Electromagnetic characterization of damage in Space Shuttle foam

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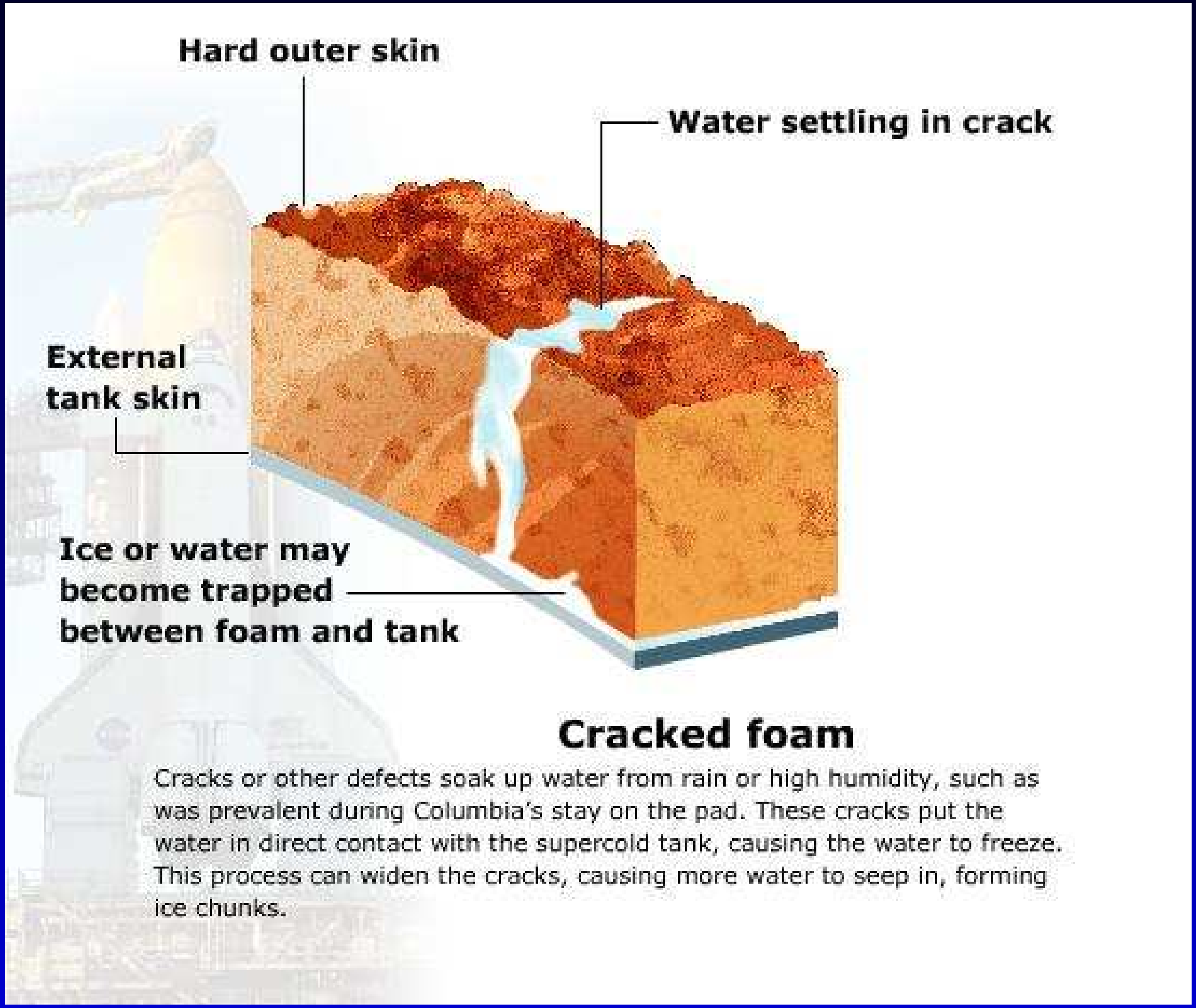
Motivating Application

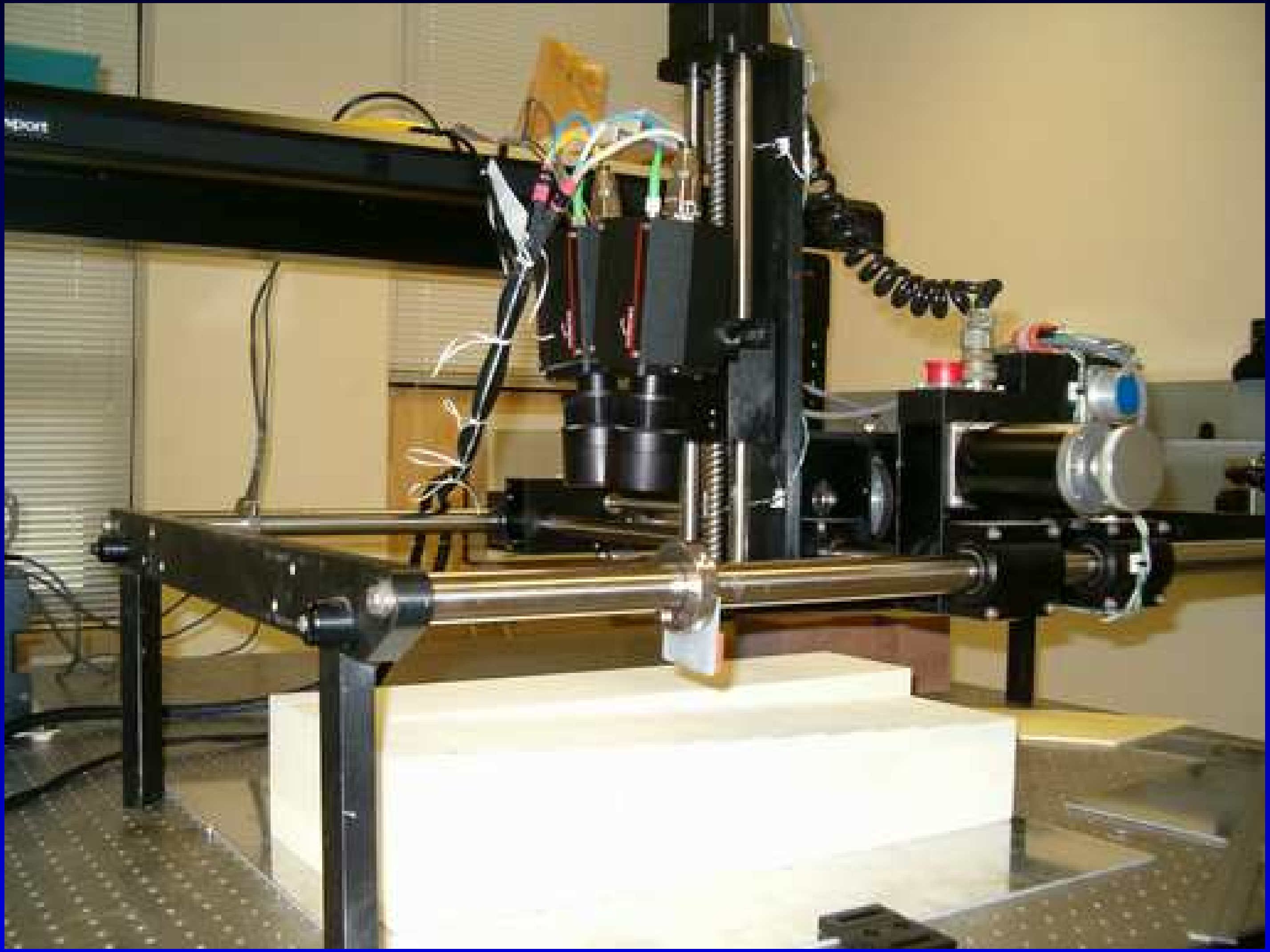


The particular motivation for this research is the detection of defects in the insulating foam on the space shuttle fuel tanks in order to help eliminate the separation of foam during shuttle ascent.

