MATH 341

SAMPLE PROBLEMS FOR FINAL PART II

The Final Exam will Consist of two Parts that will cover material from

- Chapter 4 – (Sec 4.1, 4.2.1, 4.2.2)
- chapter 5 – (all sections)
- Chapter 6 – (Sec 6.1)
- chapter 8 – (sec 8.1, 8.2, 8.3, 8.5)
- chapter 9 – (Sec 9.1, 9.2)
- Chapter 12 – (Sec 12.1.1 – 12.1.6)

Remarks: For sample problems based on pre midterm material look at Midterm Sample Problems. Part I of the Final Exam will be based on the pre-midterm material.

Below are sample problems from chapter 8, 9, 12. For sample problems from chapters 4, 5, 6 consult the midterm review problem set.

#1a) Determine whether the set of vectors

$$S_1 = \left\{ \begin{bmatrix} t \\ s \end{bmatrix} ; t, s \in \mathbb{R},\ t+s=0 \right\}$$

is a subspace of $\mathbb{R}^2$? If so, give a basis for the subspace and find its dimension.

b) Determine whether the set of vectors

$$S_2 = \left\{ (2t+u,\ t+3u,\ t+s+v,\ u)^T ; s, t, u, v \in \mathbb{R} \right\}$$

is a subspace of $\mathbb{R}^4$? If so, find a basis and determine the dimension of the subspace.

#2] Let $A = \begin{bmatrix} 1 & 3 & 1 & 3 & 4 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 & 3 \end{bmatrix}$

a) Find a basis for $N(A)$ the null space of A. What is the nullity of A?

b) Find a basis for the column space of A? What is the rank of A?

c) Check that the rank-nullity theorem holds.

#3] Given the vectors in $\mathbb{R}^3$

$x_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$, $x_2 = \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}$, $x_3 = \begin{bmatrix} 1 \\ 5 \\ 5 \end{bmatrix}$, $x_4 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

a) Are x_1, x_2, x_3 and x_4 linearly independent in $\mathbb{R}^3$? explain?

b) Do x_1, x_2 span $\mathbb{R}^3$? explain?

c) Do x_1, x_2, x_3 span $\mathbb{R}^3$? Are they linearly independent? Do they form a basis for $\mathbb{R}^3$? Explain?

d) Do x_1, x_2, x_4 span $\mathbb{R}^3$? Are they linearly independent? Do they form a basis for $\mathbb{R}^3$? Give reasons for your answers.

#4] Let $L: \mathbb{R}^3 \to \mathbb{R}^3$ be defined by the map

$$L(x) = \begin{bmatrix} x_2 - x_1 \\ x_3 - x_2 \\ x_3 - x_1 \end{bmatrix} ; \forall\, x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3$$

Show that L is a linear operator on $\mathbb{R}^3$ and determine $\ker(L)$ and range(L).

#5] Let L be a linear operator on $\mathbb{R}^n$. Suppose that $L(x) = 0$ for some $x \neq 0$. Let A be the standard matrix representation of L. Show that A is singular.

#6] Let L be a linear operator that reflects each vector x about the line $x_1 = x_2$, doubles the length and then rotates the vector in $\mathbb{R}^2$ by 90° in the counterclockwise direction.

a) Find the standard matrix representation of the operator L

b) Give a different interpretation of operator L

#8] Consider the 2×2 matrix

$$A = \begin{bmatrix} 2 & -3 \\ 2 & -5 \end{bmatrix}$$

(a) Find the eigenvalues of A.

(b) Find the subspaces of eigenvectors

for each eigenvalue (i.e., eigenspaces)

c) Find the algebraic and geometric multiplicity of each eigenvalue. Is A a defective matrix?

d) Diagonalize A, ie find a nonsingular matrix X and diagonal matrix D so that $X^{-1}AX = D$.

#8] State if TRUE OR FALSE

a) If A is $n \times n$ and 0 is an eigenvalue of A then A is singular.

b) If $A = X^{-1}BX$, A, B, X $n \times n$ matrices $\det(A) = \det(B)$.

c) If A is $m \times n$ then
$$\text{rank}(A) + \text{nullity}(A) = m$$

d) Let $L: \mathbb{R}^n \to \mathbb{R}^n$ be a linear operator. If A is the standard matrix representation of L, then
$$\text{Ker}(L) = N(A)$$

(Recall: $N(A) \equiv$ null space of A Also called $\text{Ker}(A)$)

e) If $x_1, x_2, \ldots, x_k$ are vectors in a vector space V and
$$\text{Span}(x_1, x_2, \ldots, x_k) = \text{Span}(x_1, x_2, \ldots, x_{k-1})$$
then $x_1, x_2, \ldots, x_k$ are linearly dependent.

(f) Suppose $T: \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation and A_T is its standard matrix representation. If $x_1, x_2 \in \mathbb{R}^n$ are solutions to the linear system $A_T x = b$, for $b \in \mathbb{R}^m$, then $\exists$ a vector $y \in N(A_T)$ such that $x_2 = x_1 + y$.

#9] Consider matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Compute det(A) using Elimination. Represent each row operation using elementary matrices. You should compute det(A) by using the determinants of the elementary matrices and a reduced form of A (either echelon or reduced echelon form).

(This problem was on the sample midterm problem set. However, elementary matrices were not included. For the Final, elementary matrices are included and you should know how to do this.)