# Flexible Decision Variables in Multi-Objective Reservoir Operation

Parnian Hosseini[a], Nathan, L.Gibson[a] , Duan Chen [b] and Arturo S. Leon[c]

[a] Oregon State University, Corvallis, OR, USA;[b] Changjiang River Scientific Research Institute, China; [c] Florida International University, Miami, FL, USA

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**ABSTRACT**
This study explores optimal control in the case when certain input uncertainties cannot be well quantified. In these scenarios the decision maker prefers to have the most flexibility, which is defined to be the largest range of options for decision variables, and still achieve the objectives of the operation while satisfying all constraints.

Each decision variable is generalized to be a range of potential actions. These ranges are modeled with random variables, thus uncertainty quantification techniques are employed to compute expected values of objectives and probabilities of chance constraints. The proposed framework determines optimal probability densities for the decision variables by treating the amount of flexibility as an additional objective. There is a clear trade-off between the amount of flexibility that can be allowed and the resulting expected values of the other objectives. A dimension reduction technique is employed to ensure a reasonable dimension for the search space.

**KEYWORDS**
Flexibility of decision variables; Multi-objective optimal control; Optimization; Reservoir operation

## 1. Introduction

Many important management and control problems in applications can be phrased as multi-objective, constrained optimization under uncertainty problems. Sources of uncertainty are classified as: knowledge deficiency or natural variability. Uncertainties caused by knowledge deficiency can be decreased by gathering more data. To mitigate the effects of uncertainties due to natural variability, robust optimization can be implemented to find solutions that are less sensitive to small deviations in inputs. In order to consider the potential implications of additional uncertainties which are unaccounted for, as yet unknown, or even impossible to account for, the decision making problem may be recast into a bi-level optimization with an upper level including all quantifiable uncertainties, and the lower level using the upper level solutions as constraints. For example, in hydropower planning, reasonable forecasts for water inflow exist for the short term (e.g., 48 hours), but the scheduling problem for this time frame depends on wind power generation and power demands, which can fluctuate significantly [14, 27].

We further assume that the upper level optimization is too time consuming to be solved on a short time scale. In this case, a fixed point approach cannot be applied. Decisions need to be made in real time with very short term forecasts (e.g., two hours).

CONTACT Nathan L. Gibson. Email: gibsonn@oregonstate.edu

One common approach is to solve the planning optimization problem in advance, and use this solution to guide the scheduling problem. In this context, a scheduling operator would want sufficient flexibility in decision making as to be able to adjust the control in order to accommodate an unforeseen realized event. We focus here on the upper level optimization problem. The main contribution of this paper is a precise mathematical formulation of this upper level optimization problem using a novel definition of flexibility.

### *1.1. Framework*

We consider the potential implications of additional uncertainties which are unaccounted for, or even impossible to account for. In this context, an operator would want sufficient flexibility in decision making as to be able to adjust the control in order to accommodate an unforeseen realized event. We model the flexibility in decisions explicitly as a range of potential actions represented by random variables. We note that this approach is different from the common use of random variables to model uncertainty associated with system parameters or inputs. In fact, our approach is an indirect way of accounting for unquantifiable input uncertainty. Further, instead of minimizing uncertainties, we maximize the variance of the distributions in order to provide the largest possible flexibility of controls. Specifically, the amount of flexibility becomes an additional objective. Also, the constraints become probabilistic with respect to the randomness in the decision variables. The decision variable is a stochastic process, or after a suitable discretization, a random vector. Thus we seek to find the optimal probability distribution function for feasible decisions.

### *1.2. Application*

We explore the application of this framework to the particular problem of optimal hydropower reservoir operation. Some sources of uncertainty are not well predicted or quantified. For short-term hydropower planning, with a typical window period between one and two weeks, hydropower demand and wind power production are significant sources of uncertainty [14, 27]. Additionally, unexpected transmission disruptions can occur suddenly [22]. The importance of considering flexibility in short-term optimization of hydropower systems was also discussed by [16]. Therefore, the operator may need to modify some of the assumed optimal decision variables in response to the actual varying circumstances. These modifications may result in violations of some constraints or deterioration of some objectives from their predicted optimal values.

A flexible framework will provide the operator with a range of *nearly optimal* decisions which are *highly likely* to be feasible. Reservoir operation is concerned with how to operate and release water from a reservoir system over time to maximize the goals and benefits, such as hydropower production, while satisfying water demands (e.g., irrigation). Most of these objectives are conflicting [17] thus, an optimal trade-off of solutions must be found. Optimization of reservoir operation under uncertainty was investigated in [21]. Consideration of input uncertainty was addressed by several studies (e.g., [8–10, 15, 18]). Robust optimization treating decision variability as the source of uncertainty was studied [2], however, the variability was decreased to ensure robustness, which is contrary to the approach presented in this study.

Flexibility in decision making is a fairly recent approach in water resources management problems. The importance of flexibility in decision making and the consideration of decision maker preference in the trade-off between performance and robustness, has been discussed extensively [7, 12, 19]. A more complete review of the literature can be found in [13]. A methodology for a flexible design in water distribution systems was formulated by [1]. They represented possible future demand scenarios by random variables. Flexible designs were also studied by [19]. They used a real option methodology to find flexible design variables in a water distribution network. In both of these studies, a decision tree was used to represent different possible outcomes that might occur. For each of these outcomes, a probability of occurrence was assigned based on expert opinion or data. Certain scenarios for realizations that may happen were considered, and the optimal design for each scenario was found, but only a few scenarios were examined. Moreover, they evaluated constraints discretely. The proposed method in this paper attempts to find a range of options for each decision variable and the constraints are calculated in a continuous probability space.

