Name: ______________________________

Instructions:

- One 3×5 in. notecard (front and back) may be used.
- A basic scientific calculator may be used for numerical computations. However, sufficient work or explanation must be presented to receive credit.
- **NO PROGRAMMING OR GRAPHING CALCULATORS ARE ALLOWED**

1. Let $O = (0,0)$ denote the origin, P be the point with rectangular coordinates $(1,2)$, and Q the point with rectangular coordinates $(-2,-1)$.
   (a) (15 points) On a set of rectangular coordinate axes accurately draw the vector $\mathbf{u} = \overrightarrow{OP}$, the vector from O to P, $\mathbf{v} = \overrightarrow{PQ}$, the vector from P to Q, and $\text{proj}_{\mathbf{v}}\mathbf{u}$ (the orthogonal projection of $\mathbf{u}$ onto $\mathbf{v}$). **Make sure to label the vectors.**
   (b) (10 points) Compute $\text{proj}_{\mathbf{v}}\mathbf{u}$ and $\text{scal}_{\mathbf{v}}\mathbf{u}$ (the scalar component of $\mathbf{u}$ in the direction of $\mathbf{v}$). **Check: Do your computations and sketch agree?**

2. Given $\mathbf{v} = \mathbf{i} + 4\mathbf{k}$ and $\mathbf{w} = 3\mathbf{i} + 2\mathbf{k}$
   (a) (5 points) Find the dot product $\mathbf{v} \cdot \mathbf{w}$.
   (b) (5 points) Find the angle between $\mathbf{v}$ and $\mathbf{w}$.
   (c) (5 points) Find a vector orthogonal to both $\mathbf{v}$ and $\mathbf{w}$.

3. (10 points) A projectile is launched from the origin at an angle of α radians to the horizontal and with an initial speed of 125 ft/sec. Assume that the x-axis is the horizontal, the y-axis is vertical, and the only force acting on the object is gravity. Find the position function $\mathbf{r}(t)$ for this projectile.

4. Consider the helix $\mathbf{r}(t) = \langle 4\cos t, 4\sin t, 3t\rangle$, $-\infty < t < \infty$.
   (a) (5 points) Find the velocity $\mathbf{v}(t)$
   (b) (5 points) Find the speed
   (c) (5 points) Find the distance traveled along the curve in one unit of time
   (d) (5 points) Find the unit tangent vector $\mathbf{T}(t)$
   (e) (5 points) Find the acceleration $\mathbf{a}(t)$
   (f) (5 points) Compute the principle unit normal vector $\mathbf{N}(t)$
   (g) (5 points) Compute the curvature $\kappa(t)$
   (h) (5 points) Compute the components of acceleration in the direction of $\mathbf{N}(t)$ (a_N), and the direction of $\mathbf{T}(t)$ (a_T)

5. (10 points) Find the arc length of the spiral curve $r = e^{\theta}$ for $0 \le \theta \le 2\pi$.

6. (10 points) Consider the circle $\mathbf{r}(t) = \langle R\cos t, R\sin t\rangle$, for $0 \le t \le 2\pi$, where $R > 0$. Show that the curvature $\kappa = 1/R$.