BI-LEVEL FLEXIBLE-ROBUST OPTIMIZATION FOR ENERGY ALLOCATION PROBLEMS

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ABSTRACT

A common issue in energy allocation problems is managing the trade-off between selling surplus energy to maximize short term revenue, versus holding surplus energy to hedge against future shortfalls. For energy allocation problems, this surplus represents resource *flexibility*. The decision maker has an option to sell or hold the flexibility for future use. As a decision in the current period can affect future decisions significantly, future risk evaluation of uncertainties is recommended for the current decision in which a traditional robust optimization is not efficient. Therefore, an approach to Flexible-Robust Optimization has been formulated by integrating a Real Options Model with the Robust Optimization framework. In the energy problem, the real options model evaluates the future risk, and provides the value of holding flexibility, whereas the robust optimization quantifies uncertainty and provide a robust solution of net revenue by selling flexibility. This problem is solved using Bi-level programming and a complete general mathematical formulation of *Bi-Level Flexible-Robust Optimization* model is presented for multi-reservoir systems and results shown to provide an efficient decision making process in energy sectors. To reduce the computational expense, mathematical techniques have been used in the proposed model to reduce the dimension in the quantification and propagation of uncertainties.
Keywords: Bi-Level Optimization, Flexible-Robust Optimization, Real Option Analysis, KL-expansion, Stochastic Collocation.

1. INTRODUCTION

Renewable energy is a boon to developed countries. There are various sources of renewable energy, such as Solar, Hydro, Wind, etc. With the help of advanced technologies, we can convert these energies efficiently into electricity. Day by day, the population is increasing significantly which leads to an increase in the use of electricity. Therefore, research has been conducted on how to increase the efficiency of generation of electricity from these renewable resources and to allocate it optimally. This will increase the revenue of the energy sectors, and also minimize power failures. In this paper, we will focus on one particular renewable source of energy: Hydro energy. Over many decades, there has been continuous development of water resource management for the economic benefit of electricity industries. Various studies are being conducted in a wide range of domains, such as water allocation, infrastructure capacity expansion, water quality, drought control mitigation, flood control and conservation of aquatic ecosystems [1]. A key consideration for the power sector is identifying optimal strategies for buying and selling electricity, thereby using the water optimally to maximize revenue and also meet demand. Fundamental ideas of engineering have been studied in the water allocation optimization problem; literature related to hydro-economic optimization models are widely available [1]–[3]. Like all the renewable sources, Hydro energy also has many sources of uncertainty, thereby making the energy allocation problem very complex. For example, we have low uncertainty in the water level and demand in an initial time period, as we will have a good idea of the inflows and demands on the current day; however, we are not as sure about these quantities on future dates, and therefore uncertainty in the system increases. This uncertainty could make a significant impact in the decision making of optimal allocation of electricity generated from hydro energy. Thus, the *Robust Optimization* approach is necessary in these problems to account for uncertainty in the system. Though Robust Optimization quantifies uncertainty in the system, it does not have an efficient method to value resource *flexibility*. Resource flexibility is the defined as the surplus hydro energy after meeting the demand and is expressed in energy units (MWh.). As we will focus only on resource flexibility in this paper, we will simply call this flexibility. The value of flexibility refers to the economic value created by the ability to move this hydro energy generation from one-time period to another (e.g. save water today to use tomorrow). The Robust Optimization approach provides a robust solution of the optimal generation of electricity; however, in energy allocation problems considering all the uncertainties, we need to decide whether to allocate the flexibility in the current period or to hold it for the future to overcome any negative shocks (due to uncertainties) in the energy market. Therefore, the valuation of flexibility is required to realize such decisions for selling or holding the flexibility, which traditional Robust Optimization is not capable of providing. For general allocation problems, economic valuation is done by various economic models for any optimal decision making in investment problems, often using a *Real Options* (RO) valuation approach. This valuation helps in making choices on whether to invest now or later. We can relate this similar scenario to our problem, where during each period we are making choices whether to use the flexibility or hold it for future. Thus, we present an approach to formulate a *Flexible-Robust Objective* by integrating the real options model within a robust optimization framework. However, the energy sectors deal with many complex constraints and guidelines on the large scale multi-reservoir systems. These need to be considered in energy allocation problems in multi-reservoir system, which introduces compatibility issues with RO model in the model framework. The operational model has many constraints on the operations, but these are difficult to apply to the economical valuation model. Therefore, an approach to *bi-level Flexible-Robust Optimization* for optimal energy allocation problems in a complex multi-reservoir system is presented to divide the whole framework into two levels, focusing upon two different strategies as shown in Fig. 1. Similar approaches have been attempted previously in solving other domain problem, such as Strategic Offensive and Defensive Military structure design (left of Fig. 1), where the upper level is the offensive strategy and lower level is the defensive strategy [4]. In this problem, one would choose the best offense, assuming one was facing an optimized defense. In the right side of Fig. 1 is our proposed model where the upper or top level will be the *economic strategy* and the lower or bottom level will be *operational strategy*. Though our research focuses upon the energy domain, the idea of *bi-level flexible-robust optimization design* is not restricted to only energy allocation problems and can be applicable to the optimization problems of different domains. To summarize, the list of research topics in this paper is provided below:

