Valuation of Operational Flexibility: A Case Study of Bonneville Power Administration

I. Introduction

The four US federal power marketing administrations (PMAs) operate electric systems in 33 states. On the one hand, they sell the electricity generated from federally owned and operated hydroelectric dams, which constitutes over 40% of hydroelectric generation in US. As federal marketing administrations, PMAs are required by law to set rates to cover costs at the lowest possible rates to consumers and forgo any profit (EIA, 2013). This provides the institutional incentive for PMAs to maximize the sales revenue of hydroelectricity. On the other hand, they also serve the role as balancing authorities (BAs) for their respective regions. As balancing authorities, these PMAs need to balance the electricity supply and demand in real time in order to maintain the safe and reliable operation of the power system and avoid blackouts. However, because electric flows are instantaneous and electricity is non-storable on a large scale, any imbalance between the demand and supply must be offset on the order of seconds. Therefore, it is necessary for PMAs to have some backup resources. In other words, it is important for PMAs to maintain some operational flexibility in order to cope with unexpected future imbalances. Hydroelectric generation therefore becomes more valuable because water is storable in reservoirs and the marginal cost of hydroelectric generation is small. Even though it has been a common practice to use hydroelectric capacity as backup resources, the quantitative valuation of this operational flexibility, the hydroelectric capacity as a backup resource, remains mystical. In the essence, the value of operational flexibility originates from the necessity to balance the uncertain future quantities in the electricity demand and supply.

Applying the real option theory, we resolve the puzzle by developing a theoretical framework for the valuation of operational flexibility. It explicitly models the intertemporal tradeoffs embodied in the two management roles of PMAs. That is, an increase of the current sales revenue reduces operational flexibility, the stock of the backup resource that can be used to buffer against future electricity shortages. As a result, an increase in current sales of hydroelectricity increases the probability of electricity shortage in the future and therefore tends to increase the value of operational flexibility. We argue that the foregone value of operational flexibility should be considered as foregone opportunity cost in the optimization problem of PMAs.

Existing literature on the value of operational flexibility dates back to the pioneering work of Arrow and Fisher (1974), which shows that an irreversible investment decision incurs the cost due to the loss of options. Brennan and Schwartze (1985) formulate the value of a mine allowing for operational flexibility in the form of costly closing or reopening the mine. The value of operational flexibility is interpreted as “convenience yield” that “arise either from local price variation or from the ability to maintain a production process” (p.139). To date, the real option theory has been applied in many subfields of environmental and resource economics, as reviewed by Mezey and Conrad (2010) Fernandes et al. (2011), Cesena et al. (2013) and Kozlova (2017), like forestry (Insley, 2002; Insley and Lei 2007), fishery (Nøstbakken, 2006; Murillas and Cahmorro, 2006), water resources, nonrenewable resources (Hazra et al., 2019; Insley, 2017; Marmer and Slade, 2018; Zhang et al., 2015; Zhang et al., 2018; Muehlenbachs, 2015), and conservation. All of these applications cover a wide spectrum of binary choices including the option to defer, the option to stage, the option to abandon, the option to change scale, the option to stop/restart, the option to grow, the option to change inputs/outputs[1] (Trigeorgis, 1996). In contrast to the focus on binary choices in these previous studies, this paper is the first that applies real option theory to a continuous choice variable. In particular, the real option theory is applied for the optimal management of hydroelectric production: that is, how much of the current production capacity should be used so as to maximize current revenue without mitigating the ability of balancing the future electricity demand and supply. While the proposed theoretical framework focuses on hydroelectric production, the underlying tradeoff between current reward and irreversible change on future uncertainty is relevant to many other resource and environmental issues.

With this extension to the continuous choice variable, we are able to quantify the economic value of managerial flexibility and illustrate how it varies with incremental changes in flexibility. This is in sharp contrast to the binary choice between using either all or none of the flexibility. It is possible that the binary choice model may generate dramatically different management policy recommendations as compared to the continuous choice model.

With the ability to quantify the option value for incremental changes in flexibility, it becomes practically feasible for the application to real-time trading in the wholesale electricity market and real-time pricing in the retail electricity market (Borenstein and Holland, 2005). In practice, electricity prices generally reflect the cost for energy production and transmission (EIA, 2019), which provides a practical benchmark for real-time trading and pricing. Even though experienced real-time BPA traders acknowledge that their transaction decisions change the future exposure to electricity shortages, they do not have a quantifiable framework or tool to guide their transaction decision. In this paper, we propose such a framework and a quantification tool. We show that the option value of operational flexibility is an important factor to consider in the pricing of electricity for BPA, which should be considered in the real-time trading in the wholesale electricity market, as well as the possible future real-time pricing in the retail electricity market.

Last but not the least, as an application of the proposed framework, we simulate the impact of the Wind Vision of the U.S. Department of Energy for 2050. We show that the increased installed capacity of wind energy may temporarily reduce BPA's current electricity supply due to the increased uncertainties in the system.

Model Setup

Background

Among the four PMAs, we focus on Bonneville Power Administration (BPA), which has the largest installed capacity (22,363 MW). It markets electrical power from 31 federal hydroelectric projects in the Northwest and one nuclear plant, which accounts for 28 percent of the electric power in the Northwest (Idaho, Oregon, Washington, some parts of Montana, California,

[1] This refers to the ability to change input materials or output products, not the quantity of inputs or outputs.

Nevada, Utah and Wyoming). [2] We focus on the hydroelectric projects because water is storable and can be used to serve BPA's role as both PMA and BA. Water stored in BPA-operated reservoirs provides the water reserves for hydroelectric generation. Precipitation and melting snow packs provide additional inflows into BPA's water reserves. This water reserve is then fed into a short-term hydropower generation model (Bashiri et al., 2017) to generate the maximum hydroelectric generation capacity C_t, after considering various technological (e.g. turbine capacity and fore-bay elevation), environmental (e.g. wildlife preservation) and legal (e.g. flood control) constraints. As a BA, BPA must satisfy the current demand (D_t) including pre-existing contracts. The remaining production capacity is referred to as the operational flexibility, defined as:

(1) $F_t = C_t - D_t$.