Outline

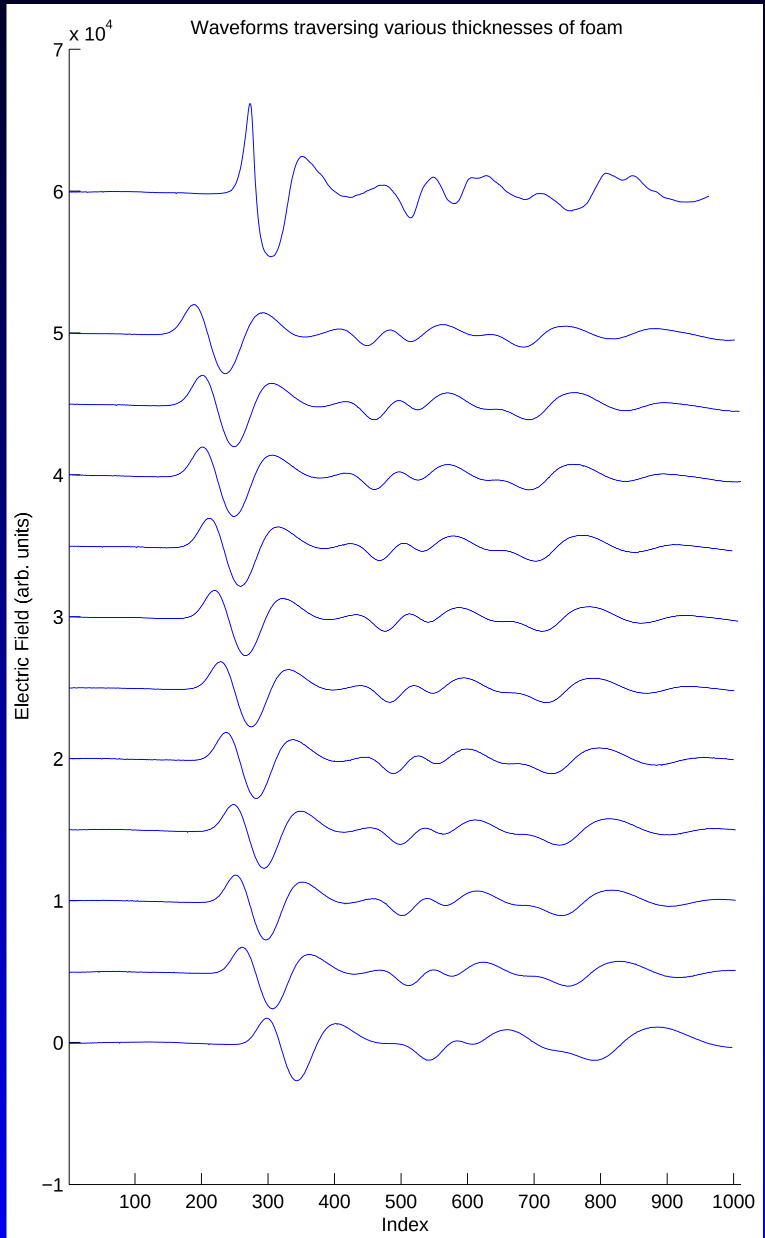
- 1D Gap Detection Model
- Numerical Methods for Forward Problem
- Inverse Problem
  - Ordinary Least Squares
  - Alternate Approach
  - Computational Results

Picometrix T-Ray Setup



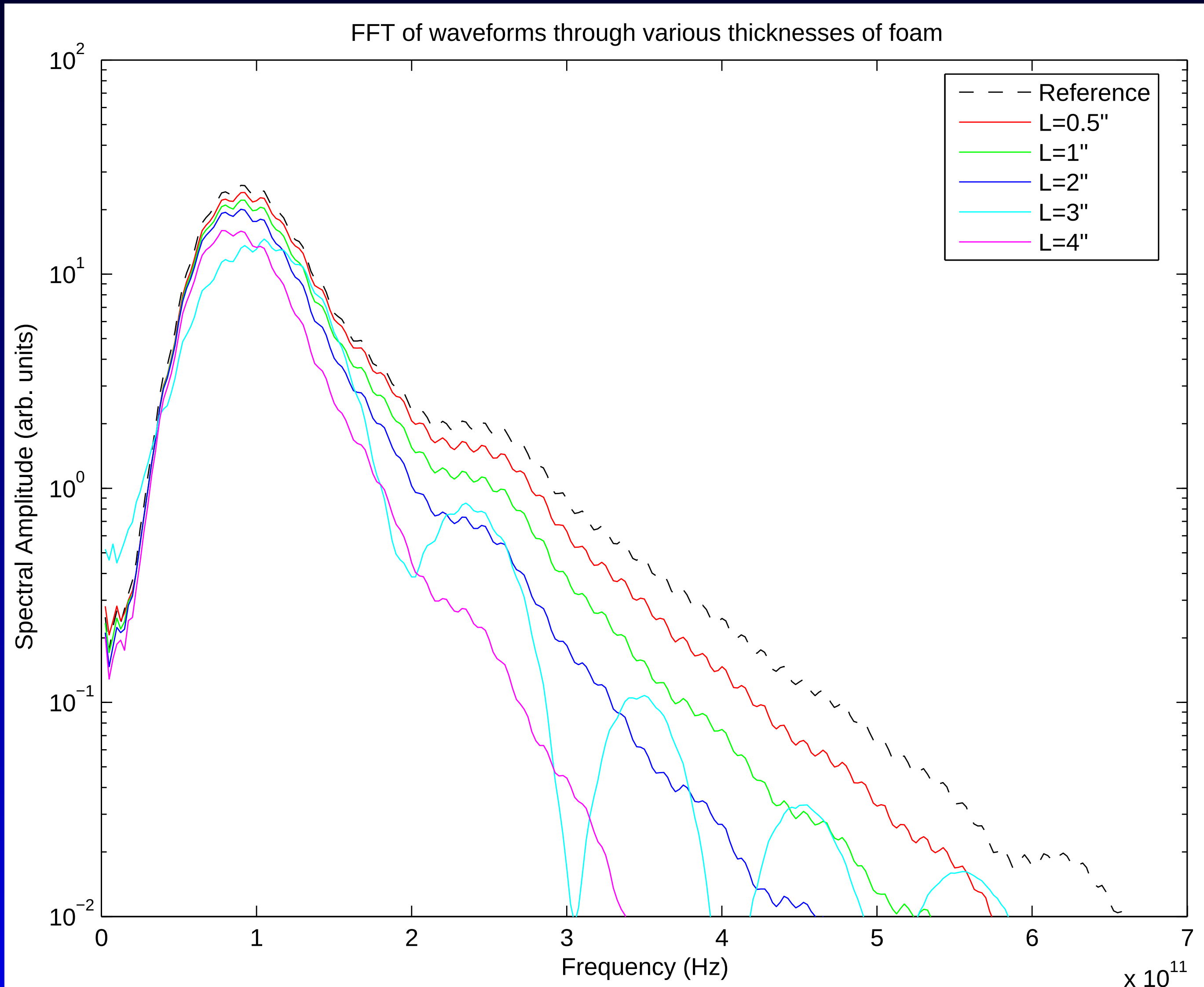
Step-block can be used to interrogate varying thicknesses of foam. It can be turned upside down to sample varying gap sizes.