We note that the flexibility concept proposed in this paper is not the same as finding multiple optimal solutions for a multi-objective optimization problem (called non-dominated, non-inferior or Pareto solutions). In fact flexible decision variables can be found for each of the Pareto solutions in a multi-objective optimization problem. It is also different from sensitivity analysis in which the variation of an objective due to uncertainty of decision variables is studied. Interval arithmetic [11] is another method to consider the effect of a prescribed interval for each input. Unlike these methods, the goal of the proposed approach is to find feasible ranges for decision variables by optimization.

The main contributions of this paper are the development of a framework for incorporating flexibility in decision variables within constrained optimization problems, and the assessment of the influence of flexible decision variables on the resulting Pareto-optimal solutions of multi-objective optimization problems. The main ideas of the methodology are introduced in Section 2. In Section 3.1 we demonstrate the effectiveness of the approach on a test problem. A genetic algorithm (NSGA-II) [6] is used to determine the Pareto frontier for the multi-objective optimization problem. As the use of a random variable for each time step results in a very large dimension for the search space, in Section 4, we describe the application of dimension reduction techniques to arrive at a tractable problem to which we then apply our methodology. Finally, conclusions of the approach are given in Section 5.

## 2. Flexible Decisions at Discrete Times

We consider the following bound constrained, multi-objective (multi-criteria) optimization problem for decision $x(t)$:

$$\min_{x\in\mathcal{X}} \qquad f_1(x) \text{ and } f_2(x) \text{ ... and } f_M(x) \tag{1}$$

$$\text{subject to} \quad a(t) \le x(t) \le b(t), \quad \forall t \in [0,T]$$

where $\mathcal{X} = C(0,T)$, each objective function $f_i : \mathcal{X} \to \mathbb{R}$ for $i = 1,\dots,M$, and $a$ and $b$ are lower and upper bounds, respectively, for $x$.

In order to incorporate flexibility in decision making, we model $x$ as a range of options, i.e., a stochastic process $\xi(t,\omega)$, with $\omega \in \Omega$, where $\Omega$ is a sample space.

We then attempt to maximize the some measure (possibly weighted) of the standard deviation of $\xi$, $\sigma(t)$, over the fixed time interval in order to provide the largest amount of flexibility possible (or equivalently, minimize the negative of the variance).

We note that the bound constraints on the controls become chance constraints with respect to the probability distribution of the flexibility, however we exclude infeasible controls from our flexible options by restricting to include only distributions with bounded support within the feasible space. However, in Section 3.3, we also include system constraints which become chance constraints. These are enforced by setting a threshold for the probability of failure. Optimal solutions with probabilities of failure less than the threshold, e.g., "likely feasible", are still referred to as "feasible" for convenience.

The bound constrained, $M$-objective optimization problem above for an optimal function $x$ is replaced by a bound constrained, $(M+1)$-objective optimization problem for an *optimal probability distribution* $F$, e.g., (1) is replaced by

$$\min_{F\in\mathcal{F}} \quad \mathbb{E}_F[f_1(\xi(\omega))] \text{ and } \mathbb{E}_F[f_2(\xi(\omega))] \text{ ... and } \mathbb{E}_F[f_M(\xi(\omega))] \text{ and } -\|\sigma(t)\| \tag{2}$$

where $\mathcal{F}$ represents the space of all probability distributions for $\xi$ with support within the feasible set and $\mathbb{E}_F$ is the expected value. The objectives $f_1$ through $f_M$ are the objectives for the non-flexible multi-objective formulation, and $\|\sigma(t)\|$ some measure of the standard deviation of $\xi$, e.g., an $L_2$ norm over the time interval.

In order to solve the optimization problem numerically, we discretize in time. The process $\xi(t)$ can be approximated by $\vec{\xi} = [\xi_1, \ldots, \xi_{Nd}]$ at discrete times $[t_i]_{i=1}^{Nd}$, where $\xi_i$ are assumed independent, specifically, here we let $\xi_i \sim \mathcal{U}[l_i, u_i]$. While the independence assumption ignores auto-correlation of the process, which may be justifiable for large time steps, we fix this deficiency below in Section 4. We define the vector of upper and lower bounds by $\vec{u}$ and $\vec{l}$, respectively, and means and standard deviations by $\vec{\mu}$ and $\vec{\sigma}$. The third objective in (2) becomes $\|\vec{\sigma}\|_2$. To describe the joint probability distribution for $\vec{\xi}$, it is sufficient to determine the lower and upper bounds of each random variable. This doubles the number of decisions in the optimization problem over the non-flexible approach. The infinite dimensional optimization problem over $\mathcal{F}$ in (2) is now replaced by a finite dimensional optimization problem for the parameters of a joint probability distribution of a random vector. With the understanding that we have restricted the search space, we will still refer to solutions of the finite dimensional optimization problem as "optimal probability distributions". It is important to clarify that while the distribution of controls may be optimal, any given sample from the distribution is very likely not. The main point of the approach is that the optimal probability distribution should contain many *nearly optimal* solutions which are all *likely feasible.*

We note here that the choice of a uniform distribution of decisions is in some sense like an uninformative prior distribution on the set of nearly optimal controls returned by the procedure. The only thing that it imposes is a bound on the control values. Any other compactly supported distribution (e.g., beta) could also be used if preference needs to be given to certain control values (e.g., midpoints). These random variables form a basis for representing the set of decisions that could potentially be chosen by a decision maker.