If $F_t > 0$, there is usable water reserve in the BPA-operated reservoirs which can be used to generate hydroelectricity. As a PMA, BPA can sell the hydroelectricity in the wholesale market to generate additional revenues. If $F_t < 0$, then electricity demand exceeds supply. As a BA, BPA is obliged to purchase on the wholesale market to balance the excess demand and ensure the safe and reliable operation of the power system. In a world with many sources of uncertainties, future flexibility is volatile. Current sales of the remaining capacity, or operational flexibility F_t, reduce the water reserve and increase the probability of future electricity shortages. In other words, as water reserve decreases, BPA has less operational flexibility to buffer future shortages. [3]

Repurchasing is not a problem if the purchase price equals the sales price on average. However, it is believed that BPA's purchase price on average exceeds its sales prices. This can be attributed to the following reasons. Firstly, BPA is required by law to set rates to cover costs "at the lowest possible rates to consumers consistent with sound business principles", because it relies mainly on hydroelectric power generated from federally owned dams (FERC, 2013). However, when BPA needs to buy electricity, it may often be charged the higher cost of thermoelectric generation or at a rate set by profit-driven entities. The seasonality makes the trading conditions even worse. For an example, as the raining season comes, BPA and other hydroelectricity providers in the region may want to sell the hydroelectricity at the same time. It is likely that all of them receive the price reflecting the generation cost of hydroelectricity. In the dry season, BPA and other hydroelectric power providers are in short of water reserves for power generation, therefore, when faced with unexpected demand, the purchase prices likely reflect the more costly thermoelectric power generation.

Secondly, the transaction price in the spot market is given by the locational marginal pricing (LMP) which reflects "the marginal cost of serving load at the specific location, given the set of

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[2] BPA, BPA Facts. Accessed online on Feb. 22, 2018: https://www.bpa.gov/news/pubs/GeneralPublications/gi-BPA-Facts.pdf.

[3] In reality, if BPA fails to balance the excess electricity demand, there are much larger institutional and legal costs for the BPA which are difficult to quantify and therefore not included in the following analyses. However, given careful quantification of such losses, the proposed framework can quantify its induced economic value for flexibility.

generators that are dispatched and the limitation of the transmission system" (FERC, 2015, p.60). LMP includes three elements: an energy charge, a congestion charge and a charge for energy losses in transmission. The last two elements add to the purchase price paid by BPA but not the sale price received.

Thirdly, because BPA contributes approximately 30 percent of the electric power in the regional market, the fact that BPA, as a BA, wants to sell (buy) electric power on the spot market implies an unexpected excessive supply (demand), the market price will go up (down).

To summarize, BPA needs to balance the current increase in sales revenue and the possibility of buying it back at a higher price in the future. Such a tradeoff gives the water reserve in the reservoirs an option value: holding on to the water reserve is equivalent to holding on the operational flexibility as the right but not the obligation to use it for hydroelectric generation. The use of flexibility, running water through the hydropower-generating turbines, is irreversible in nature. Once the water reserves are used, it will be economically very costly to replenish the reservoirs, even though pumping the downstream water back into the upstream reservoirs is technically possible. The irreversible use of flexibility is similar to a financial put option, which gives the option holder the right but not the obligation to sell an asset at pre-specified price within certain amount of time. If the stock price is lower (higher) than the pre-specified price, the put option has positive (zero) value. The operational flexibility gives BPA the right but not the obligation to sell the flexibility, that is, selling the hydroelectricity generated using water reserve in the reservoirs. The operational flexibility has positive (zero) economic value when electricity shortage (does not) occurs, because having operational flexibility allows BPA to avoid open market purchase at a higher price. The key difference between the financial put option and the operational flexibility option is that the former is designed to buffer downward risk in stock price (price goes below the pre-specified price) whereas the latter is intended to buffer the downward risk in quantity (flexibility goes below zero). In the following sections, we propose to quantify the economic value of operational flexibility using financial option theory. Like an American option, BPA may choose to exercise the operational flexibility in any period before it expires.

Formulation of BPA's Management Problem

At the beginning of a period, BPA is endowed with operational flexibility denoted as F_0, which is the remaining capacity for power supply after satisfying the power demand in the current period. If electricity shortage occurs ($F_t < 0$) in any period t, BPA is required by law to purchase on the open market in order to meet the unsatisfied demand. This costs BPA a total of $P_t^b F_t$, where P_t^b denotes the purchase price in the period.[4] If $F_t > 0$, BPA has some operational flexibility. It may sell all of it. Alternatively, it may choose to sell a part of F_t to generate some revenue in the current period and hold on to the remaining to buffer the future electricity shortages. Denote the amount sold as $S_t \in [0, F_t]$ and the sale price as P_t^s. The current period reward function is therefore summarized as:

[4] Additional penalty cost can also be incorporated into this framework.

$$(2)\ R(F_t, S_t) = \begin{cases} P_t^s S_t & \text{if } F_t > 0 \\ P_t^b F_t & \text{if } F_t \leq 0 \end{cases}$$

The operational flexibility available at the beginning of the next period is a controlled Markov process:

$$(3)\ F_{t+1} = \begin{cases} F_t - S_t + \epsilon_{t+1} & \text{if } F_t > 0 \\ \epsilon_{t+1} & \text{if } F_t \leq 0 \end{cases}$$

where ϵ_{t+1} is the net increase in the operational flexibility due to factors like water inflows into the reservoirs or decreases in electricity demand. Assume that ϵ_{t+1} follows normal distribution $(\mu_{t+1}, \sigma_{t+1}^2)$. Then F_{t+1} is also normal with the same variance and mean $F_t - S_t + \epsilon_{t+1}$. Denote the density function as $\phi_{t+1}(F_{t+1})$. The probability that electricity shortage occurs in the next period $t + 1$ can be expressed as:

$$(4)\ q_{t+1} = \int_{-\infty}^{0} \phi_{t+1}(F_{t+1}) dF_{t+1}.$$

Because $S_t \in [0, F_t]$, as $F_t \to 0$, $S_t \to 0$. F_{t+1} is continuous in F_t. $\lim_{F_t \to +\infty} q_{t+1} = 0$, $\lim_{F_t \to -\infty} q_{t+1} = 1$. If the system has more water reservoirs in the current period, the probability of future shortage is smaller, i.e. $\partial q_{t+1}/\partial F_t \leq 0$ almost everywhere.[5]

The objective of BPA is to maximize the discounted sum of the net revenues from hydroelectricity generation:

$$(5)\ \max_{S_t} E_0\{\textstyle\sum_{\tau=0}^{T} (1 + r)^{-\tau} R(F_\tau, S_\tau)\}$$

where r is the interest rate and $R(F_\tau, S_\tau)$ is the current period revenue defined in equation (2).

In order to focus on the operational flexibility, we abstract away from the price volatilities and assume that purchase price P_t^b and sale price P_t^s do not change over time. Therefore, we can simply denote them as P^b and P^s respectively.

