MTH 351 – Lab 5

1. Approximate each of the following integrals using the composite trapezoidal rule:

   i.
   $$\int_0^2 e^{-x^2}\,dx$$

   ii.
   $$\int_0^4 \frac{1}{1+x^2}\,dx$$

   iii.
   $$\int_0^{2\pi} \frac{1}{2+\sin(x)}\,dx$$

   iv.
   $$\int_0^1 \sqrt{x}\,dx$$

   (a) For each integral, create a table of values $T_n(f)$ for $n = 2, 4, 8, \ldots, 512$. Also compute the difference between successive iterates $T_{2n}(f) - T_n(f)$, and the ratio between successive differences

   $$\frac{T_{2n}(f) - T_n(f)}{T_{4n}(f) - T_{2n}(f)}.$$

   Your table should look something like:

n	Approximation Difference Ratio

   Note: In order to see all significant figures, it is helpful to use something similar to the following when you output values in Matlab:

```
a=0;
b=2;
n0=2;
f='exp(-x^2)';
[inT,diT,raT]=trapezoidal(a,b,n0,f);
fprintf('n \tIntegral \tError \t\tRatio\n')
for i=1:length(inT),
  fprintf('%d\t%0.12f\t%0.5e\t%g\n',n0*2^(i-1),inT(i),diT(i),raT(i))
end
```

   (b) Comment if the trapezoidal rule performed worse or better than expected for each integral. Explain what may be the cause.

2. Repeat 1 using composite Simpson's rule. Compare to trapezoidal with respect to accuracy and efficiency.

   Note: In order to see all significant figures, it is helpful to use something similar to the following when you output values in Matlab:

```
[inS,diS,raS]=simpson(a,b,n0,f);
fprintf('n \tIntegral \tError \t\tRatio\n')
for i=1:length(inS),
  fprintf('%d\t%0.12f\t%0.5e\t%g\n',n0*2^(i-1),inS(i),diS(i),raS(i))
end
```

3. Regarding integral (i.), the asymptotic error formula for Simpson's rule estimates that the number of subdivisions required to achieve an accuracy of $\epsilon = 10^{-10}$ is at least $n = 160$. For integral (ii.) $n = 396$ is required for an accuracy of $\epsilon = 10^{-12}$. Comment on whether your computational results agree or disagree with the asymtotic error formula. (See 5.2 Problem 6 in the text.)

4. Repeat 1 using Gaussian quadrature rule. Compare to trapezoidal and Simpson's with respect to accuracy and efficiency.

   Note: In order to see all significant figures, it is helpful to use something similar to the following when you output values in Matlab:

```
[inG,diG,raG]=gausstable(a,b,n0,f);
fprintf('n \tIntegral \tError \t\tRatio\n')
for i=1:length(inG),
  fprintf('%d\t%0.12f\t%0.5e\t%g\n',n0*2^(i-1),inG(i),diG(i),raG(i)))
end
```

5. **Optional:** Improve $T_{512}(f)$ for each integral above by using the *corrected trapezoidal rule* (if it applies). Compare this approximation ($CT_{512}(f)$) to the Simpson's rule approximation ($S_{512}(f)$) with respect to accuracy and efficiency.

6. **Optional:** Improve $T_{512}(f)$ for each integral above by using the *Richardson's extrapolation formula*. Compare this approximation ($R_{512}(f)$) to the Simpson's rule approximation ($S_{512}(f)$) with respect to accuracy and efficiency.