To compute the expectations of objectives, the Stochastic Collocation (SC) method is used, which deterministically samples the random variables at strategically chosen points [10, 23]. SC is non-intrusive, which means that it can be applied to complex systems without requiring the systems to be augmented or modified in any way. See [24] for detailed description and analysis of SC, including its relation to standard Gaussian quadrature. A set of collocation points in each dimension is chosen as the roots of an appropriate orthogonal polynomial. The complete set of sample points is the tensor product of these sets. The tensor product, in this context, is all possible combinations of the sets of collocation points in all dimensions [25]. For higher random dimensions, one would use a sparse grid subset of these collocation points for efficiency [24]. The expectation is then calculated with weighted Gaussian quadrature on the collocation points [18].

In order to ensure that the optimal solutions are feasible, we calculate the probability of failure (PF) of each constraint [20]. Rather than sampling the system again for the purpose of computing PF, we recycle the samples we already have to build a polynomial interpolant. Then we use this polynomial as a surrogate function which is much cheaper to sample. Specifically, a polynomial surrogate of each constraint is constructed using the constraint values already calculated at the collocation points, and then these surrogates are sampled to determine the probability of failure efficiently [10].

## 3. Simple test problems with flexible decision variables

In this study the concept of flexibility of decision variables is discussed and tested on simple test problem and then is extended to a simplified reservoir problem which is the main test case of this study.

### *3.1. Test 1: Single-Objective Mathematical Test Problem*

The goal is to find the optimal values with flexible decision variables for a quadratic function using the proposed methodology. We start with the following deterministic problem.

Problem 1A: Find $\vec{x}$, in order to

$$\text{Minimize} \quad f(x) = \sum_{i=1}^{m}(x_i^2) \tag{3}$$

$$\text{subject to} \quad 0 \leq x_i \leq 1 \quad \text{for all} \quad i$$

where $x_i$ is the $i^{th}$ decision variable and $m$ is the number of decision variables of the quadratic function. The known deterministic optimal decision variables and the resulting optimal objectives are $x_i = 0$ and $f(x_i) = 0$, respectively.

To find the optimal flexible decision variables, the bounds of the uniform random variables representing each decision variable are found by optimization. We state this as the following problem:

Problem 1B: Find $\vec{l}$ and $\vec{u}$, where $\xi_i \in [l_i, u_i]$, in order to

$$\text{Minimize} \quad \mathbb{E}[f_1(\tilde{\xi})] = \mathbb{E}[\sum_{i=1}^{m}(\xi_i)^2] \tag{4}$$

$$\text{Maximize} \quad f_2(\tilde{\xi}) = ||\tilde{\sigma}|| \tag{5}$$

$$\text{subject to} \quad 0 \leq l_i \leq u_i \leq 1, \quad \text{for all} \quad i.$$

We demonstrate the approach on the test problem for $m = 2$. The optimal results for two conflicting objectives can be demonstrated as a trade-off. The first objective is the expected value of the quadratic function of the random variables (4) and the second objective is the norm of the standard deviations of the random variables, which represents the flexibility of each Pareto solution calculated by (5).

We note that these numerical results match well with the analytical solution of $\mathbb{E}[f_1] = 4f_2^2$ [13] since

$$\mathbb{E}[f_1] = \sum_{i=1}^{m} \mathbb{E}[\xi_i^2] = \sum_{i=1}^{m} \left(\sigma_i^2 + \mu_i^2\right) = f_2^2 + \sum_{i=1}^{m} \mu_i^2 \tag{6}$$

and the constraints on $l_i$ and $u_i$ imply that $\mu_i \geq \sqrt{12}\sigma_i/2$, i.e., $\mu_i^2 \geq 3\sigma_i^2$. Thus minimizing $\mathbb{E}[f_1]$ requires $\mu_i^2 = 3\sigma_i^2$. For example, $(l_i, u_i) = (0, 0)$ gives $(\mathbb{E}[f_1], f_2) = (0, 0)$ and minimizes $\mathbb{E}[f_1]$, while $(l_i, u_i) = (0, 1)$ gives $(\mathbb{E}[f_1], f_2) = (2/3, 1/6)$ and maximizes $f_2 = ||\vec{\sigma}||$. Each $(\mathbb{E}[f_1], f_2)$ value in between is attainable.

In this first test problem, the genetic algorithm population size is 20 and the maximum number of generations is 10000. The comparison of the deterministic solution of the quadratic function (Problem 1A) and the Pareto solutions with flexible decision variables (Problem 1B) shows that the higher the flexibility of one scenario, the farther it will deviate from the deterministic optimal solution (Figure 1 A). The solutions with warmer colors (light colors in gray scale), have more flexibility in decision variables while the expected objective values are higher than the deterministic minimum solution. The flexible decision variables (the mean and the range of options) corresponding to each of the Pareto solutions, demonstrate that the ranges of options are greater for the solutions with warmer colors (light colors in gray scale) (Figure 1 C, D). Due to randomness of the flexible decision variables, the value of the first objective may vary from its mean value, as is shown by horizontal lines in Figure 1 B.