THz through foam

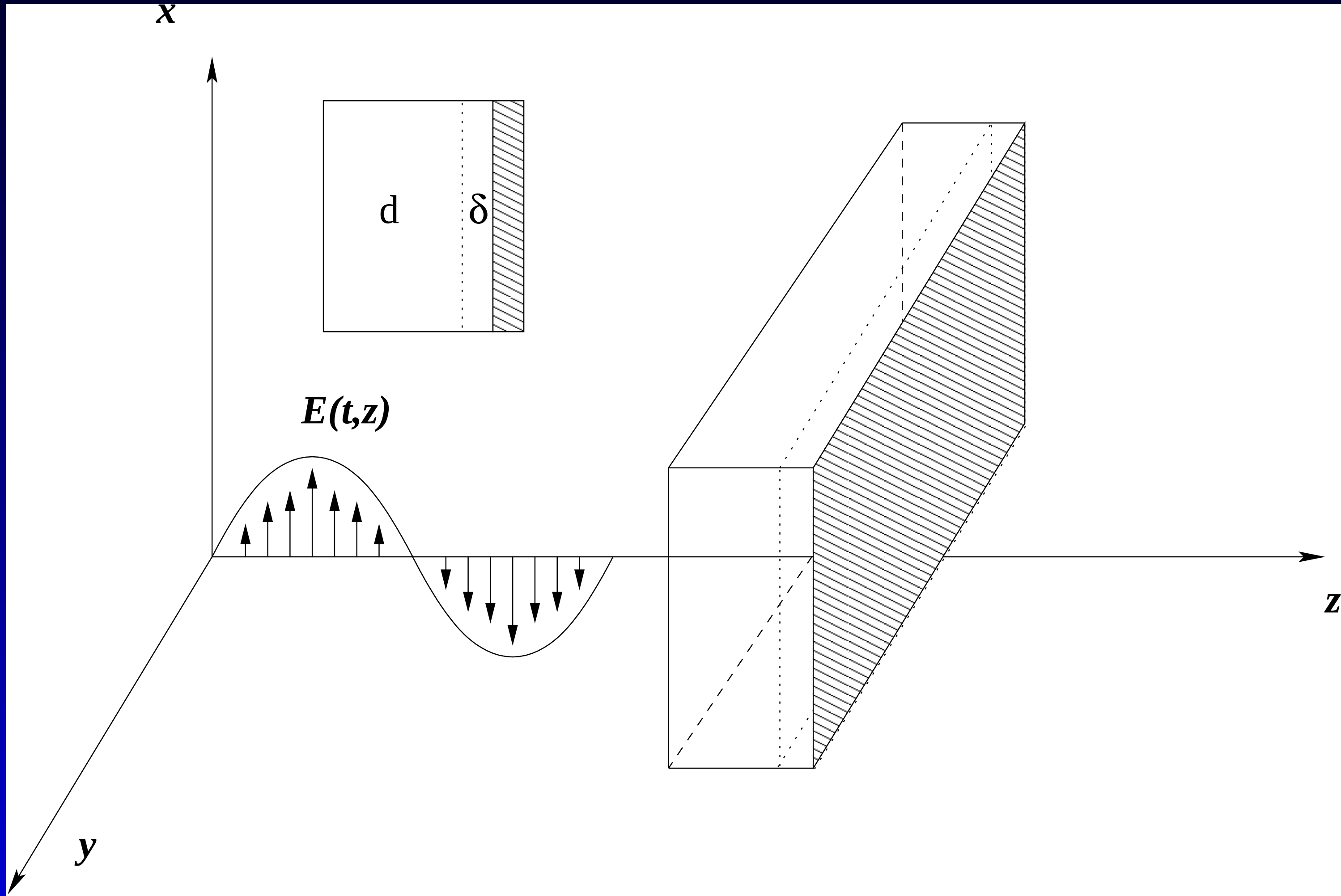


THz signal recorded after passing through foam of varying thickness, in a pitch-echo experiment.

FFT of THz Signal



Gap Detection Problem



Model

$$\mu_0\epsilon_0\epsilon_r\ddot{E} + \mu_0 I_\Omega \ddot{P} + \mu_0\sigma I_\Omega \dot{E} - E'' = -\mu_0 \dot{J}_s \qquad \text{in } \Omega \cup \Omega_0$$

$$\tau\dot{P} + P = \epsilon_0(\epsilon_s - \epsilon_\infty)E \qquad \text{in } \Omega$$

$$[\dot{E} - cE']_{z=0} = 0$$

$$[E]_{z=1} = 0$$

$$E(0,z) = \dot{E}(0,z) = 0$$

$$P(0,z) = 0$$

where

$$J_s(t,z) = \delta(z) sin(\omega t) I_{[0,t_f]}(t)$$

and

$$\epsilon_r = (1 + (\epsilon_\infty - 1)I_\Omega).$$

Numerical Discretization

- Second order FEM in space
  - piecewise linear splines
- Second order FD in time
  - Crank-Nicholson (P)
  - Central differences (E)
  - $e_n \longrightarrow p_n \longrightarrow e_{n+1} \longrightarrow p_{n+1} \longrightarrow \cdots$
- E equation implicit, LU factorization used

Finite Element Method in Space

The resulting system of differential equations in semi-discrete form can be written

$$M_1\ddot{e} + M_2\dot{e} + M_3 e + \lambda^2\bar{p} = \eta_0 J \quad (1)$$

$$\dot{\bar{p}} + \lambda\bar{p} = \epsilon_d \lambda M^\Omega e. \quad (2)$$

where $\eta_0 = \sqrt{\mu_0/\epsilon_0}$, $\epsilon_d = \epsilon_s - \epsilon_\infty$, $\lambda = 1/c\tau$, e and p are vectors representing the approximate values of E and P respectively at the nodes z_i.

$\bar{p} = M^\Omega p$ where M^Ω is the mass matrix integrated only over Ω.

Finite Difference in Time (p)

Our finite difference approximation for (2) is

$$\bar{p}_{n+1} = \bar{p}_n + \frac{\lambda \Delta t}{1 + \lambda \Delta t \theta} (\epsilon_d M^\Omega e_{n+\theta} - \bar{p}_n) \qquad (3)$$

where $[e_n]_j = E(t_n, z_j)$, $[\bar{p}_n]_j = M^\Omega P(t_n, z_j)$, $z_j = jh$.

The value $e_{n+\theta} = \theta e_n + (1-\theta) e_{n+1}$ is a weighted average of e_n and e_{n+1} for relaxation to help with stability of the method.

Note: we take $\theta = 1/2$.

