Mdt. sol 1

1.

I: $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-\frac{u^2}{c^2}}}$; $\Delta t_0 = 15$ years, $u = 0.8c$

$\sqrt{1-(0.8c)^2/c^2} = \sqrt{1-0.64} = \sqrt{0.36} = 0.6$

Hence, $\Delta t = 15\text{ years}/0.6 = 25$ years ;

$1993 + 25 = 2018 \Rightarrow$ D

II: $\lambda = \frac{h}{p}$ and $p = \sqrt{2mK}$

Since all have the same K, the one with the largest mass has the largest p, and hence the smallest λ. Sodium atom has the largest mass $\Rightarrow$ A

III. $\lambda_{max} \cdot T = \text{const} \Rightarrow$ if T increases 4-fold then λ_{max} must decrese 4-fold. $5.6 \div 4 = 1.4$ so C is the right answer.

IV. $E = \hbar\omega_0(n+\frac{1}{2})$ The intervals between consecutive energy levels are all the same, so, after 1.5 eV for n=1, the next, for n=2 is 2.5 eV. So, A

Problem 2:

(a) and (b) Initially there are two particles moving along the x axis. Their momenta are then:

$$\vec{p}_1 = (p_1, 0, 0) \text{ and } \vec{p}_2 = (p_2, 0, 0).$$

After the collision, there are two photons propagating along the y axis. So, their moments are:

$$\vec{p}_{\gamma_1} = (0, p_{\gamma 1}, 0) \text{ and } \vec{p}_{2\gamma} = (0, p_{2\gamma}, 0).$$

The momentum is conserved, so that the total momentum before the collision, and the total momentum after the collision must be equal:

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_{1\gamma} + \vec{p}_{2\gamma}$$

Which can be written as:

$$(p_1 + p_2, 0, 0) = (0, p_{1\gamma} + p_{2\gamma}, 0)$$

if two vectors are equal: $\vec{A} = (A_x, A_y, A_z) =$

$= \vec{B} = (B_x, B_y, B_z)$, it means that:

$$A_x = B_x \ ; \ A_y = B_y \ ; \text{ and } A_z = B_z$$

Accordingly, from the equity of the momentum vector, we obtain:

$$p_1 + p_2 = 0 \implies p_2 = -p_1 \qquad (*)$$

and

$$p_{1\gamma} + p_{2\gamma} = 0 \implies p_{1\gamma} = -p_{2\gamma} \qquad (**)$$

The photon momentum magnitude $|\vec{p}_\gamma|$ is:

$$|\vec{p}_\gamma| = \hbar|\vec{k}| = \hbar\frac{2\pi}{\lambda} = \frac{h}{2\pi}\cdot\frac{2\pi}{\lambda} = \frac{h}{\lambda} =$$

$$= \frac{h}{c/\nu} = \frac{h\nu}{c} = \frac{E_\gamma}{c}$$

From Eq. (**) it follows that the magnitude of the momenta of the two photons are equal, so their energies also must be equal: $E_{\gamma 1} = E_{\gamma 2}$.

Before the collision, the momenta of the two particles, as follows from (*), are of opposite signs, but of the same magnitude:
$|p_2| = |p_1|$.

$$p_1 = \frac{mV_1}{\sqrt{1 - V_1^2/c^2}} = \frac{0.8\,mc}{\sqrt{1-0.64}} = \frac{0.8\,mc}{\sqrt{0.36}} = \frac{0.8\,mc}{0.6}$$

$$= \frac{4}{3}mc \cong 1.333\,mc$$

The momentum of particle 2 is thus

$$|p_2| = \frac{4}{3}mc, \text{ or } p_2 = -\frac{4}{3}mc$$

(c). The total energy is conserved. The total energy before the collision can be obtained from the formula: $E_{TOT} = \sqrt{(pc)^2 + (mc)^2}$ for 1 particle.

relativistic equation for total particle energy:

$$E = \sqrt{(pc)^2 + (mc^2)^2}$$

In the present case, there are two particles with masses m and 2m, and both have the same momentum $\frac{4}{3}mc$.

So:

$$E_{TOT} = \sqrt{\left(\frac{4}{3}mc^2\right)^2 + (mc^2)^2} + \sqrt{\left(\frac{4}{3}mc^2\right)^2 + (2mc^2)^2}$$

$$= \sqrt{\frac{16}{9}m^2c^4 + m^2c^4} + \sqrt{\frac{16}{9}m^2c^4 + 4m^2c^4}$$

$$= mc^2\sqrt{\frac{16}{9} + \frac{9}{9}} + mc^2\sqrt{\frac{16}{9} + \frac{36}{9}} =$$

$$= mc^2\sqrt{\frac{25}{9}} + mc^2\sqrt{\frac{52}{9}} = \frac{5}{3}mc^2 + \frac{2}{3}mc^2\sqrt{13}$$

$$= \frac{mc^2}{3}\left(5 + 2\sqrt{13}\right) = 4.07\, mc^2$$

Because the energy is conserved, ~~this ener~~ and each photon has the same energy, the photon energy value is:

$$E_{\gamma 1} = E_{\gamma 2} = \frac{4.07}{2}mc^2 = 2.035\, mc^2$$

If $m = 1.022\, MeV/c^2$, we get: $E_{\gamma 1} = E_{\gamma 2} = 2.08\, MeV$.

Mdt. sol 5

Problem 3:

(a) Photon: $E = h\nu = h\cdot\frac{c}{\lambda}$

for $\lambda = 455\ nm = 455\cdot10^{-9}\ m$: $E = \frac{4.136\times10^{-15}\ eV\cdot s\cdot 3\times10^{8}\ m/s}{455\times10^{-9}\ m}$

$= 2.727\ eV$

$K_{max} = h\nu - \phi \Rightarrow \phi = h\nu - K_{max} = 2.727\,eV - 1.227$

$= 1.50\ eV.$ $\boxed{\phi = 1.50\,eV}$

(b) For $\lambda = 575\ nm$: $E = 2.158\ eV$

so $K_{max} = E - \phi = 2.158\,eV - 1.50\,eV = 0.53\,eV$

(c) $h\nu = 1.50\ eV$

$\lambda = \frac{4.136\times10^{-15}\,eV\cdot s\ \cdot 3\cdot10^{8}\,m/s}{1.50\,eV} = 827\ nm$