### *3.2.* Test 2: Multi-Objective Mathematical Test Problem

The multi-objective test problem ZDT1 suggested by [28] is considered. The reason for choosing this specific test problem is the similarity of its objective functions with the reservoir problem test case (Test 3). ZDT1 has two objectives that should be minimized and the Pareto is convex. We state the deterministic (non-flexible) test problem as follows:

Problem 2A: Find $\vec{x}$ in order to

$$\text{Minimize} \quad f_1(x) = x_1, \tag{7}$$

$$\text{Minimize} \quad f_2(x) = g(x)[1 - \sqrt{x_1/g(x)}], \tag{8}$$

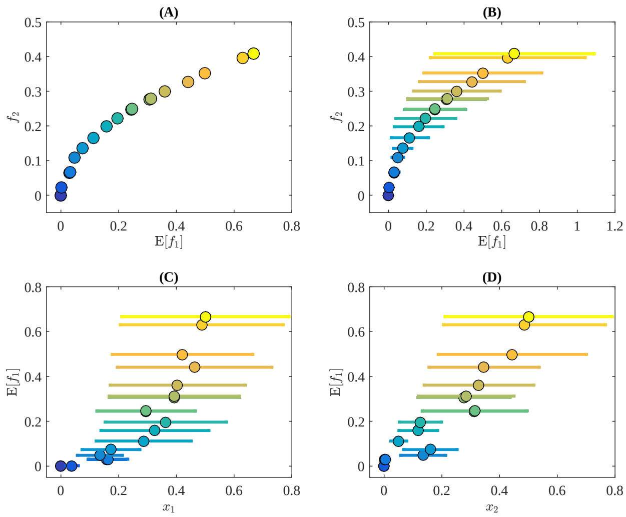


**Figure 1.** Optimal solutions for single-objective test problem with two decision variables A) Pareto solutions; B) The range objective due to flexibility; C) Flexible bounds for $x_1$; and D) Flexible bounds for $x_2$ (Note that these ranges are determined by the standard deviations, $(\mu_i \pm \sigma_i)$, these are not the entire range $[l_i, u_i]$)

$$g(x) = 1 + 9(\sum_{i=2}^{m} x_i)/(m-1) \tag{9}$$

$$\text{subject to} \quad 0 \leq x_i \leq 1, \quad \text{for} \quad \text{all} \quad i.$$

To find flexible decision variables for ZDT1 problem, with an additional objective for flexibility, the problem becomes:

Problem 2B: Find $\vec{l}$ and $\vec{u}$, where $\xi_i \in [l_i, u_i]$, in order to

$$\text{Minimize} \quad E[f_1(\tilde{\xi})] = E[\xi_1], \tag{10}$$

$$\text{Minimize} \quad E[f_2(\tilde{\xi})] = E\left[g(\tilde{\xi})[1 - \sqrt{\xi_1/g(\tilde{\xi})}]\right], \tag{11}$$

$$\text{Maximize} \quad f_3(\tilde{\xi}) = ||\tilde{\sigma}|| \tag{12}$$

$$\text{subject to} \quad 0 \leq l_i \leq u_i \leq 1, \quad \text{for} \quad \text{all} \quad i.$$

The 3D surface shown in Figure 2 A is created from the Pareto solutions (solid dots) for visualization purposes. The closer the flexible solutions are to the deterministic Pareto solutions (Figure 2 B), the smaller are the ranges of flexible decision variables. For example, solutions shown with warmer colors are closer to the deterministic Pareto solutions and the decision variables have less flexibility (Figure 2 C, D); to have more flexibility the solutions shown with green color, have wider range of options in both decision variables, however the corresponding Pareto solutions are dominated

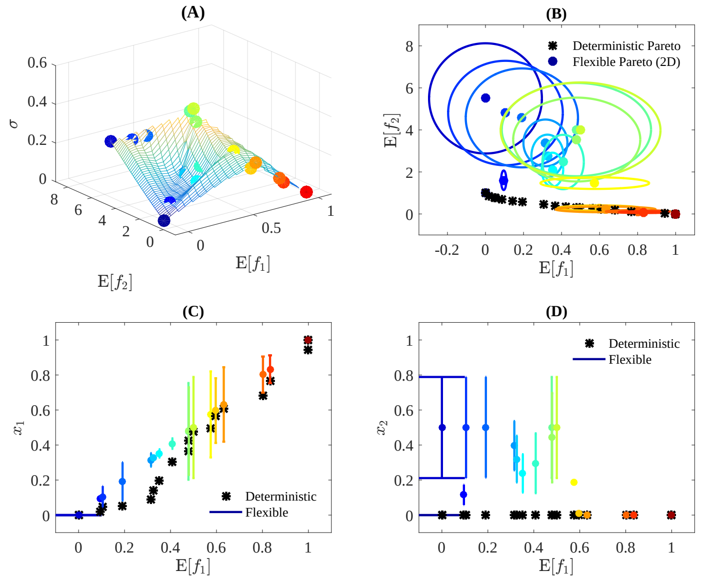


**Figure 2.** Comparing scenarios for multi-objective test problem with two decision variables for deterministic and optimal solutions with flexible decision variables A)3D Pareto solutions B) Comparison of deterministic scenarios and the flexible solutions in 2D; The range of changes of the objective due to flexibility is shown with ellipses; C) Flexible bounds for $x_1$ and the deterministic $x_1$; and D) Flexible bounds for $x_2$ and the deterministic $x_2$

by the deterministic Pareto solutions. Optimizing three objectives in Problem 2B is the reason for the difference in the shape of Pareto solutions in Problem 2A and 2B (Figure 2 B). The decision makers can make their decisions based on the flexibility for each decision variable (Figure 2 C, D) and the corresponding Pareto solution and its variation (Figure 2 B). Each Pareto solution value in objective space corresponds to the expected value of the objectives with respect to the flexibility of the decision variables. Both objective values may vary, and the standard deviation of each objective is depicted by the ellipse radius in each dimension in Figure 2 B.