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Finite Difference in Time (e)

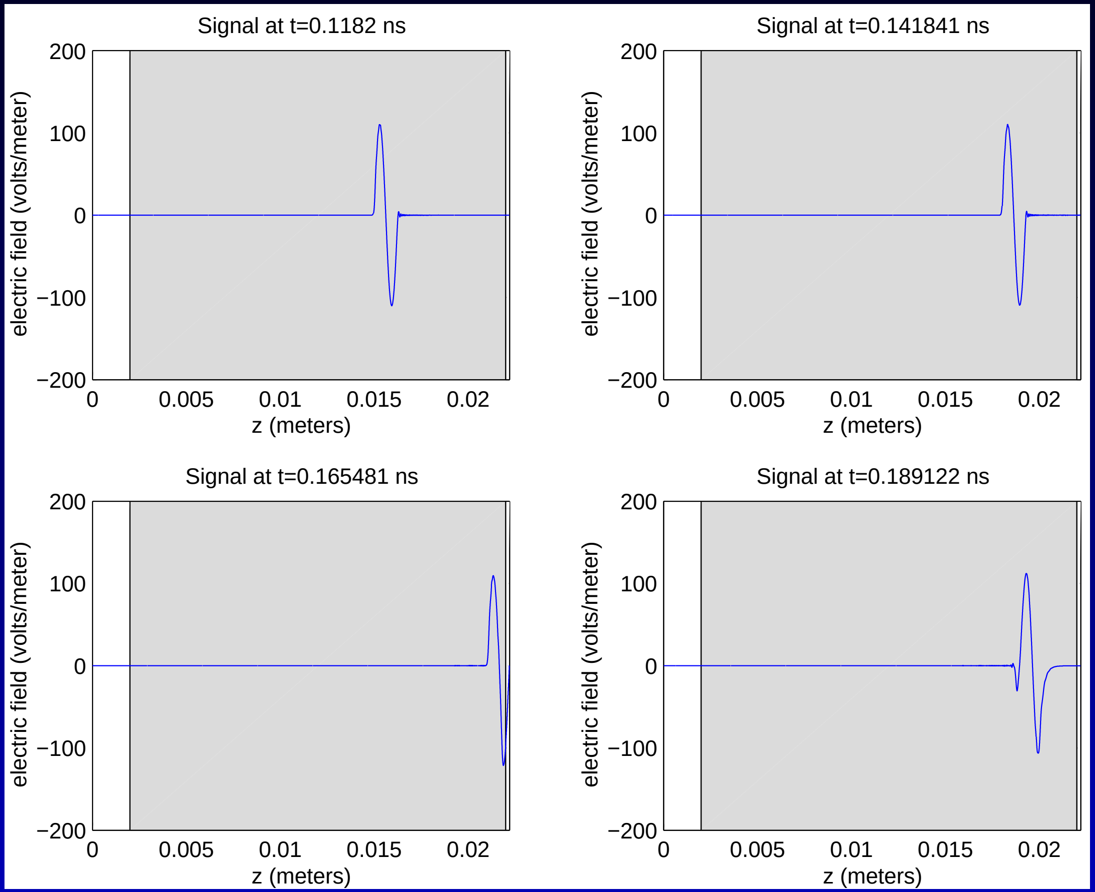
Applying second order central differencing with averaging to (1) gives

$$A_1 e_{n+2} = A_2 e_{n+1} + A_3 e_n + \Delta t^2 \eta_0 J_{n+1} - \lambda^2 \Delta t^2 \bar{p}_{n+1}. \tag{4}$$

As $\bar{p}_{n+1}$ depends explicitly on e_n and e_{n+1}, we could substitute (3) here and have one implicit equation for the update of e.

Note: we use LU factorization as A_1 does not change over time.

Sample Problem



Computed solutions at different times of a windowed electromagnetic pulse at $f{=}100GHz$ incident on a Debye medium with a crack $\delta{=}.0002m$ wide located $d{=}.02m$ into the material.

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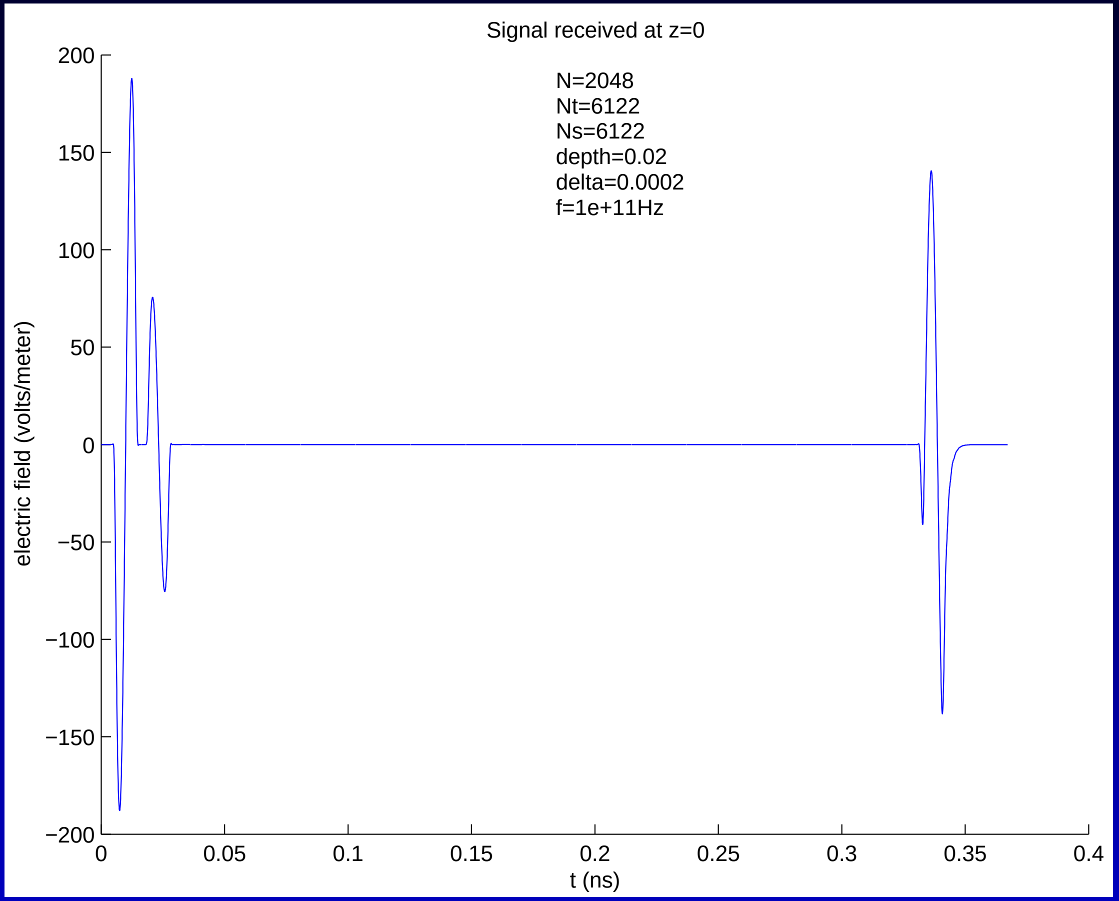
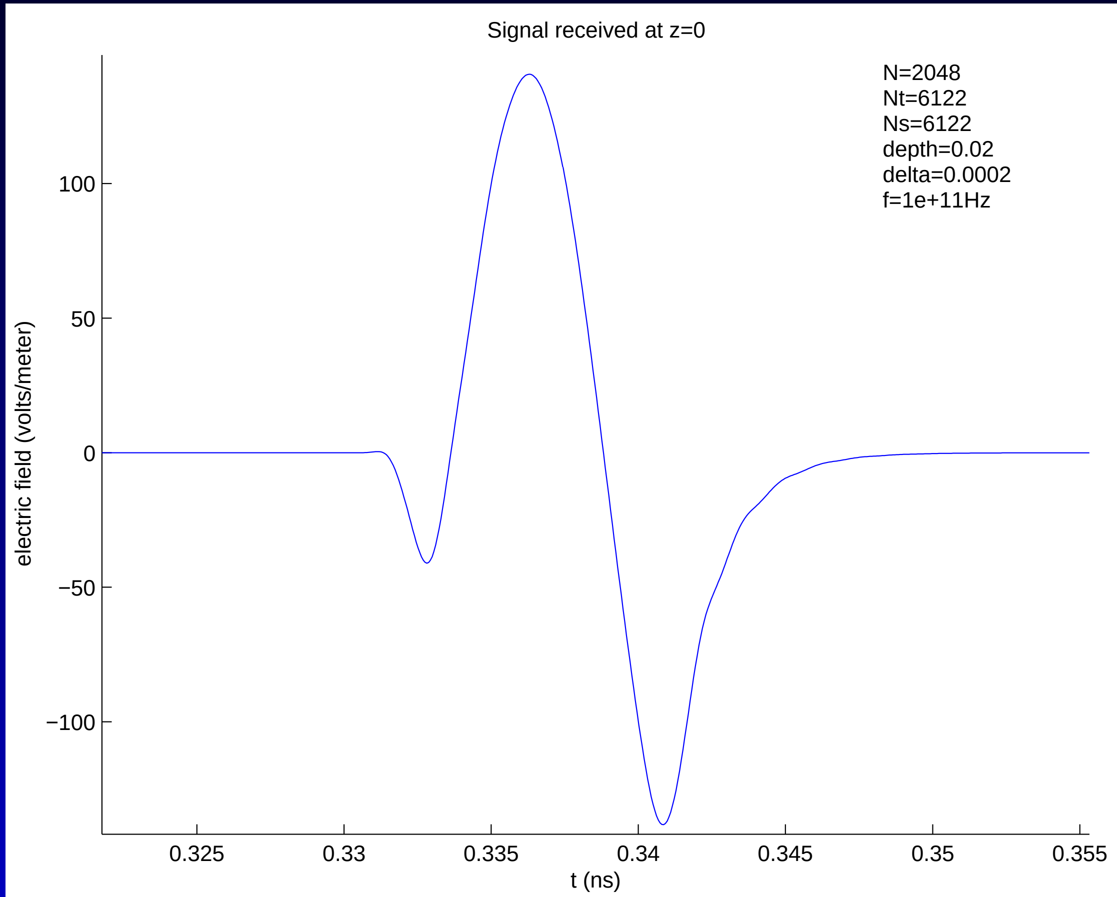
Sample Problem (Cont.)



