

MTH 452/552 – Homework 4

1. (10 points) [Derivation of Adams-Moulton]

Determine the coefficients $\beta_0, \beta_1, \beta_2$ for the third order, 2-step Adams-Moulton method using the expression for the local truncation error (from Section 5.9.1)

$$\tau(t_{n+r}) = \frac{1}{k} \left(\sum_{j=0}^r \alpha_j u(t_{n+j}) - k \sum_{j=0}^r \beta_j u'(t_{n+j}) \right)$$

and taking a Taylor expansion of each function around t_n .

2. (15 points) [Characteristic polynomials and LTE for LMM]

Determine the characteristic polynomials $\rho(\zeta)$ and $\sigma(\zeta)$ for any **three** the following linear multistep methods. Also, verify that

$$\sum_{j=0}^r \left(\frac{1}{q!} j^q \alpha_j - \frac{1}{(q-1)!} j^{q-1} \beta_j \right) = 0$$

for $q = 1..p$, and is not zero for $q = p + 1$, where p is the order of the method.

- (a) The 2-step Adams-Bashforth method

$$U_{n+2} = U_{n+1} + \frac{k}{2} (-f(U_n) + 3f(U_{n+1}))$$

- (b) The 2-step Adams-Moulton method

$$U_{n+2} = U_{n+1} + \frac{k}{12} (-f(U_n) + 8f(U_{n+1}) + 5f(U_{n+2}))$$

- (c) The 2-step Nyström method (explicit midpoint)

$$U_{n+2} = U_n + 2kf(U_{n+1})$$

- (d) The 2-step Milne-Simpson method (implicit Nyström)

$$U_{n+2} = U_n + \frac{k}{3} (f(U_n) + 4f(U_{n+1}) + f(U_{n+2}))$$

- (e) The 2-step Backward Differentiation Formula method (BDF)

$$U_{n+2} = \frac{4}{3}U_{n+1} - \frac{1}{3}U_n + \frac{2k}{3}f(U_{n+2})$$

3. (15 points) [Characteristic polynomials for LMM]

For the following three methods determine the first characteristic polynomial $\rho(\zeta)$ and find if the method is zero-stable, and if so, strongly or weakly stable.

- (a) The 2-step Nyström method (explicit midpoint)

$$U_{n+2} = U_n + 2kf(U_{n+1})$$

- (b) The 2-step Backward Differentiation Formula method (BDF)

$$3U_{n+2} = 4U_{n+1} - U_n + 2kf(U_{n+2})$$

- (c) The 3-step Backward Differentiation Formula method (BDF)

$$11U_{n+3} = 18U_{n+2} - 9U_{n+1} + 2U_n + 6kf(U_{n+3})$$

4. (10 points) [Predictor-corrector methods]

Show that the one-step Adams-Bashforth-Moulton method is actually an RK method (which one?). What is the order of this method? Is this consistent with expected order for a predictor-corrector method?