### *3.3.* Test 3: Reservoir Operation Problem

A simplified model of the Grand Coulee reservoir on the Columbia River is used as a test case, which is based on the problem studied by [3, 4]. The desirable decision variables are the daily turbine outflows. To simplify the reservoir test problem, a period of two days was considered. Minimizing the deviation of forebay elevation (water surface elevation at the dam) at the end of the optimization period is the first objective. This objective is required to maintain consistency with long-term planning. To further restrict the deviation of the forebay elevation at the end of the optimization period, this deviation is constrained as well. The second objective is to maximize the revenue due to hydropower generation. The difference of generated hydropower and demand is called net electricity. The net electricity multiplied by the hydropower price determines the revenue at each time-step and the summation of the produced revenue at all time-steps is considered as the second objective value. The daily hydropower price for this period is considered deterministic and is pre-determined by an economic model [3].

The non-flexible and flexible problems are stated as follows:
Problem 3A: Find $\vec{Q} = [Q_n]_{n=1}^{N_t}$ in order to

$$\text{Minimize} \quad f_1(\tilde{Q}) = \frac{(\left|FB_{end}(\tilde{Q}) - FB_{target}\right|)}{U_r - L_r}, \text{and} \tag{13}$$

$$\text{Minimize} \quad f_2(\tilde{Q}) = -\left(\frac{\sum_{n=1}^{N_t}(PG_n(\tilde{Q}) - PL_n) * Pr_n}{\sum_{n=1}^{N_t}(Pr_n * PL_n)}\right) \tag{14}$$

$$\text{subject} \quad \text{to} \quad Q^{min} \leq Q_n \leq Q^{max} \quad \text{for} \quad \text{all} \quad n$$

$$\text{subject} \quad \text{to} \quad f_1(\tilde{Q}) \leq \delta FB$$

Problem 3B: Find $[l_n]_{n=1}^{N_t}$ and $[u_n]_{n=1}^{N_t}$, in order to

$$\text{Minimize} \quad \mathbb{E}[f_1(\tilde{\xi})], \text{and} \tag{15}$$

$$\text{Minimize} \quad \mathbb{E}[f_2(\tilde{\xi})], \text{and} \tag{16}$$

$$\text{Maximize} \quad f_3(\tilde{\xi}) = ||\tilde{\sigma}|| \tag{17}$$

$$\text{subject} \quad \text{to} \quad Q^{min} \leq \xi_n \leq Q^{max} \quad \text{for} \quad \text{all} \quad n$$

$$\text{subject} \quad \text{to} \quad PF(C) \leq \alpha, \quad C : f_1(\tilde{\xi}) \leq \delta FB$$

where $\vec{\xi} \in [l_n, u_n]$ and is the set of random variables representing the decision variables, $FB_{end}$ is the forebay elevation at the end of the optimization period, which depends on the turbine outflows ($Q_n$) at times $t_n$, $FB_{target}$ is the desired forebay elevation at the end of optimization period, which is pre-determined for each reservoir ($\delta FB$), $U_r$ and $L_r$ are the maximum and minimum allowable reservoir's forebay elevation, $PL_n$ is the hydropower demand (load), $Pr_n$ is the price of hydropower and $N_t$ is the number of time-steps in the optimization. The term $PG_n = \gamma \sum_{n=1}^{N_t} Q_n H_n$ is the hydropower produced in $n^{th}$ time-step, where $\gamma$ is the efficiency and $H_n$ is related to the reservoir water head at time $n$ [26]. The reservoir storage is calculated by continuity equation and the flow balance is considered in the model as a function of the decision variables through the mass conservation equation.

$$V_{n+1} - V_n = [(Q_n^{in} + Q_{n+1}^{in})/2 - (Q_n^{out} + Q_{n+1}^{out})/2]\Delta t \tag{18}$$

where $V_n$ represents the reservoir storage at time $t_n$, $Q_n^{in}$ represents the inflow to the reservoir, which is a model input, and $Q_n^{out}$ is the outflow from the reservoir, which is the decision variable of the optimization problem. It is assumed that the water losses due to evaporation are negligible due to short-term period of optimization problem [5]. Storage levels at the reservoir at the end of each decision period are treated as constraints. The hydropower is then computed using the decision variables (i.e., outflows) and the reservoir water level. The reservoir operation is being optimized over the specified period with all the decision variables (in all the time-steps), simultaneously.

For convenience, the operational objectives of the optimization are formulated as a minimization. Therefore, the second objective is formulated as a minimization of revenue loss. The constraints of this problem are designed to maintain the outflows within the allowable boundaries ($[Q^{min}, Q^{max}] = [30, 290]$ kcfs for this problem).

For problem 3B, $\vec{\sigma}$ is the set of standard deviation of the decision variables. The probability of failure ($PF$) of the constraint is calculated and the solutions with $PF$ less than an allowable failure threshold ($\alpha$) are considered as feasible solutions. The value of $\alpha$ is chosen based on the risk attitude of the decision maker.