Marginal Value of Operational flexibility

In this subsection, we consider the value of marginal change in the operational flexibility. Denote initial amount of operational flexibility as F_0, which is measured in the amount of hydroelectricity generation capacity stored in the form of water in the reservoirs. Let h be a small positive amount of operational flexibility to be allocated: either used for electricity generation in the current period for sales revenue or as a commitment to buffer potential shortage in some future period. Define the control variable as

$$(6)\ x_t = \begin{cases} 1: & \text{sell flexibility } h, \\ 0: & \text{hold flexibility } h. \end{cases}$$

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[5] More rigorously, $q_{t+1} = prob(F_{t+1} \leq 0|F_t) = prob(F_t - S_t + \epsilon_{t+1} \leq 0|F_t) = prob(\epsilon_{t+1} \leq -F_t + S_t|F_t)$. Therefore, $\partial q_{t+1}/\partial F_t < 0$ for $F_t > 0$ and $\partial q_{t+1}/\partial F_t = 0$ for $F_t < 0$.

As a BA, the value of operational flexibility originates from the need to buffer possible future electricity shortage $F_t < 0$, which may arise due to the unstable wind power generator, the unexpected increase in electricity demand or the unpredicted decrease in stream inflow. Holding h (i.e. $x_0 = 0$) helps to maintain the flexibility to avoid the more expensive power purchase on the open market in the future. The sale of operational flexibility h (i.e. $x_0 = 1$) in the current period generates the instantaneous sales revenue but increases the likelihood of obliged future purchase if electricity shortage occurs. That is, current sales increase the expected payment in the future.

Take an arbitrary future period t as an example. If h is insufficient to cover all the electricity shortage, i.e. $F_t < -h$, the induced future purchase cost by selling h in the initial period is given by $P^b h$. That is, if h were hold on until period t, it can buffer shortage up to h. The remaining part $(h - F_t)$ has to be purchased from the market anyway. If h is sufficient to cover all the shortage, i.e. $-h < F_t < 0$, BPA can use part of the flexibility h to buffer the cost of an obliged market purchase, which saves the purchase expenditure of $P^b F_t$. The remaining can still be sold on the market at the sale price $P^s(h + F_t)$. Because the energy shortage F_t is the amount BPA needs to purchase from the market as a BA, buying additional energy will create an energy surplus that BPA has to sell anyway. Therefore, if the marginal flexibility were held to period t, its expected value is given by:

$$(7)\ \mathrm{E}R_t(h) = \left[\int_{-\infty}^{-h} P^b h\phi(F_t)dF + \int_{-h}^{0}[P^b|F_t| + P^s(h - |F_t|)]\phi(F_t)dF\right],$$

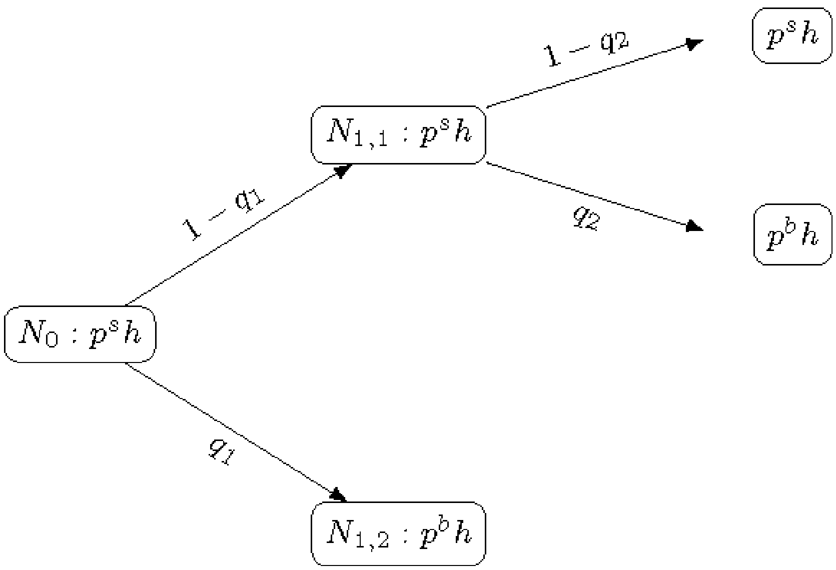
which is the foregone value of selling h in the current period. When h is sufficiently small, the second term in equation (7) is practically negligible and the above formula can be simplified as

$$(8)\ \mathrm{E}R_t(h) \approx P^b h q_t$$

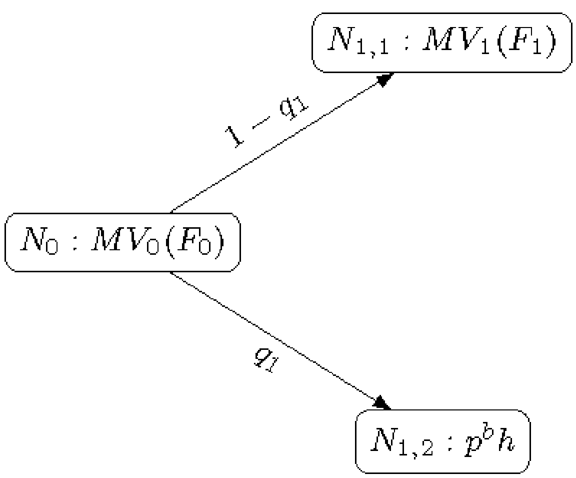
where $q_t = \int_{-\infty}^{0} \phi(F_t)dF_t$ is the probability that an electricity shortage occurs in period t.

If there is no energy shortage ($F_t \geq 0$), BPA is not obliged to purchase anything so that it can sell it in the period to get a sales revenue $P^s h$. Alternatively, it may choose to hold on to the flexibility to the next period.

The value for the h amount of operational flexibility can be illustrated using a three-period binomial tree model as shown in Figure 1. In the current period $t = 0$, denoted as decision node N_0, BPA has the right but no obligation to sell the flexibility and generate a revenue of $P^s h$. This is like the exercise value of the put option. If BPA chooses not to sell the flexibility h, BPA can hold h to the next period $t = 1$. Depending on the realization of uncertainties, the state of the world may evolve along different paths, represented by the different branches spawning from the preceding decision nodeN_0. If electricity shortage occurs in period $t = 1$, which occurs with probability q_1. BPA arrives at decision node $N_{1,2}$ as the system evolves along the lower branch. As a balancing agency, holding the h amount of flexibility saves the obliged purchase cost of $P^b h$. If there is no electricity shortage, BPA is at decision node $N_{1,1}$. It may sell the flexibility for $P^s h$. Alternatively, it may hold the flexibility to the next period $t = 2$. For simplicity, assume period $t = 2$ is the final period that BPA has to use the flexibility.[6] So that the value of h amount of flexibility is either $P^b h$ if electricity shortage occurs or $P^s h$ otherwise.



