \frac{1}{1 + \frac{u}{c}}$$

Now, we take into account the time dilation.

When a time period $\Delta t'$ passes in O', in O, which is moving away, a shorter time period passes. So, it must be:

$$\Delta t_0 = \Delta t' \sqrt{1 - \frac{u^2}{c^2}}$$

Combining everything together:

$$f' = f \frac{\Delta t' \sqrt{1 - \frac{u^2}{c^2}}}{\Delta t' \left(1 + \frac{u}{c}\right)} = f \sqrt{\frac{\left(1 + \frac{u}{c}\right)\left(1 - \frac{u}{c}\right)}{\left(1 + \frac{u}{c}\right)^2}}$$

$$\boxed{f' = f \sqrt{\frac{1 - \frac{u}{c}}{1 + \frac{u}{c}}}}$$

The formula for relativistic Doppler effect.
(Doppler frequency shift)