In this second test problem, the genetic algorithm population size is 20 and the maximum number of generations is 10000. A surface is created using the Pareto solutions in 3D (Figure 3 A) which demonstrates the spread of Pareto solutions for the 3 objectives of the optimization problem. However, this surface is not meant to be the exact representation of the Pareto surface, merely an interpolation. Each control variable's standard deviation is bounded by the maximum and minimum constraints, specifically, the maximum standard deviation can be almost 75 kcfs (in a Uniform distribution, $\sigma = \frac{u-l}{\sqrt{12}}$). Therefore, in Figure 3 A, the maximum flexibility occurs when both control variables have the maximum allowable standard deviation.

Figure 3 B shows that the Pareto solutions corresponding to flexible decision variables are *dominated* by the deterministic Pareto solutions. In other words, each solution on the deterministic Pareto is better than the flexible solutions at least for one of the objectives. As is expected, the objective values are sacrificed (in comparison to deterministic Pareto solutions) to some extent in order to allow flexibility in the decision variables. Moreover, the variations of the objectives due to random behavior of the flexible decisions are more restricted in comparison to the objective variations in Test 1 . The complex structure of the reservoir operation problem and the impact of the higher number of non-linear constraints may be the reason that the Pareto solutions are less different from the deterministic Pareto solutions. Each of the flexible Pareto optimal solutions can be desirable based on the decision maker's preference. The solutions with higher flexibility (e.g., solution with green color in Figure 3) may be preferred by a certain decision maker over the solutions with lower forebay elevation deviation or even lower revenue losses. All of the solutions are assured to have less than 1% probability of failure of their constraints.

Due to high computational cost, this methodology can be suitable for short-term operation of reservoirs with a small number of decision variables (e.g., 6) as the number of function evaluations increases exponentially with the number of decision variables. For example, the solutions depicted in Figure 2 required approximately 8 hours on a workstation and involved only two time steps. In contrast, the corresponding solutions in Section 4 involved 14 times steps but took only approximately 4 hours. Using a reduced basis representation allows improved scalibility as well as arbitrarily fine temporal resolution, and is described in the next Section.

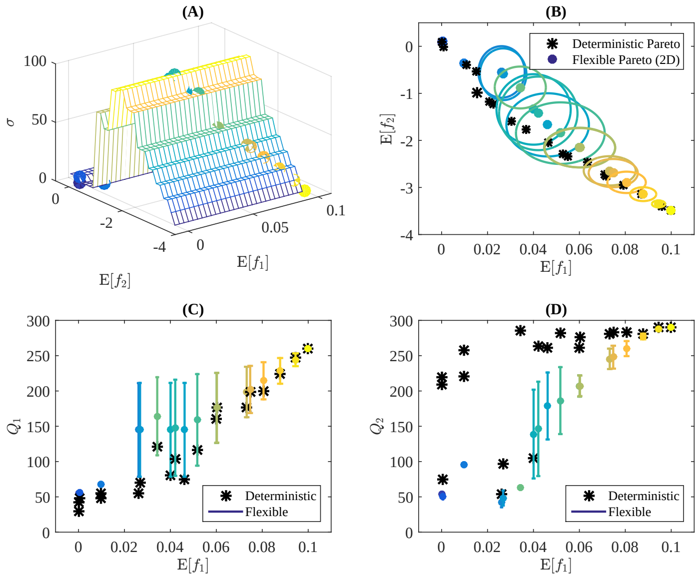


**Figure 3.** Optimal solutions for reservoir operation problem with two decision variables A) Pareto solutions; B) The range of changes of the objective due to randomness of decision variables; C) Flexible bounds for first decision variable for all the Pareto solutions; and D) Flexible bounds for second decision variable for all the Pareto solutions

## 4. Finding Flexible Decisions Using Dimension Reduction Concept

To find decision variables in a reservoir operation problem, the primary decision space can be approximated by either, a set of deterministic optimal decision variables (decisions leading to deterministic Pareto solutions in the objective space) or, a set of historical decision variables, or both.

A dimension reduction method, Karhunen-Loeve (KL) expansion, is applied to optimal decision variables to reduce the dimension of the decision space to a manageable number of random variables [10, 20]. In the proposed methodology the flexible decisions are defined by the choice of random coefficients in the KL expansion of outflows

$$Q^g(t, \vec{\xi}) = \sum_{k=1}^{N_{rv}} \lambda_k \xi_k \phi_k(t), \tag{19}$$

where the eigenpairs $(\lambda_k, \phi_k)$ are determined by the choice of representative samples of the decision space, while the random variables $([\xi_k]_{k=1}^{N_{rv}})$ describe the flexibility. Thus, the optimization method finds the optimal mean and standard deviation of each random variable in the KL expansion. These parameters are referred to as *control variables* to avoid confusion with the deterministic decision variables (which are the turbine outflows in daily time-steps in this study). The objectives of this optimization problem are the expected value of the operational objectives with an additional objective to maximize flexibility. Flexibility of control variables is represented by the sum of squares of the standard deviations of the random variables in the KL-expansion multiplied by the corresponding eigenvalues. By including the eigenvalues in this calculation, the actual influence of each random variable's standard deviation is taken into account.