(a). Three-period binomial tree for the payoffs.



(b). Two-period binomial tree for the payoffs.

Figure 1. Marginal Value of Operational flexibility and its Valuation.

The structure is similar to the binomial tree model in the standard American option theory and can be solved similarly using the backward induction method. The solution starts from the last period.

[6] The existence of the final period reflects the fore-bay elevation requirement set by the long term management goals of BPA.

The expected value in period $t = 2$ is: $q_2 P^b h + (1 - q_2) P^s h$. The corresponding present value in period $t+1$ is $\frac{1}{1+r}[q_2 P^b h + (1-q_2)P^s h]$. The value of exercising the flexibility in period $t = 1$ is $P^s h$. Because BPA can choose to use h, a marginal change in flexibility, or hold on to it , as in an American option, we calculate the marginal value of flexibility at the node $N_{1,1}$ as $MV_1(F_0) = \max(P^s h, \frac{1}{1+r}[q_2 P^b h + (1-q_2)P^s h])$. This simplifies the three-period binomial tree model in Figure (1a) into a two-period binomial tree model in Figure (1b). Repeating the same procedure, we can calculate the marginal value at decision node N_0 as $MV_0(F_0) = \max\left(P^s h, \frac{1}{1+r}[q_1 P^b h + (1-q_1)MV_1]\right)$. In general, if $F_t > 0$, the marginal value of operational flexibility $MV_t(F_t)$, i.e., the change in the value of flexibility $V_t(F_t)$ for a small change in the operational flexibility (h), is given by

$$(9)\ MV_t(F_t) = \max\left(P^s h, \frac{1}{1+r}[q_{t+1} P^b h + (1-q_{t+1})MV_{t+1}(F_{t+1})]\right), \forall \tau,$$
$$\text{and } MV_T = P^s h.$$

Proposition 1 (Properties of $MV_t(F_t)$): The marginal value of operational flexibility $MV_t(F_t)$ has the following properties:

a) $P^s h \leq MV_t(F_t) \leq P^b h$ for all $F_t \in R^+$, because $P^b > P^s$. In particular, $\lim_{F_t \to +\infty} MV_t(F_t) = P^s h$.
b) $MV_t(F_t)$ is continuous in F_t.
c) $\partial MV_t(F_t)/\partial F_t \leq 0$ for all $t = 0, 1, \ldots, T$.

Proof of (b). MV_T is continuous in F_{T-1}. According to equation (4),

$$F_T = \begin{cases} F_{T-1} - S_{T-1} + \epsilon_T & \text{if } F_{T-1} > 0, \\ \epsilon_T & \text{if } F_{T-1} \leq 0. \end{cases}$$

F_T is clearly continuous in F_{T-1} for $F_{T-1} \neq 0$. Further, because $S_{T-1} \in [0, F_{T-1}]$, $S_{T-1} \to 0$ as $F_{T-1} \to 0$. So F_T is continuous in $F_{T-1}$for all F_{T-1}. Because ϵ_T is a continuous random variable, q_T is the probability of $F_T < 0$, thus q_T is also continuous in F_{T-1}. According to equation (9), $MV_{T-1}(F_{T-1})$ is the upper envelope of two continuous functions, therefore it is also continuous in F_{T-1}. By iteration, $MV_t(F_t)$ is continuous in F_t for all t.

Proof of (c): Because $MV_T = P^s h$, $\partial MV_T/\partial F_{T-1} = 0$.

Given $F_{T-1} > 0$,

$$MV_{T-1}(F_{T-1}) = \max\left(P^s h, \frac{1}{1+r}[q_{T-1} P^b h + (1-q_{T-1})MV_T(F_T)]\right).$$

If $P^s h \geq \frac{1}{1+r}[q_{T-1} P^b h + (1-q_{T-1})MV_T]$, then $MV_{T-1}(F_{T-1}) = P^s h$, and therefore $\partial MV_{T-1}(F_{T-1})/\partial F_{T-1} = 0$.

If $P^s h < \frac{1}{1+r}[q_{T-1} P^b h + (1-q_{T-1})MV_{T-1}]$, then

$$\frac{\partial MV_{T-1}}{\partial F_{T-1}} = \frac{1}{1+r}(P^b h - MV_{T-1})\frac{\partial q_{T-1}}{\partial F_{T-1}} + \frac{1}{1+r}(1 - q_{t+\tau+1})\frac{\partial MV_T}{\partial F_{T-1}} \leq 0,$$

because $\partial q_t/\partial F_t \leq 0$ for all $t = 0,1,\dots,T$ and $\partial MV_T/\partial F_{T-1} = 0$. Given $\partial MV_{T-1}/\partial F_{T-1} \leq 0$, it can be shown iteratively that $\partial MV_t/\partial F_t \leq 0$ for all t$= 0, 1, \dots, T-1$.

Proposition 2 (Optimal policy): If $r > 0$ and $P^b > P^s(1+2r)$, there exists $F^* \in R^+$, such that the optimal policy is to hold onto the flexibility if $F_0 \leq F^*$ and to sell all the flexibility exceeding F^* if $F_0 \geq F^*$.

Proof. $\lim_{F_0 \to +\infty} q_1(F_0) = 0$, $\lim_{F_0 \to +\infty} MV_1(F_0) = P^s h$

$$\lim_{F_0 \to +\infty} \frac{1}{1+r}[q_1 P^b h + (1-q_1)MV_1] = \frac{P^s h}{1+r} < P^s h$$

Similarly, $\lim_{F_0 \to 0} q_1(F_0) = 0.5$, $\lim_{F_0 \to 0} MV_1(F_0) \geq P^s h$, and $P^b > P^s(1+2r)$,

$$\lim_{F_0 \to +\infty} \frac{1}{1+r}[q_1 P^b h + (1-q_1)MV_1] \geq \frac{P^b + P^s}{2(1+r)} h > P^s h$$

Because $MV_0(F_0)$ is continuous and monotonically non-increasing in F_0, there exists $F^* \in R$, such that

$$P^s h = \frac{1}{1+r}[q_1 P^b h + (1-q_1)MV_1(F^*)].$$

Therefore for all $F_0 \geq F^*$

$$P^s h \geq \frac{1}{1+r}[q_1 P^b h + (1-q_1)MV_1],$$

and it is optimal to sell h. In this case, optimal policy is $x_0^* = 1$, $MV_0(F_0) \geq P^s h$. For all $F_0 \leq F^*$, it is optimal to hold onto h. In this case, optimal policy is $x_0^* = 0$, $MV_0(F_0) = P^s h$.

