MTH 656

Uncertainty Quantification

Homework 3

Stochastic Collocation

Download `SCcodes.zip` from the course website. Given a deterministic (black box) simulator for a river flow, apply Stochastic Collocation to the random system resulting from a Karhunen-Loeve expansion (KLE) of an ensemble of inflow data in order to determine the expected *maximum* outflow required to keep water height (e.g., Water Stage Elevetion, or WSE) at $10m$. In particular,

- Use enough terms of the KLE in order that 99% of the variance is retained.
- Assume uniform random variables in the KLE (note that this violates the independence assumption) so as to avoid negative inflows and to make the formation of the sparse grid easier. Since the random variables must be zero mean and unit variance, they are both distributed as $\mathcal{U}[-\sqrt{3}, \sqrt{3}]$.
- Use Clenshaw-Curtis nodes, both full tensor and Smolyak sparse grids.
- Use `parfor` if you have it.

See `SCdriver.m`. Try to understand what is happening. See also Saint-VenantEquations.pdf for a description of the simulator, if you are interested.

1. Comment on the runtime for each grid. How does this change with increasing level (at least 3)? How does this change when using `parfor` (if you have it)?
2. Comment on the accuracy of each grid for various levels (as compared with the highest level you are able to run).
3. Change the code to compute the expected time that the maximum flow occurs. (Note: it is not possible to apply Stochastic Galerkin to this question.) Comment on the accuracy of each grid for various levels.
4. Use either the full or sparse grid solutions to compute the standard deviation of the outflow $Qout(t)$, and plot the expected outflow along with the expectation plus and minus two standard deviations.