MTH 351 – HW for Section 6.4

Consider the $n \times n$ matrix A from Section 6.2, Problem 21 given by:

$$A = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 \\ -1 & 2 & -1 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & -1 & 2 & -1 \\ 0 & \cdots & 0 & -1 & 2 \end{bmatrix}$$

Note that this matrix is the result of discretizing the following *two-point boundary value problem* (see Section 8.8) using finite difference derivatives (see Section 5.4):

$$y''(x) = f(x), \qquad y'(a) = 0, \qquad y(b) = 0, \qquad a \leq x \leq b.$$

1. Compute ("by hand", i.e., show each step) the LU factorization for A using
   (a) $n = 2$
   (b) $n = 3$
2. Find the pattern in the above factorizations, what do you suppose is the LU factorization for arbitrary n?
3. Verify that the i, i entry of the product LU is $A_{i,i} = 2$.
4. Verify that the $i, i+1$ entry of the product LU is $A_{i,i+1} = -1$. (Since A is symmetric, we do not need to verify $A_{i,i-1}$.)
5. Comment on the number of floating point operations (multiplications and divisions only.. assume, rather crudely, that additions, subtractions and multiplications by 1 are free!) required to solve $A\vec{x} = \vec{b}$ for this particular A
   (a) if given
   $$A^{-1} = \begin{bmatrix} n & n-1 & n-2 & n-3 & \cdots & 1 \\ n-1 & n-1 & n-2 & n-3 & \cdots & 1 \\ n-2 & n-2 & n-2 & n-3 & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 1 & 1 & 1 & 1 & \cdots & 1 \end{bmatrix}$$
   (b) if given LU from 2.