The marginal value of operational flexibility is state contingent for any period t. If $F_t < 0$, BPA has no choice but is obliged to sell the h in order to meet the unsatisfied demand. The marginal value of operational flexibility is zero. If $F_t > 0$, BPA faces with the choice whether to sell the h in period t or hold on to the flexibility for future use. If $MV_t(F_t) > P^s h$, the operational flexibility generates additional value beyond the sales value of the marginal flexibility. Subtract $P^s h$ from Equation (9) and rearrange terms, we get:

$$(10)\quad MV_t(F_t) - P^s h = -\tfrac{r}{1+r} P^s h + \tfrac{1}{1+r}[q_{t+1}(P^b - P^s)h + (1-q_{t+1})(MV_{t+1} - P^s h)].$$

The tradeoff between current period revenue increase and increased future exposure to adverse shocks becomes evident. The first term is the opportunity cost of holding onto the marginal flexibility h: the revenue difference between selling in the current and the next period. The second term, the term in the square bracket, is the discounted value of holding h to the next period. If the marginal flexibility h were sold in the current period, the revenue would increase by $\frac{r}{1+r}P^s h$. However, this action would increase the exposure to future electricity shortages and on average incur a cost that equal $\frac{1}{1+r}[q_{t+1}(P^b - P^s)h + (1 - q_{t+1})(MV_{t+1} - P^s h)]$, which includes both the cost from failing to buffer shortage in the next period and cost from failing to buffer future shortages.

The difference, $MV_t(F_t) - P^s h$, increases monotonically with the probability of future shortage (q_{t+1}) and the cost of electricity shortage ($P^b - P^s$). It decreases with interest rate (r) and selling price (P^s). These properties are self-evident from Equations (9) and (10), and are consistent with intuitive reasoning. The ability to buffer future electricity shortages is more valuable if future shortages are more likely to occur and the damage of electricity shortage is larger. The ability to buffer future electricity shortage is less valuable if the cost of holding on to the flexibility is high, which is inversely related to the interest rate and the selling price. It is also important to highlight the other implication of these results: it is possible that flexibility adds no additional value on top of the sales revenue, i.e. $MV_t(F_t) = P^s h$. The irreversibility, future uncertainty and additional damage of adverse shock ($P^b > P^s$) are not sufficient condition for $MV_t(F_t) > P^s h$.

Value of Total Operational Flexibility and Optimal Sales

Given the marginal value of operational flexibility, the value of total operational flexibility is given by

(11) $$V_0(F_0) = \int_0^{F_0} MV_0(\xi_0) d\xi_0.$$

This value can be approximated by

(12) $$V_0(F_0) \approx \sum_{k=0}^{K} MV_0(F_0 - kh),$$

where $K = F_0/h$. Starting from the operational flexibility in the current period F_0, the marginal value of using the first h amount of flexibility can be determined using the formula in the previous subsection. Now consider the value of remaining flexibility $F_0 - h$. There are two possible scenarios. In the first case $MV_0(F_0) = P^s h$, it is optimal to sell the h amount of flexibility in the current period. The remaining flexibility is given by $F_0 - h$ and the future evolution of flexibility is updated according to Equation (4) with only the modification that the remaining flexibility is $F_0 - h$ instead of F_0. The sequence of future shocks $\{\epsilon_1, \epsilon_2, \dots \epsilon_T\}$ is unaffected by the sell of flexibility in the current period. The marginal value of the second h amount of flexibility can be calculated using the modified initial flexibility $F_0 - h$ and the unmodified sequence of future shocks $\{\epsilon_1, \dots, \epsilon_t, \dots, \epsilon_T\}$.

In the second case $MV_0(F_0) > P^s h$, it is optimal to hold on to the h amount of flexibility until some future period. Because water in the reservoirs can only go through the turbines once, the h amount of flexibility can only be used in one of the future periods, either to buffer the electricity shortage or generate revenue in that particular future period. Without loss of generality, we assume the h amount of flexibility is allocated to the period with the maximum expected revenue, that is,

$$\tau = \underset{t}{\operatorname{argmax}} \frac{[q_t P^b h + (1 - q_t) P^s h]}{(1 + r)^{t-1}}.$$

This implies that the h amount of flexibility is designated to buffer the shock in period ϵ_τ. Therefore, the shock in period τ is updated as

(13) $\qquad \epsilon'_\tau = \epsilon_\tau + h.$

In this second case, the marginal value of the second h amount of flexibility can be calculated using the updated initial flexibility $F_0 - h$ and the updated future shocks $\{\epsilon_1, \dots, \epsilon_{\tau-1}, \epsilon'_\tau, \epsilon_{\tau+1}, \dots \epsilon_T\}$.

The allocation of the first h amount of flexibility, either through the sales in the current period or the allocation to the future period τ, reduces the amount flexibility available to cope with remaining future uncertainties. As the remaining flexibility is reduced from F_0 to $F_0 - h$, the probability of future shortage is increased, as shown in equation (4). With the updated initial flexibility $F_0 - h$, the value of the second h amount of flexibility can be evaluated similarly. Equation (12) therefore defines the iterative procedure to calculate the value of total operational flexibility in Equation (11). The implementation process is illustrated in the following chart.

Algorithm: Valuation of Flexibility

Initialization:

(1) initial flexibility F_0,

(2) mean flexibility over time $\boldsymbol{F_\mu} = F_0 + [0,\ \boldsymbol{\mu}]$, where $\mu_t = E\epsilon_t$, $t = 1,2, \dots T$;

(3) std. of flexibility over time $\boldsymbol{F_\sigma} = [0, \boldsymbol{\sigma}]$, where $\sigma_t = std(\epsilon_t)$, $t = 1,2, \dots T$;

(4) $V(F_0) = 0$.