### 4.1. Deterministic Optimization

This first scenario is the deterministic optimization of reservoir operation to find $\vec{Q} = [Q_n]_{n=1}^{N_t}$, in order to

$$\text{Minimize} \quad f_1(\tilde{Q}) = \frac{(\left|FB_{end}(\tilde{Q}) - FB_{target}\right|)}{U_r - L_r}, \text{and} \tag{20}$$

$$\text{Minimize} \quad f_2(\tilde{Q}) = -\left(\frac{\sum_{n=1}^{N_t}(PG_n(\tilde{Q}) - PL_n) * Pr_n}{\sum_{n=1}^{N_t}(Pr_n * PL_n)}\right) \tag{21}$$

$$\text{subject} \quad \text{to} \quad Q^{min} \leq Q_n \leq Q^{max} \quad \text{for} \quad \text{all} \quad n,$$

$$\text{subject} \quad \text{to} \quad f_1(\tilde{Q}) \leq \delta FB,$$

$$\text{subject} \quad \text{to} \quad |Q_n - Q_{n+1}| \leq Q^{ramp} \quad \text{for} \quad \text{all} \quad n.$$

where $\vec{Q}$ is the set of turbine outflows from the reservoir for $N_t$ time-steps. This problem has constraints for maximum and minimum allowable turbine outflows ($[Q^{min}, Q^{max}] = [30, 290]$ kcfs for this problem). Also, a constraint (22) is included to further restrict the maximum allowable forebay elevation deviation, $\delta FB$.

The dimension reduction method is applied to the optimal deterministic decision variables. The eigenvalues decrease exponentially when they are sorted in descending order (Figure 4). Therefore, the effect of the first few eigen-pairs are the most important. The comparison of the randomly generated realizations ($Q^g$) using only a few, versus all of the eigenpairs, demonstrates that even 3 random coefficients can be sufficient for the purpose of representing the decision space and generating flexible decision variables (Figure 5 C, D).

To investigate the effect of the generated realizations on the objective space, the original Pareto solutions are compared to a randomly generated solution set using complete and truncated KL-expansions [13]. Because of the random nature of the KL-expansion the generated decision variables may fall outside the boundary of the original data. Therefore, there are scenarios in the objective space in addition to the original deterministic Pareto solutions. However, they may be infeasible and are disqualified and eliminated in the process of optimization.

### 4.2. Flexible Decision Variables Using Dimension Reduction

The second scenario explored involves optimizing expected values of operational objectives while also maximizing flexibility in decisions. The control variables are the statistical parameters of the random variables in the KL-expansion representation of the decision variable.

The problem is stated as follows: Find $\{\mu_k\}_{k=1}^{N_{rv}}$ and $\{\sigma_k\}_{k=1}^{N_{rv}}$ in order to

$$\text{Minimize} \quad \mathbb{E}[f_1(Q^g(t, \tilde{\xi}))], \tag{22}$$

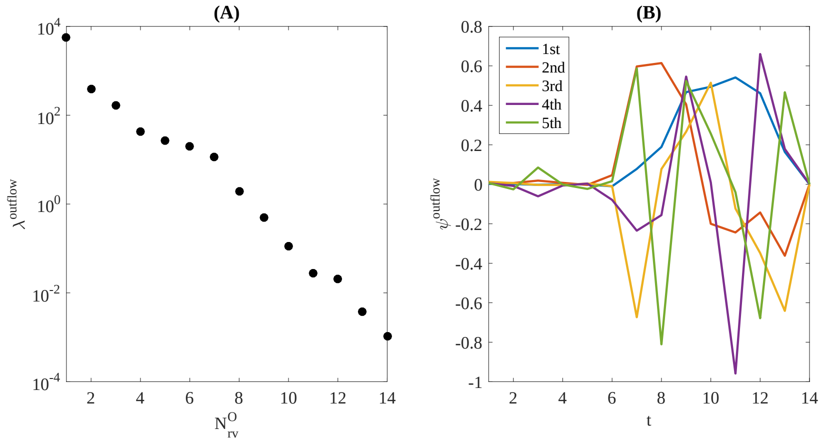


**Figure 4.** (A) Eigenvalues and (B) eigenfunctions of the original (deterministic) decision variables (logarithmic scale)

$$\text{and} \quad \mathbb{E}[f_2(Q^g(t,\tilde{\xi}))], \tag{23}$$

$$\text{Maximize} \quad f_3(\tilde{\xi}) = ||\Lambda\sigma|| \tag{24}$$

subject to

$$PF(C_1, C_2, C_3) \leq \alpha;$$

$$C_1 : Q^{min} \leq Q^g_n \leq Q^{max} \quad \text{for} \quad \text{all} \quad n,$$

$$C_2 : f_1(Q^g) \leq \delta FB,$$

$$C_3 : |Q^g_n - Q^g_{n+1}| \leq Q^{ramp} \quad \text{for} \quad \text{all} \quad n,$$

where the expected values are approximated using SC at points $\vec{X} = \{X_l\}_{l=1}^{N_l}$, with $N_l = {N_c}^{N_{rv}}$ and $N_c$ is the number of collocation points in each dimension. The points $X_l$ are determined according to the parameters of the random variables ($\mu_k$ and $\sigma_k$). In the above, $\Lambda$ is the matrix comprised of $\lambda_i$ along the diagonal.

