Mth 254 Sample Midterm Problems

1. Let $O = (0, 0)$ denote the origin, P be the point with rectangular coordinates (1, 2), and Q the point with rectangular coordinates $(-2, -1)$.

   (a) On a set of rectangular coordinate axes accurately draw the vector $\mathbf{u} = \overrightarrow{OP}$, the vector from O to P, $\mathbf{v} = \overrightarrow{OQ}$, the vector from O to Q, and $\text{proj}_{\mathbf{v}}\mathbf{u}$, the orthogonal projection of $\mathbf{u}$ onto $\mathbf{v}$.

   (b) Compute $\text{proj}_{\mathbf{v}}\mathbf{u}$ and $\text{scal}_{\mathbf{v}}\mathbf{u}$, the scalar component of $\mathbf{u}$ in the direction of $\mathbf{v}$.

2. Consider the points $P(-1, 0, 3), Q(0, 3, -6)$. Let O denote the origin.

   (a) Find the sum vector $\mathbf{r} = \mathbf{OP} + \mathbf{OQ}$.

   (b) Find a vector that is orthogonal to $\mathbf{OP}$, and $\mathbf{OQ}$.

   (c) Find the area of the triangle formed by the points O, P, Q.

3. An object moving in space is subject to an acceleration at time t given by

$$\mathbf{a}(t) = \langle t, e^{-t}, 1\rangle = t\mathbf{i} + e^{-t}\mathbf{j} + \mathbf{k} \quad \text{m/sec}^2.$$

   Assuming that its initial velocity is $\mathbf{v}(0) = \langle 0, 1, 1\rangle = \mathbf{j}+\mathbf{k}$ m/sec and its initial position is $\mathbf{r}(0) = \langle 4, 1, 0\rangle = 4\mathbf{i} + \mathbf{j}$ m, find the position $\mathbf{r}(t)$, the velocity $\mathbf{v}(t)$ and the distance travelled $s(t)$ of the object at all times $t \geq 0$.

4. A golf ball is hit from the point $\langle x_0, y_0\rangle$ at an angle of 30^o with an initial speed of 150 ft/sec. Find the time of flight, range of the object and maximum height of the object.

5. A particle travels along the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1.$ in such a manner that its position at time t is given by

$$\mathbf{r}(t) = \langle 3\cos t, 4\sin t\rangle = 3\cos t\mathbf{i} + 4\sin t\mathbf{j}.$$

   (a) Find the velocity $\mathbf{v}(t)$, speed $v(t)$, acceleration $\mathbf{a}(t)$, unit tangent vector $\mathbf{T}(t)$, the principle unit normal vector $\mathbf{N}(t)$, and the curvature $\kappa(t)$.

   (b) Compute $\mathbf{a}(t) \cdot \mathbf{T}(t)$. How is this related to speed?