MTH 323: Mathematical Modeling

Lab 3

Due: Jun 3 @ 5PM

This lab will help you visualize solutions to the traffic flow model using both constant and linear velocity-density relationships.

The PDE model for traffic density can be written

$$\frac{\partial \rho}{\partial t} + \frac{dq}{d\rho}\frac{\partial \rho}{\partial x} = 0$$

for $t > 0$ and $x \in \mathbb{R}$. A numerical solution is computed in the file `pdex6.m` (for reference, see `pdex1.m` through `pdex5.m` built-in to MATLAB). Each of these examples uses the `pdepe` solver.

1. Run `pdex6.m` for the first example initial density profile

$$p_0(x) = \exp\left(-(x-\xi)^2\right)$$

(where $\xi > 0$ is some fixed value) using the constant velocity-density relationship.

The number of spatial nodes in the discretization n has been set to 101 for convenience (i.e., quick runtime). Increase this to, say 501, if any numerical errors bother you.

(a) Rotate the surface plot (rotate3d should be on, just click and drag the plot around) so that you can clearly see the initial profile (i.e., the $x\rho$-plane). Then rotate around so that you can clearly see the final profile (again, ignore any numerical error). The second figure also shows the initial and final profile.

Describe in words the initial and final density profiles. Give a physical scenario where you might observe the initial profile. Discuss the implications of the final profile.

(b) Finally, rotate the surface plot to look straight down (xt-plane). Each color represents a certain constant density (red $= 1$, blue $= 0$, etc.). Comment on the lines of constant density (look straight?, are parallel?, maximum/minimum observed slope?, etc.)

2. Repeat 1 using the linear velocity-density relationship:

$$u(\rho) = u_{max}\left(1 - \frac{\rho}{\rho_{max}}\right).$$

(i.e., comment out the constant velocity-density line in function `pdex6pde` of the code, and uncomment the linear velocity-density line).

3. Repeat 1 for the second example

$$p_0(x) = \begin{cases} x & \text{if } x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

using both the constant and then the linear velocity-density relationship (i.e., comment out the first profile in function `pdex6ic` and uncomment the second option.)

Again, attempt to ignore the numerical error, however, the higher you can run n the better (MATLAB does not like discontinuous functions!).

4. Repeat 1 for the third example

$$p_0(x) = \begin{cases} 1 & \text{if } 1 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

using both the constant and then the linear velocity-density relationship (i.e., comment out the second profile in function `pdex6ic` and uncomment the third option.)

Again, attempt to ignore the numerical error, however, the higher you can run n the better. Discuss what you think the exact solution to the final profile should look like.