Each Pareto solution for the flexible decision variables is the expectation of all the flexible scenarios [13]. Therefore, each Pareto solution is a representation of what the expected value would be; the variation of the objective value in the objective space is not shown in this figure. To show the variability of the objective values due to random samples of KL-expansion, a few examples of the flexible decision variables (20 random samples) and the corresponding objectives and their standard deviations are shown (Figure 6C and 6D). For clarity, only two of the Pareto solutions are selected to demonstrate the flexibility in decision space and the resulting variation in objective space. Each optimal control variable found by the proposed methodology, corresponds

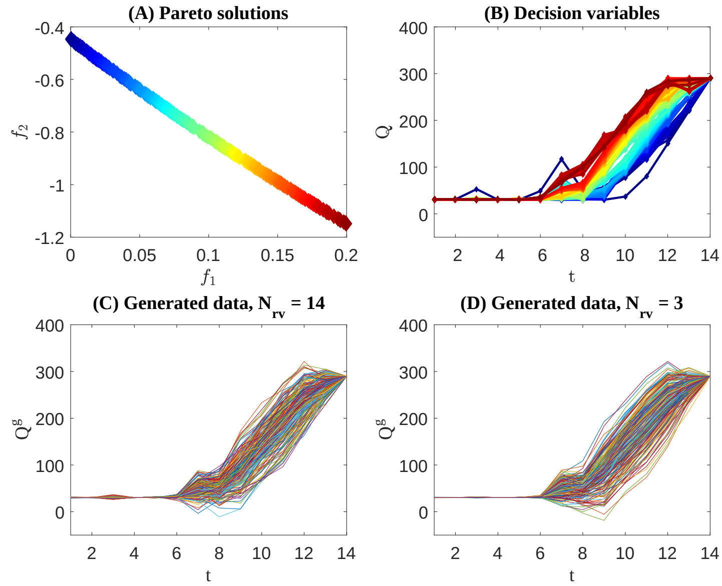


**Figure 5.** (A) The original (deterministic) optimal Pareto solutions, (B) the corresponding decision variables, (C) the generated decision variable realizations using 14 terms of the KL expansion, (D) the generated decision variable realizations using 3 terms

to a cluster of possible decision variables that gives the decision maker flexibility in decision space.

## 5. Conclusion

The goal of this study is to rigorously incorporate flexibility into the decision making process. The major contribution of the work is the representation of decision variables in the upper level of a decoupled bi-level optimization problem using continuous random variables. Each decision variable is represented by a range and the operator can then choose any value within the optimal range and be certain of the feasibility of the option. The proposed methodology finds the largest possible range of optimal flexible decision variables with a pre-specified probability of failure. The methodology is used to find flexible decision variables for a reservoir operation problem that give decision makers more options in the process of optimization in order to account for input uncertainties which are unquantifiable.

The first methodology is only suitable for problems with few (e.g., 6) decision variables, such as short-term reservoir operation. It has been included in this study as an intermediate step in order to allow a direct explanation of the proposed approach for representing flexibility and solving the corresponding multi-objective problem. However, exploiting dimension reduction techniques allows for large scale problems to be handled by this approach as well. For those problems which have a reasonable quantification of all input uncertainties, the flexibility approach developed here is unnecessary.

We note that here we considered only the unquantifiable sources of uncertainty, however additional sources may be incorporated as well. In particular, stream inflows are significant sources of uncertainty and should be treated explicitly and independently of other sources. Inflow uncertainty is not addressed in this manuscript in order

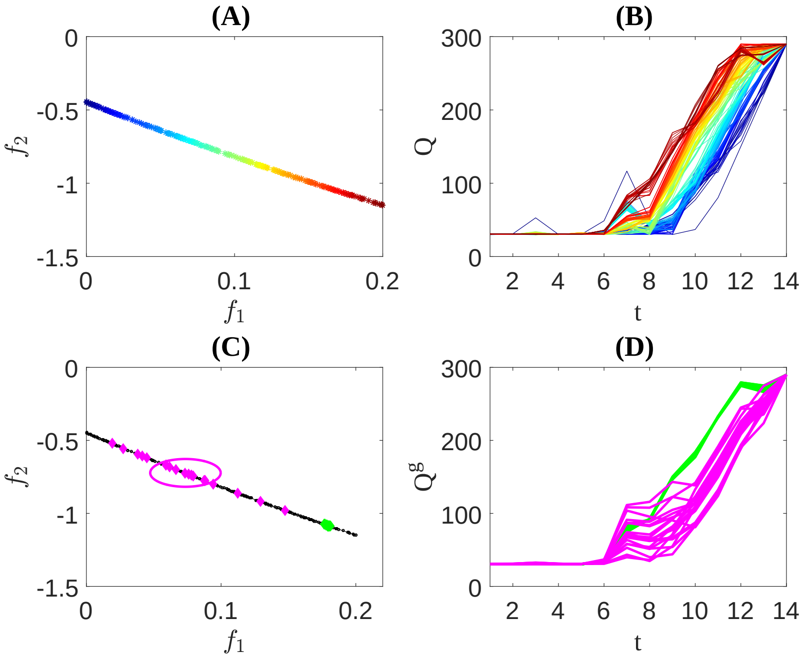


**Figure 6.** (A) The original deterministic optimal Pareto solutions, (B) the corresponding decision variables (with the same color as A), (C) comparison of the original deterministic Pareto solutions and the samples corresponding to 2 of the flexible Pareto solutions; each ellipse represents the standard deviation of $f_1$ and $f_2$ by its radii in horizontal and vertical dimension, respectively, (D) samples of the decision variable (outflow) realizations for 2 samples showed in C

to focus on the explanation of proposed method for handling flexibility, however, it is the topic of ongoing research by the authors.

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