(5) Probability of power shortage:

$$q_t = cdf\big('normal', -F_\mu(t)/F_\sigma(t)\,, 0, 1\big), t = 1, 2, \dots T;\ q_0 = 0;$$

Valuation:

$x_T = 1$ (sell h in the last period);

$MV_T(F_T) = p^s h$ (revenue from selling h in the last period);

$K = floor(F_0/h)$;

for $k = 1{:}\,K$

for $t = T{:} -1{:}\, 0$

$$[MV_t(F_t),\ x_t] = \max\left(P^s h,\ \frac{1}{1+r}[q_{t+1}P^b h + (1 - q_{t+1})MV_{t+1}(F_t)]\right);$$

end

$V(F_0) = V(F_0) + MV_0(F_0)$;

$\tau = \underset{k}{\operatorname{argmax}}\, [q_{t+\mathrm{k}}P^b h + (1 - q_{t+\mathrm{k}})P^s h]/(1 + r)^{t+k-1}$;

$F_0 = F_0 - h;\ F_\mu = F_\mu - h$;

if $x_1 == 2,\ F_\mu(\tau) = F_\mu(\tau) + h$; end

end;

An Application

As an application, we apply the hydraulic routing model (Bashiri et al., 2017) that simulates real-time operational flexibility based on historical inflow data obtained from BPA. The distribution of F_t $(t = 0, 1, \dots, 14)$ is plotted in Figure 2. We normalize the flexibility, both the mean and the standard deviation, with the maximal flexibility in the simulation period. As shown in the figure, the mean value of flexibility, the solid line in Figure 2, grows in the early periods and decreases gradually from period five and onwards. This characterizes the transition period from the raining to dry season. The uncertainty about the flexibility is measured by the standard deviation of flexibility. Figure 2 plots the two standard deviations from the mean value with the dashed lines. The increase in the standard deviation over time reflects the increasing uncertainty in the prediction about the future. Both the means and the standard deviations are calibrated using the historical inflow data. As is evident from Figure 2, the likelihood of electricity shortages in the early periods are negligible due to the abundant precipitation in the raining season. However as time proceeds, it is likely that electricity shortages may occur toward the end of the planning period. The interest rate is assumed to be 2%, calibrated using overnight US Dollar LIBOR interest rate.[7] The price for selling hydro-electricity is normalized to be 1. The price for purchase is assumed to be 3, because the cost of power production by gas turbine is three times of the cost for hydropower.[8] Figure 3 shows that when h is sufficiently small, the value of flexibility (F_0) becomes insensitive to further decreases in h.

Before the discussion of the value of flexibility as defined in Equation (11) and the corresponding optimal policy, it is necessary to discuss the value of flexibility and the optimal policy when the use of flexibility is considered as a binary choice: either to use all of the flexibility F_0 or none at all. In this binary choice model, for the initial flexibility $F_0 = 0.74$, the value of flexibility $V(F_0) = 0.74$. In Figure 3, the point at the lower right corresponds to the case .$h = F_0$. The optimal policy is to sell all of the flexibility F_0. This result can be explained by the risk of power shortage illustrated in Figure 2. Essentially, if BPA chooses to hold on to F_0, the probability of power shortage is almost negligible, with the highest probability equal to 0.08 in period 13. As shown in Equation (9) with h replaced by F_0, the potential gain from holding on to the total flexibility F_0 has to be discounted and then compared to the foregone opportunity cost. Initially, holding on to the total flexibility (the second term in Equation 9) generates higher present value than the sales revenue (the first term in Equation 9). However, after 7 periods, the discount effect is strong enough that the sales revenue becomes larger. Therefore, it is better to sell all of the flexibility F_0 than none of it.

[7] https://www.global-rates.com/interest-rates/libor/american-dollar/usd-libor-interest-rate-overnight.aspx, accessed Sept. 6, 2019.
[8] US Energy Information Administration, https://www.eia.gov/electricity/annual/html/epa_08_04.html, accessed Sept. 6, 2019.

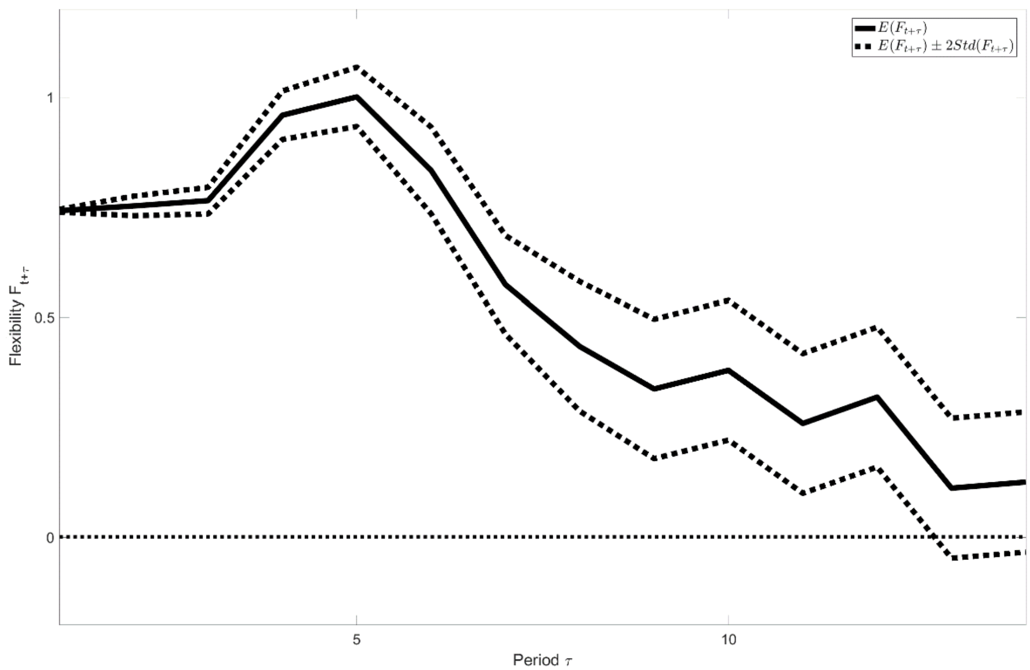


Figure 2. The distribution of future flexibility F_t $(t = 0, 1, \ldots, 14)$.

