

MTH 452/552 – Homework 1

1. (40 points) [Method of Undetermined Coefficients]

- (a) Use the method of undetermined coefficients to set up and solve the 3×3 Vandermonde system that would determine a centered finite difference approximation to $u'(\bar{x})$ of the highest order possible based on 3 equally spaced points, i.e.,

$$D_{-1:1}u(\bar{x}) := c_{-1}u(\bar{x} - h) + c_0u(\bar{x}) + c_1u(\bar{x} + h) \approx u'(\bar{x}).$$

(MATLAB notation for the set of numbers from a to b by 1 is $a : b$.) Find the coefficients c_{-1} , c_0 , and c_1 .

Note: the resulting formula is called the *centered 3-point approximation for the first derivative* (eventhough the center point does not appear explicitly...hint).

- (b) Check your answer using the MATLAB code `fdstencil.m` available from the author's website.
- (c) Repeat (b) for 5 points, i.e., find

$$D_{-2:2}u(\bar{x}) := c_{-2}u(\bar{x} - 2h) + c_{-1}u(\bar{x} - h) + c_0u(\bar{x}) + c_1u(\bar{x} + h) + c_2u(\bar{x} + 2h) \approx u'(\bar{x}).$$

(Note: although the algebra is tedious, one can find an explicit formula for these coefficients for any arbitrary number of points in a centered stencil.)

- (d) Download and run the m-file `fderror.m`. Describe what it is doing.

On your own, try changing h to 10^{-1} , 10^{-2} , 10^{-3} , \dots , 10^{-10} . You should observe the predicted accuracy for larger values of h . For smaller values, numerical cancellation in computing the linear combination of u values impacts the accuracy observed. In fact, the higher order methods are more sensitive to the size of h .

- (e) Test the finite difference formulas from 1a and 1c to approximate $u'(1)$ for $u(x) = \sin(x)$ with values of h from 2^{-1} , 2^{-2} , 2^{-3} , \dots , 2^{-8} . Make a table of the approximate error vs. h for several values of h , as well as ratios of errors. Compare the observed order of accuracy against the predicted error from the leading term of the expression printed by `fdstencil`. Also produce a log-log plot of the absolute value of the error vs. h and verify (by eye) that the curves have the expected slopes.

You may either modify the m-file `fderror.m` or make your own script.

2. (10 points) (552:) [Higher Order Centered Differences] Note that the 5-point formula in 1c can be written as a linear combination of 3-point formulas with different step sizes, i.e.,

$$D_{-2:2} = c_1D_{-1:1} + c_2D_{-2:2:2}$$

where $D_{-2:2:2}$ is the 3-point formula from 1a with stepsize $2h$. (MATLAB notation for the set of numbers from a to b by k is $a : k : b$.) Find the coefficients c_1 and c_2 .

(Note: although the algebra here is even more tedious, one can find an explicit formula for these coefficients for any arbitrary (even) order of accuracy.)