MTH 351 Fall 2006 – Lab 4

Due: Before class November 15

1. Approximate each of the following integrals using the trapezoidal rule:

   i.
   $$\int_0^2 e^{-x^2} dx$$

   ii.
   $$\int_0^4 \frac{1}{1+x^2} dx$$

   iii.
   $$\int_0^1 \sqrt{x} dx$$

   (a) For each integral, create a table of values $T_n(f)$ for $n = 2, 4, 8, \ldots, 512$. Also compute the difference between successive iterates $T_n(f) - T_{n-1}(f)$, and the ratio between successive differences $(T_n(f) - T_{n-1}(f))/(T_{n-1}(f) - T_{n-2}(f))$. Your table should look something like:

n	Approximation Difference Ratio

   Note: In order to see all significant figures, it is helpful to use something similar to the following when you output values in Matlab:

   ```
   [inT,diT,raT]=trapezoidal(a,b,n0,indexf);
   for i=1:length(inT),
      disp(sprintf('%d \t%0.12f \t%0.5e \t%g', ...
         n0*2^(i-1),inT(i),diT(i),raT(i)))
   end
   ```

   (b) Comment on whether the trapezoidal rule performed as well as expected for each integral. If it did not, explain what may be the cause.

2. (a) Repeat (1a) using Simpson's rule.

   (b) Regarding integral (i.), the asymptotic error formula for Simpson's rule estimates that the number of subdivisions required to achieve an accuracy of $\epsilon = 10^{-10}$ is at least $n = 160$. For integral (ii.) $n = 396$ is required for an accuracy of $\epsilon = 10^{-12}$. Comment on whether your computational results agree or disagree with the asymtotic error formula.

3. **Bonus:** Improve $T_{512}(f)$ for each integral (i.), (ii.), and (iii.) by using either the *corrected trapezoidal rule* or *Richardson's extrapolation formula*. Compare this approximation (either $CT_{512}(f)$ or $R_{512}(f)$) to the Simpson's rule approximation ($S_{512}(f)$).

Turn in a hard copy of the Matlab output, including tables, along with your answers to the questions for each problem. Email a copy of the script which runs all the commands. If you prefer to create one file with commands and comments, then begin your comment lines with %.