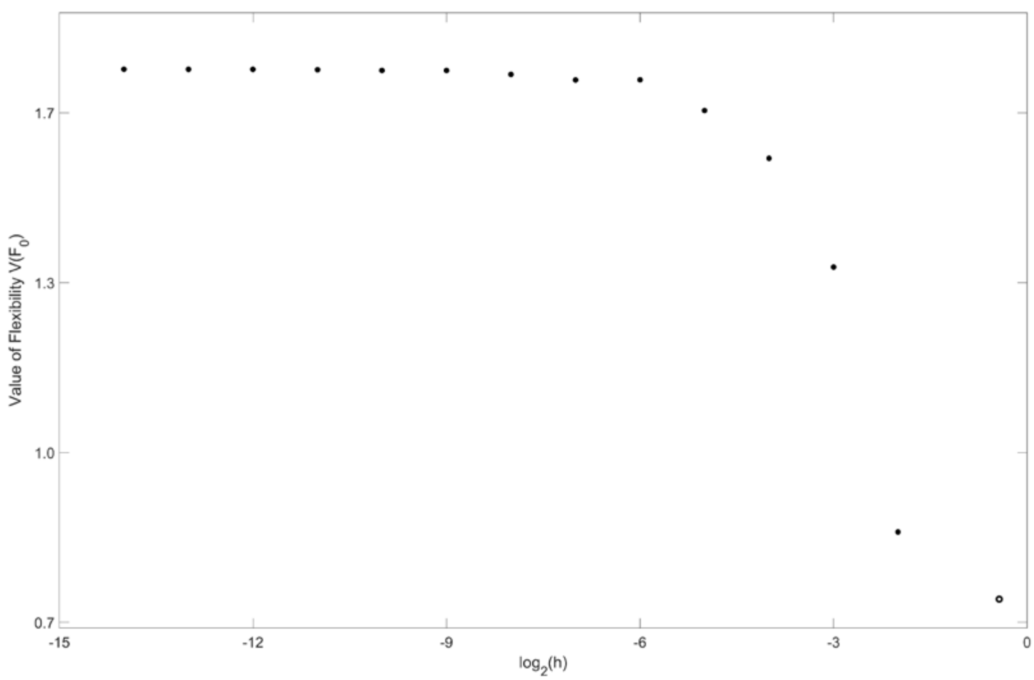


Figure 3. The value of flexibility and the choice of h.

If partial usage of the flexibility is allowed, as proposed in this paper, both the value of flexibility and the optimal policy can be very different from those in the binary choice model. In fact, we find that $F^* \approx 0.73$ when partial usage of flexibility is allowed. Despite the fact that the expected flexibility remains positive for all periods in Figure 2, it is wise to store a significant amount of flexibility to buffer future electricity shortages. When $F_0 > F^*$, BPA sells the surplus $F_0 - F^*$ and hold on to the remaining F^*. For example, in the case $F_0 = 0.74$, BPA sells only 0.01 of the flexibility, which is almost opposite to the optimal policy to hold onto the flexibility in the binary choice model. When $F_0 > F^*$, the value of flexibility $V(F_0)$ in the continuous choice model increases linearly with flexibility F_0 with the slope equal to the sale price, $MV(F_0) = p^s$, as illustrated by the solid line in Figure 4. For the initial flexibility $F_0 = 0.74$, the value of the flexibility is $V(F_0) = 1.67$, more than twice the sales value of 0.74. When $F_0 \leq F^*$, BPA sells nothing in the current period, holding on to whatever it has. As shown in Figure 4, when the initial flexibility is below F^*, the marginal value of flexibility $MV(F_0)$ is higher than the sales price, because a decrease in flexibility for $F_0 < F^*$ not only reduces the sales revenue but also increases the likelihood of future electricity shortage. However, the slope of $V(F_0)$ always stays below the per unit purchase price ($p^b = 3$), because the probability of electricity shortage is less than one and because of the discount of future values.

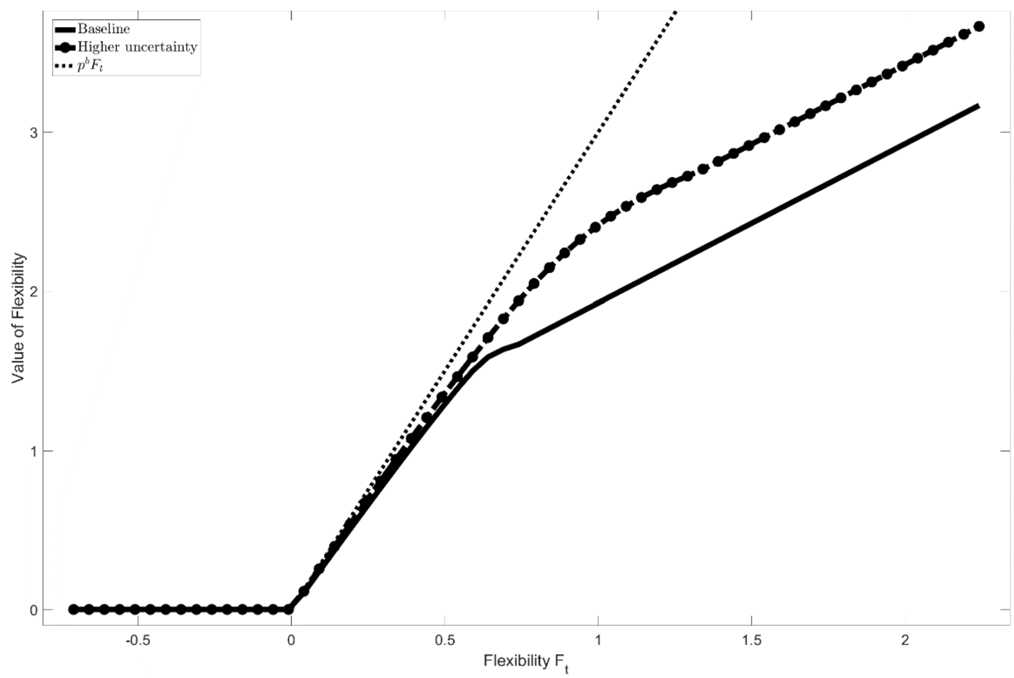


Figure 4. Value of flexibility and the impact of uncertainty.

We also vary the uncertainty in the flexibility. Under normal distribution of flexibility $N(\mu_F, \sigma_F)$, the probability of shortage is given by $Prob(F_t \leq -\mu_F/\sigma_F) = \Phi(-\mu_F/\sigma_F)$. As σ_F increases, the impact on the probability of electricity shortage hinges on the sign of μ_F. If $\mu_F > 0$, an increase in σ_F increases the probability of electricity shortage. If $\mu_F < 0$, the effect is the opposite. In the baseline case, $E(F_t) > 0$ for all τ (See Figure 2). Further increases in σ_F monotonically increase the probability of electricity shortage, the value of flexibility increases accordingly. Moreover, because of the increase in future electricity shortages, the threshold level of flexibility F^* also increases. Figure 4 plots the $V(F_0)$ with the increased uncertainty as a broken line which lies above the $V(F_0)$ in the baseline case. To further illustrate the relationship between $V(F_0)$ and the uncertainty, we first change only the uncertainty in the last period σ_{F_T} and calculate the corresponding $V(F_0)$. The results are plotted as the solid line in Figure 5, which exhibit positive monotonicity.

