Sample Questions for Midterm
MATH 341

#1] Find all solutions of the linear system

$$\begin{aligned} x_1 - x_2 + 3x_3 + 2x_4 &= 1 \\ -x_1 + x_2 - 2x_3 + x_4 &= -2 \\ 2x_1 - 2x_2 + 7x_3 + 7x_4 &= 1 \end{aligned}$$

#2 (a) Given a linear system consisting of two equations in three unknowns what is the possible number of solutions?

(b) Given a homogeneous linear system ($Ax = 0$) consisting of two equations in three unknowns, how many solutions will it have? Explain.

#3] Let A be a 3×3 matrix and let

$$b = 3a_1 + a_2 + 4a_3$$

Will the system $Ax = b$ be consistent?

Explain

#4] Let A be a 3×3 matrix and suppose that

$$a_1 - 3a_2 + 2a_3 = 0 \text{ (the zero vector)}$$

Is A nonsingular? Explain

#5] True or False. Give reasons

a) If A is nonsingular then A^T is nonsingular and

$$(A^T)^{-1} = (A^{-1})^T$$

(b) If $\det(A) = \det(B)$ then $A = B$

#6] Let

$$A = \begin{bmatrix} x & 1 & 1 \\ 1 & x & -1 \\ -1 & -1 & x \end{bmatrix}$$

a) Compute det(A) using cofactors (Your answer should be a function of x).

b) For what values of x will the matrix be singular? Explain

#7] Consider matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ 0 & -1 & 0 \end{bmatrix}$

a) Compute det(A) using Elimination (and not Cofactor expansion). Represent each row operation using Elementary matrices. You Should Compute det(A) by using the determinants of the elementary matrices & a reduced form of A.

b) Is A nonsingular? If so Compute A^{-1} by performing Elimination on the augmented matrix $[A \mid I]$