Reflected signal received at z=0.

Sample Problem (Cont.)



Close up look at reflected signal received at z=0
Shows "important" parts of the signal.

Gap Detection Inverse Problem

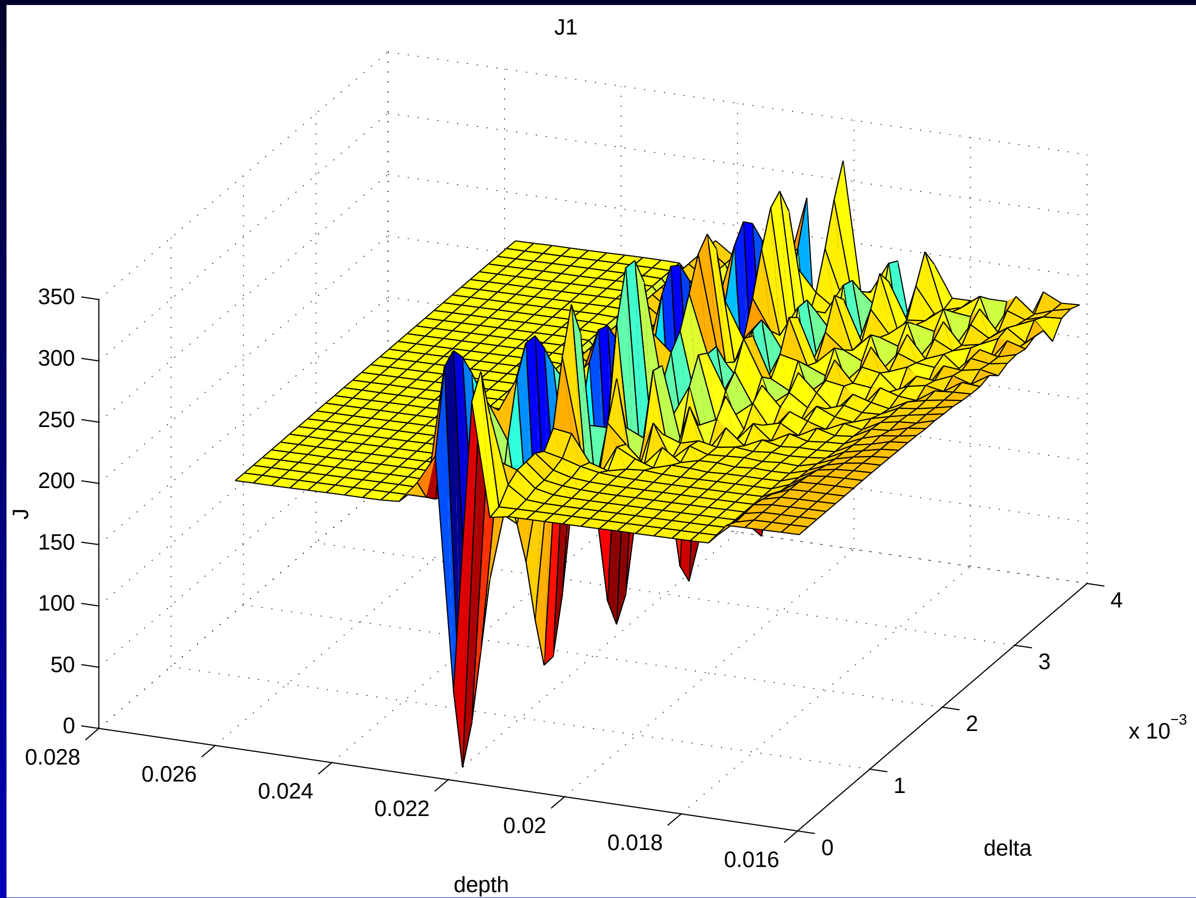
- Assume we have data, $\hat{E}_i$, recorded at z=0
- Given d and δ we can simulate the electric field
  (Need a fast numerical method)
- Estimate d and δ by solving an inverse problem:

Find q=$(d, \delta) \in Q_{ad}$ such that the following objective function is minimized:

$$\mathcal{J}_1(q) = \frac{1}{2S} \sum_{i=1}^{S} |E(t_i, 0; q) - \hat{E}_i|^2$$

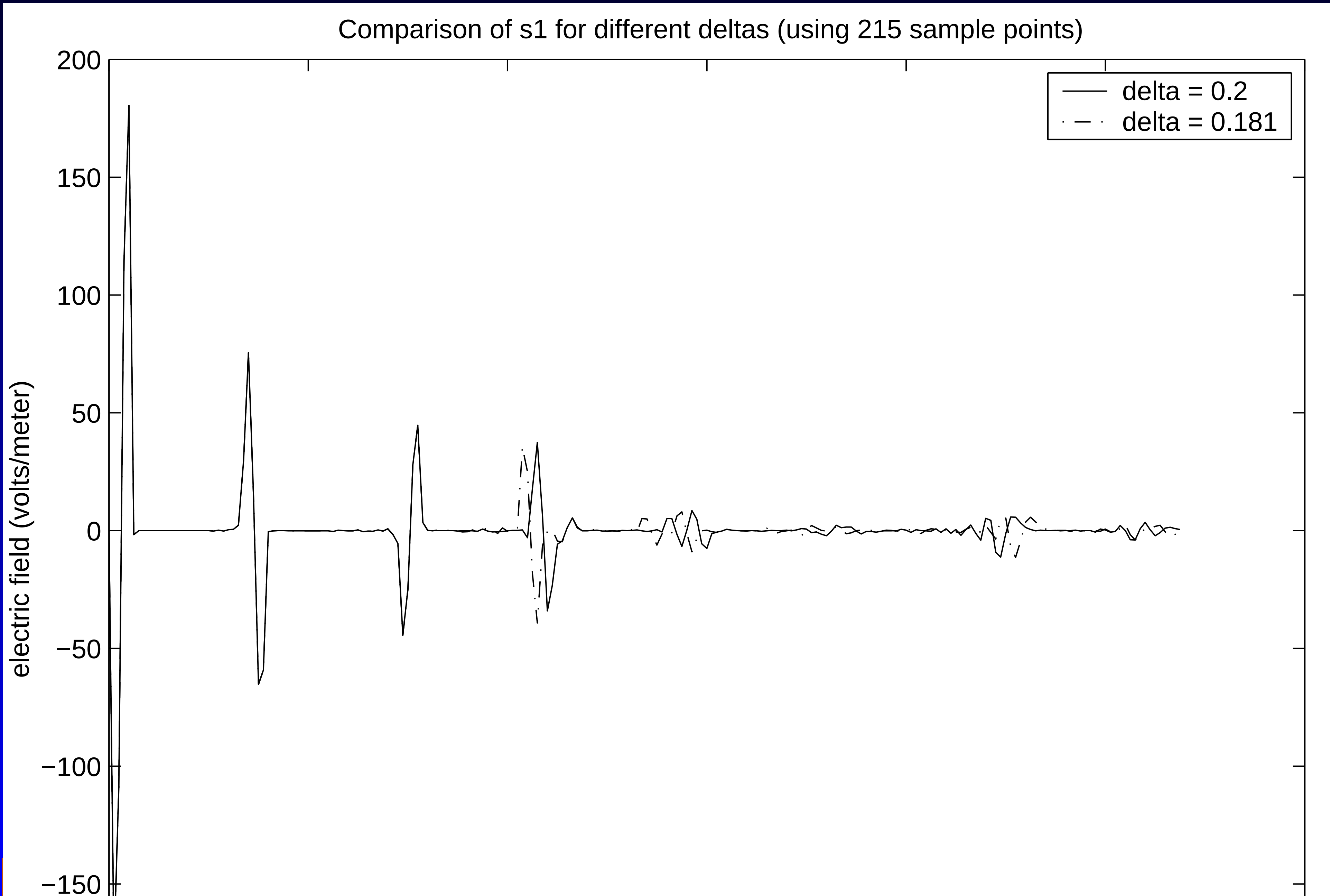
i.e., find the value of q that results in $E(q)$ which is a “best match” to the data $\hat{E}$ (in a least squares sense).

$\mathcal{J}_1(q)$ Surface Plot



Surface plot of the Ordinary Least Squares objective function demonstrating peaks in $\mathcal{J}_1$, and exhibiting many local minima.

Out of phase



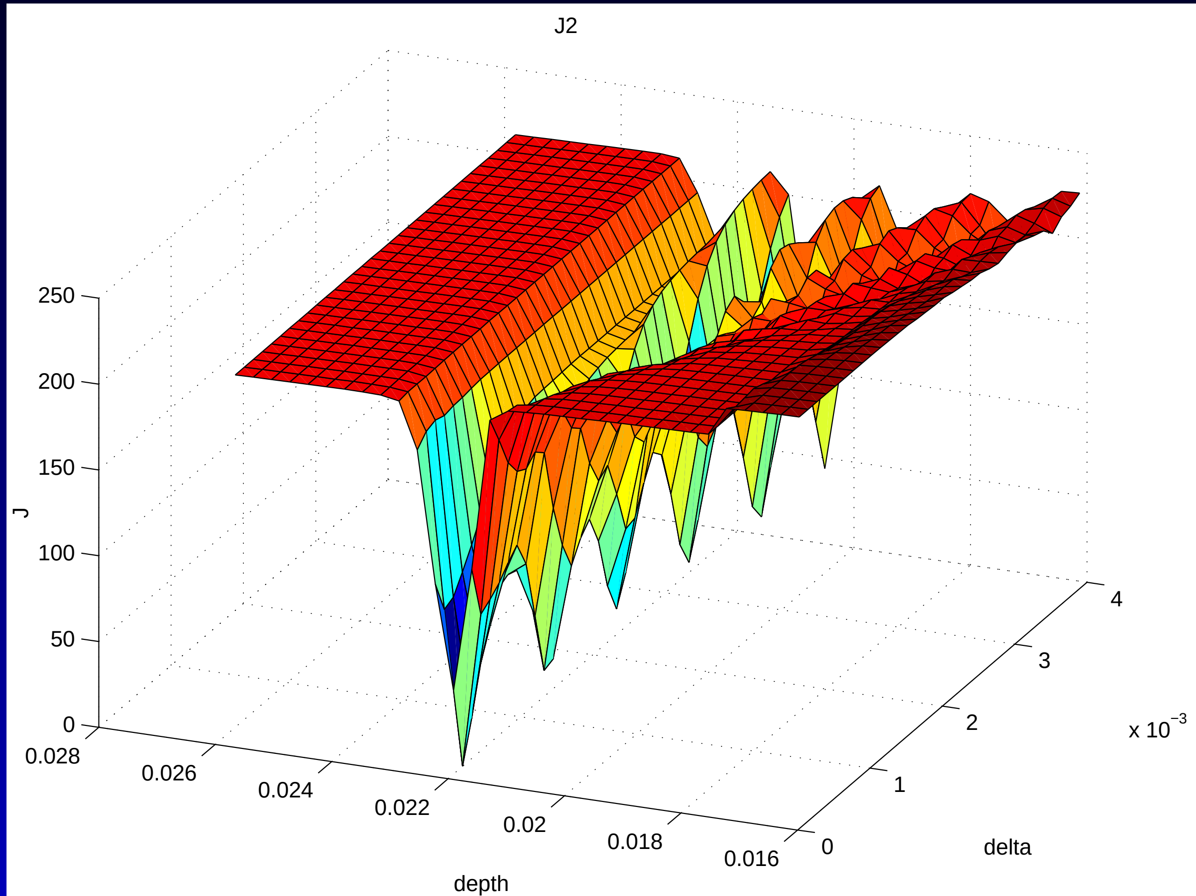
Comparison of s1 for different deltas (using 215 sample points)
200
150
100
50
0
−50
−100
−150
electric field (volts/meter)
delta = 0.2
delta = 0.181

Improved Objective Function

Consider the following formulation of the Inverse Problem:
Find q=$(d, \delta) \in Q_{ad}$ such that the following objective function is minimized:

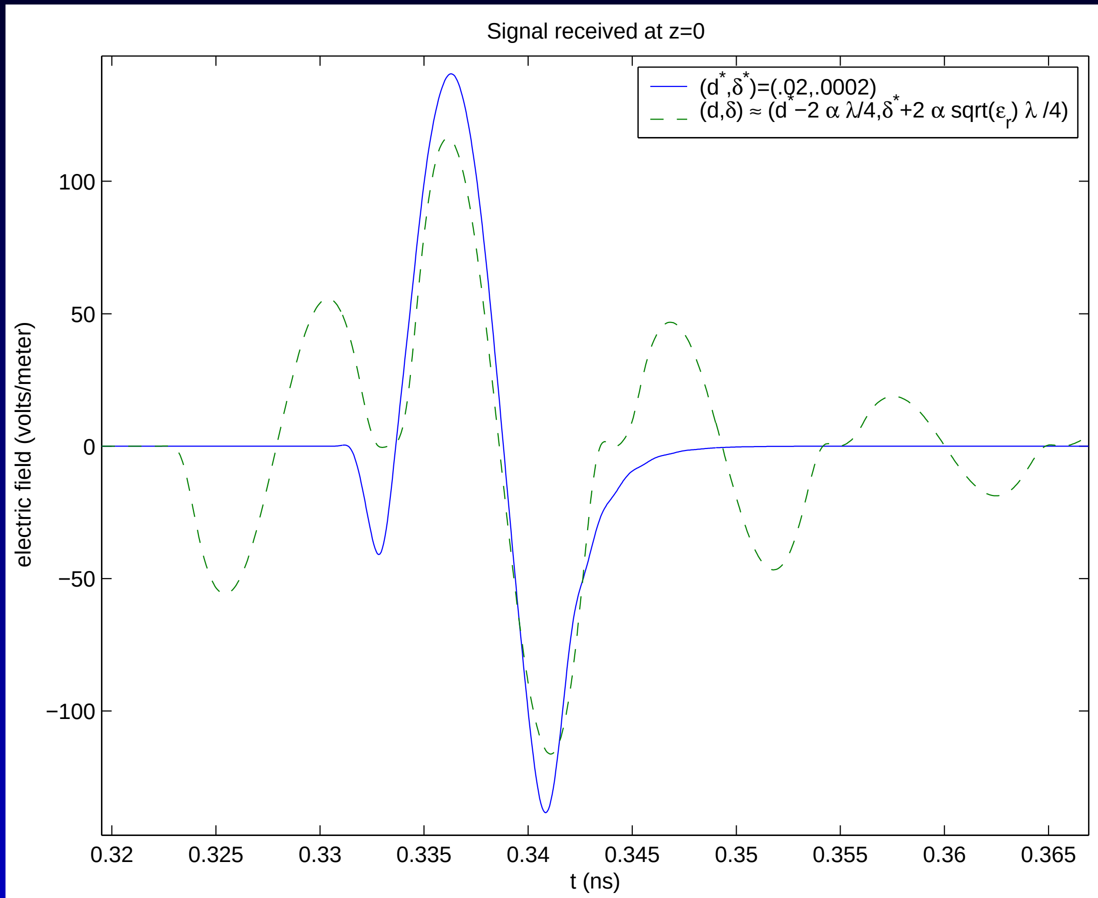
$$\mathcal{J}_2(q) = \frac{1}{2S} \sum_{i=1}^{S} \Big| |E(t_i, 0; q)| - |\hat{E}_i| \Big|^2 .$$

$\mathcal{J}_2(q)$ Surface Plot



Close up surface plot of our Modified Least Squares objective function demonstrating lack of peaks in $\mathcal{J}_2$, but still exhibiting many local minima.

Erroneous Local Minima



Data from (d^*, δ^*) and a simulation from the "check point" $(d^* - 2\alpha\frac{\lambda}{4}, \delta^* + 2\alpha\sqrt{\epsilon_r}\frac{\lambda}{4})$. The simulated signal's largest peak matches with that of the data.

Check Point Method

The diagonal “trench” occurs approximately along the line

$$d = -\frac{1}{\sqrt{\epsilon_0}}(\delta - \delta^*) + d^*.$$

Also, the minima occur every $\frac{\lambda}{4}m$ along this line. Therefore, if our optimization routine detects a local minima, we test $\frac{\lambda}{4}$ on either side of the local minima to see if there is a smaller minima nearby. If so, we restart our optimizer at the new smallest point.

Levenberg-Marquardt Method

We re-write the objective function as

$$J(q) = \frac{1}{2S} R^T R$$

where R_i=$(|E(t_i, 0; q)| - |\hat{E}_i|)$ is the residual. To update our approximation to q we make the Gauss-Newton update step q_+=$q_c + s_c$ where

$$s_c = -\left(R'(q_c)^T R'(q_c) + \nu_c I\right)^{-1} R'(q_c)^T R(q_c).$$

is the step, q_c is the current approximation, and q_+ is the resulting approximation. The value ν_c is called the Levenberg-Marquardt parameter.

Confidence Intervals for d $(\nu_r = 0)$

δ	$d^* = .02\ (N = 2048)$
.0002	$(2.00005 \pm 9.30284 \times 10^{-7}) \times 10^{-2}$
.0004	$(2.00001 \pm 6.50411 \times 10^{-7}) \times 10^{-2}$
.0008	$(2.00001 \pm 4.91232 \times 10^{-7}) \times 10^{-2}$

δ	$d^* = .04\ (N = 4096)$
.0002	$(4.00013 \pm 1.62162 \times 10^{-6}) \times 10^{-2}$
.0004	$(4.00001 \pm 1.19064 \times 10^{-6}) \times 10^{-2}$
.0008	$(4.00002 \pm 9.05240 \times 10^{-7}) \times 10^{-2}$

Confidence intervals for the OLS estimate of d when the data is generated with no noise (i.e., ν_r=0.0).

Confidence Intervals for d $(\nu_r = .1)$

δ	$d^* = .02\ (N = 2048)$
.0002	$(2.00000 \pm 4.72903 \times 10^{-5}) \times 10^{-2}$
.0004	$(2.00003 \pm 3.39327 \times 10^{-5}) \times 10^{-2}$
.0008	$(2.00003 \pm 2.79911 \times 10^{-5}) \times 10^{-2}$

δ	$d^* = .04\ (N = 4096)$
.0002	$(4.00014 \pm 5.48283 \times 10^{-5}) \times 10^{-2}$
.0004	$(4.00002 \pm 3.87474 \times 10^{-5}) \times 10^{-2}$
.0008	$(4.00003 \pm 3.19526 \times 10^{-5}) \times 10^{-2}$

Confidence intervals for the OLS estimate of d when the data is generated with noise level ν_r=0.1.

Confidence Intervals for δ ($\nu_r = 0$)

δ	$d^* = .02\ (N = 2048)$
.0002	$(1.99272 \pm 0.000182978) \times 10^{-4}$
.0004	$(4.00035 \pm 0.000201885) \times 10^{-4}$
.0008	$(7.99833 \pm 0.000136586) \times 10^{-4}$

δ	$d^* = .04\ (N = 4096)$
.0002	$(1.98142 \pm 0.000317616) \times 10^{-4}$
.0004	$(4.00737 \pm 0.000369841) \times 10^{-4}$
.0008	$(8.00332 \pm 0.000251291) \times 10^{-4}$

Confidence intervals for the OLS estimate of δ when the data is generated with no noise (i.e., ν_r=0.0).