Because the theory predicts that the uncertainty increase has the opposite effect if $\mu_F < 0$, we then modify the distribution of $\{F_t\}_{t=0}^{T}$ so that the expected flexibility in the last period is negative, i.e., $\mu_{F_T} < 0$. The distribution in all other periods are unchanged. This is equivalent to the scenario in which demand is expected to increase or inflow is expected to decrease dramatically in the last period so that the expected flexibility in the last period becomes negative. In this case, increasing uncertainty (σ_{F_T}) decreases the probability of electricity shortage. As a result, the value of flexibility decreases with uncertainty as illustrated by the downward sloping broken line in Figure 5. .

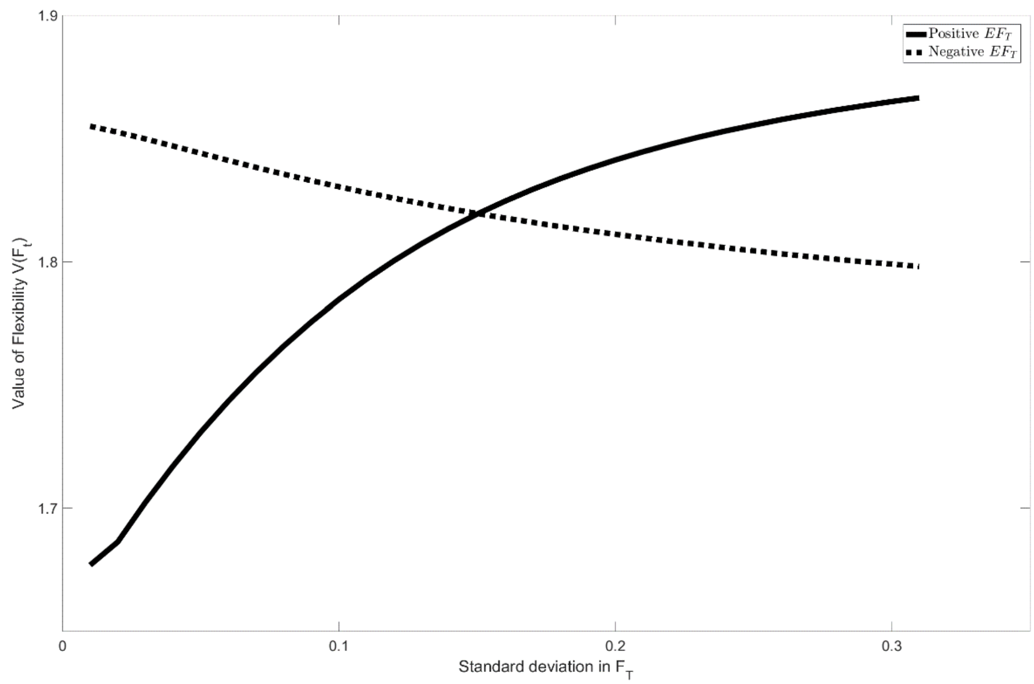


Figure 5. The impact of uncertainty on the value of flexibility.

The Wind Vision of the U.S. Department of Energy (DOE) is to have 35% wind energy by 2050 (NREL, 2018). Currently, wind power accounts for around 7.7% of the total electricity generated in the BPA control area. To simulate the impact of the wind vision, we assume the flexibility (F_t)

is increased by 27.3% and the increase is only through wind energy.[9] Because wind energy generation is more volatile than hydropower generation, to approximate the uncertainty in the wind generation, we use 2018 "total wind generation in BPA control area (5-mintue increments)" to calculate its standard error. Even though the amount of wind energy varies from month to month, the magnitude of the standard error is about the same size as the monthly mean. We therefore increase both the mean and the standard error of the flexibility by 0.273. While the increase in wind energy increase the capacity of electricity supply on average, it also increases the uncertainty in the system. In the baseline case, the increase of uncertainty due to wind energy results in an increased likelihood of future electricity shortage. This pushes up the value of flexibility $V(F_0)$ as shown in Figure 4. To cope with the increased future uncertainty, BPA has to increase the holding of current flexibility. The threshold value F^* increases from 0.73 to 1.21. The increase in the additional electricity generation capacity is embodied in the increase of the expected flexibility, which moves the value of flexibility $V(F_0)$ rightward along the broken line in Figure 4. Therefore, the increasing share of wind energy may lead to temporary reduction in the energy supply because of the increase future uncertainties. The actual impact on the use of flexibility will depend on the seasonal changes in the future distribution of flexibility. But it is possible that in spite of the increase electricity generation capacity due to the installation of wind power plants, BPA chooses to decrease the electricity supply so as to have larger reserves to buffer future uncertainties. As a major electricity provider in the northwest, this may drive up the electricity price in the short run.

Conclusion

In this paper, we apply the real option theory to a model with continuous choice variable: the optimal management of hydroelectric production. While the proposed theoretical framework focuses on hydroelectric production, the underlying tradeoff between current reward and irreversible change on future uncertainty is relevant to many other resource and environmental issues. In our valuation of the operational flexibility, the right but not the obligation to buffer future electricity shortages, we focus on the uncertainty in quantities. We have deliberately abstracted away from many other sources of uncertainties, like price uncertainties, which are interesting and important topic for future research.

With the proposed framework, we are able to quantify the value of the operational flexibility and analyze its properties. The value of operational flexibility can be a substantial part of the value of water in the reservoirs, especially with substantial likelihood of future shortages. Therefore, it is important for BPA and PMAs in general to consider the value of operational flexibility in their trading and pricing of hydroelectricity. We also show that under fairly general context, the optimal management strategy for BPA is a simple triggering policy, to store flexibility up to some threshold level, or triggering level. If the flexibility is above the triggering level, it is optimal to sell the surplus. Finally, we show that implementation of the wind vision of DOE

[9] In reality, the target is 33% of the total energy generation. In the simulated case, we assume 33% of the flexibility, the total energy generation minus the scheduled demand. In this sense, this simulation underestimates the impact of fulfilling the Wind Vision.

could potentially lead to temporary reduction in the electricity supply and an increase in the price because of the increased uncertainty associated with the wind energy.

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