Confidence Intervals for δ $(\nu_r = .1)$

δ	$d^* = .02\ (N = 2048)$
.0002	$(2.00017 \pm 0.00932701) \times 10^{-4}$
.0004	$(4.00070 \pm 0.0105331) \times 10^{-4}$
.0008	$(7.99698 \pm 0.00778563) \times 10^{-4}$

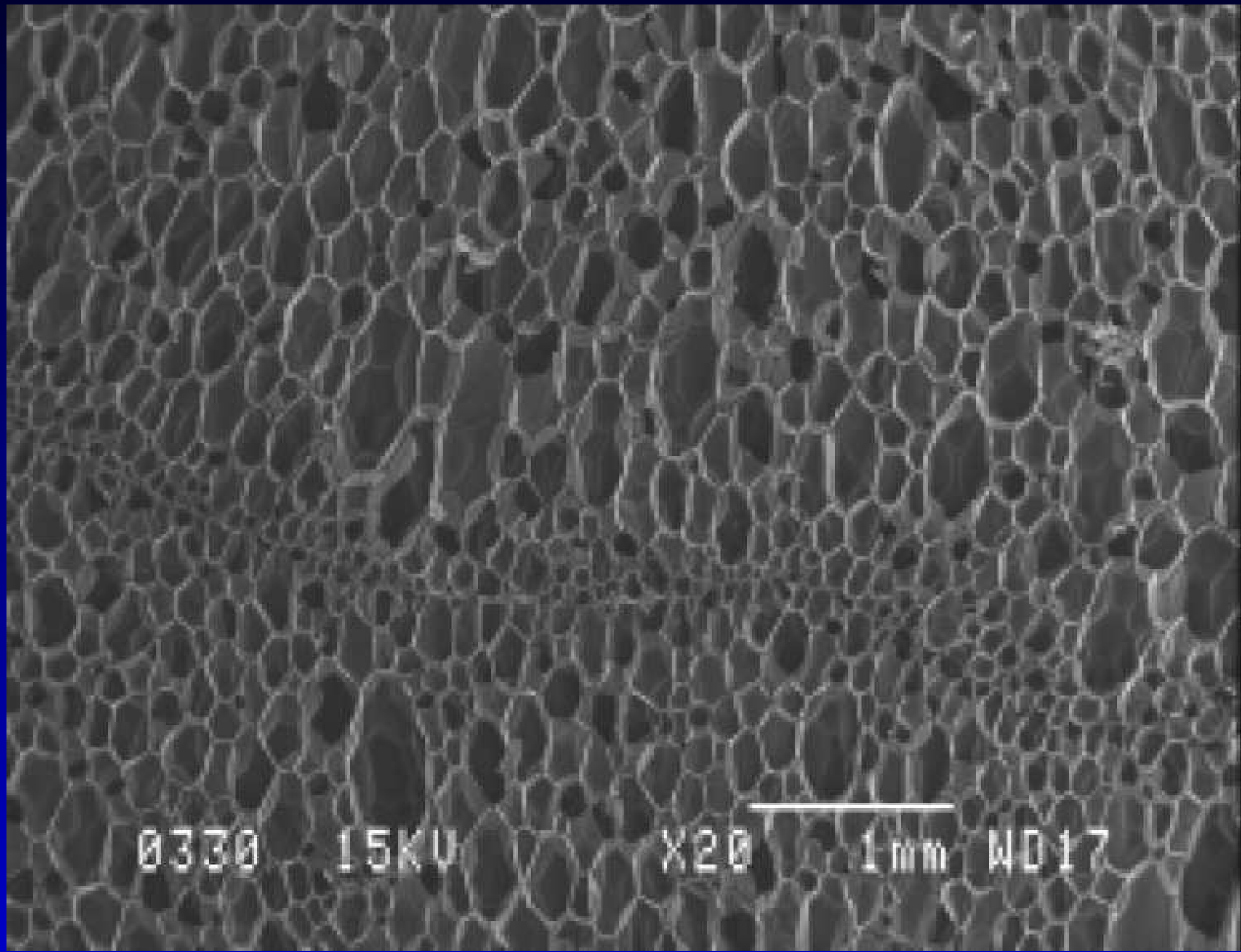
δ	$d^* = .04\ (N = 4096)$
.0002	$(1.97674 \pm 0.0107203) \times 10^{-4}$
.0004	$(4.01229 \pm 0.0120445) \times 10^{-4}$
.0008	$(8.00361 \pm 0.00886925) \times 10^{-4}$

Confidence intervals for the OLS estimate of δ when the data is generated with noise level ν_r=0.1.

Comments on 1D Gap Problem

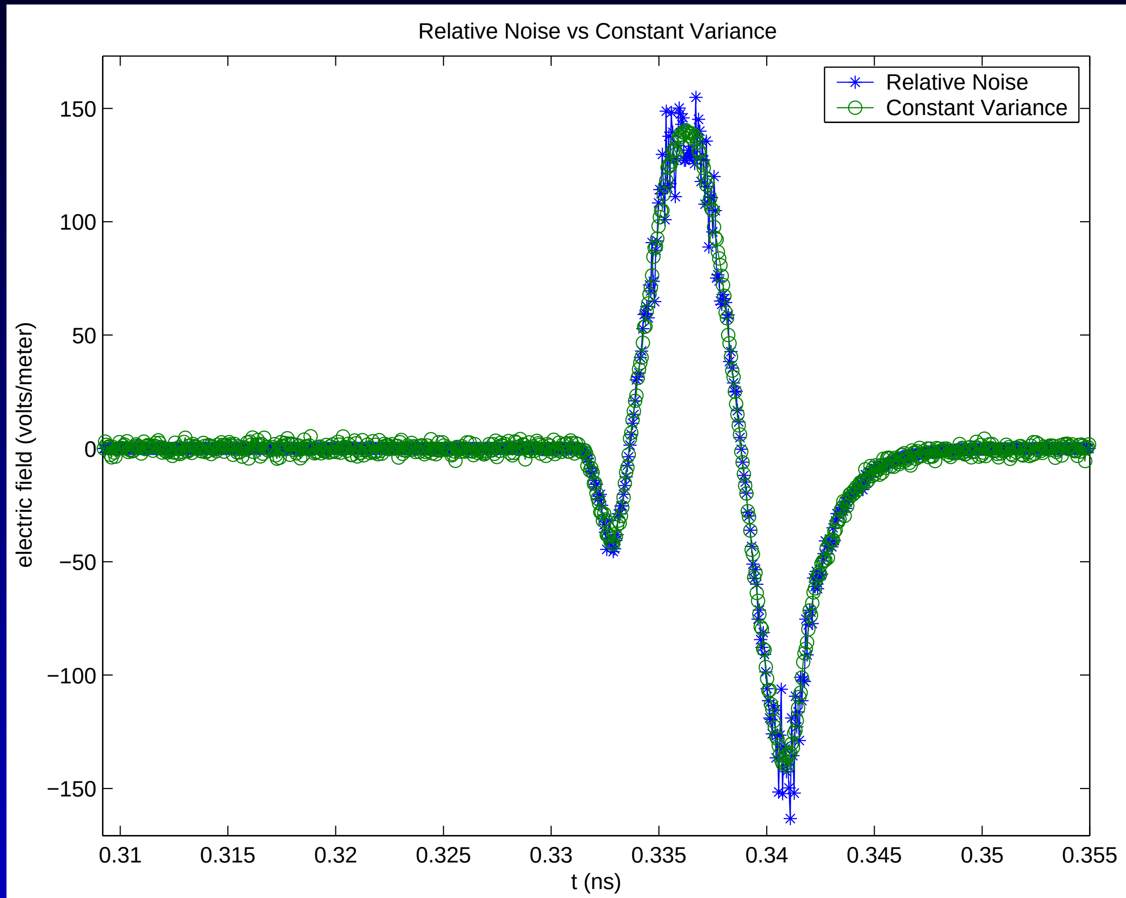
- Our modified Least Squares objective function "fixes" peaks in $\mathcal{J}$
- Can test on both sides of detected minima to ensure global minimization
- We are able to detect a $.2mm$ wide crack behind a $20cm$ deep slab
- Even adding random noise (equivalent to 20% relative noise) does not significantly hinder our inverse problem solution method, and only slightly broadens the confidence intervals in a sensitivity analysis

SOFI under 20X magnification



Material heterogeniety may have significant effects on the output of an interrogating signal, especially pulsed UWB signals. Consider distributed parameters, homogenization, or distributions.

Relative/Constant Variance Noise



The difference between data with relative noise added and data with constant variance noise added is clearly evident when E is close to zero or very large.

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