1. **Integration of the Real Options model with Robust Optimization** to resolve the trade-off between getting revenue now versus holding water to overcome future risks. Other work considers either standalone Robust Optimization or only Real Option Analysis.
2. **Implementation of the bi-level programing** in the model to enforce the complex operational constraints and to enable estimation of the outflows for the entire reservoir system to meet target flexibility allocations.
3. **Computation efficiency of solving the problem**. We have implemented the KL-expansion to reduce the dimension of the problem, stochastic collocation and sparse grid method are then used to improve the computational efficiency when the dimension is reduced.

Section 2 provides the literature review on the Real Option model, Robust Optimization and Bi-Level Optimization framework. Section 3 provides the detail problem description and the mathematical formulation of Two Stage Bi-level Flexible Robust optimization model for multi-reservoir system. Section 4 discuss the methodologies implemented in our model to reduce the dimension of the problem and increase the computational efficiency of the model. Section 5 provides the result of our case study involving a complex multi-reservoir system. Section 6 concludes our paper with final thoughts and future scope of research.

2. BACKGROUND

To implement a method to optimize operations and formulate the bi-level flexible-robust optimization framework (Fig. 1), real options analysis, robust optimization and bi-level optimization framework are investigated, and a literature review is provided.

2.1 Real Options in Valuing Flexibility in Large Scale Systems

The structure of energy markets, which face an increase in competition and a goal of improved economic efficiency, face various risks and uncertainties. As the level of risk and uncertainty increases, traditional deterministic discounted cash flow (DCF) modeling approaches used for capacity investment planning need to be complemented by other, more sophisticated methods to deal with the potential fluctuations in both demand and price, among others. The real options (RO) approach to investment decision planning provides an attractive opportunity to evaluate investment alternatives in power generation in a deregulated market environment [5]. Kumbaroğlu et al. [5] presented a policy planning model which can guide policy planning in the electricity supply sector, and is based on the real options approach to investment. Several other studies have been conducted using the real options approach to investment problems in power sectors for the valuation of flexible renewable energy where uncertainties are high [6], [7]. Marreco and Carpio [8] present a valuation study of operational flexibility using Real Option Theory in order to determine the fair premium to be paid by the thermal capacity installed and applied in the complex Brazilian Power System, considering uncertainties in natural affluences.

In the case of renewable energy facilities (e.g. hydroelectric), the system is highly dependent upon hydrological conditions; therefore, uncertainties in inflows, weather forecast, market electricity demands and prices are significant, and chances of negative shocks are high. It has been a problem for decision makers in these facilities to determine how to allocate remaining water after meeting the contracted demands while considering uncertainty. This can lead to a wrong decision which can decrease revenue significantly and, in a worst scenario, can cause environmental damage. For example, if a facility empties storage due to high market prices and suddenly there are shortages in the next days, they have to forcefully buy electricity from the market to meet the demand, which will decrease revenue. Also, if they decide to allocate the flexibility later, and if inflows suddenly increase due to unpredicted rain, reservoirs may spill and cause flooding. Thus, valuing the flexible water is necessary to address potential negative impacts. In other words, it is beneficial for these facilities to understand the future value of allocated flexibility to help them determine better scheduling plan: whether to generate and sell the electricity now with the flexible water or to hold the water for future use. Real Option Theory is an appropriate technique to determine the value of flexibility.

In this paper, a Real Option (RO) model is proposed for valuation of the flexible water, by adopting real option theory. This real option model is integrated with a stochastic optimization framework as part of the objective function in the proposed *Flexible-Robust Objective*. Details of flexible robust objective will be explained in Problem description.

2.2 Robust Optimization

In this section, we provide a brief discussion about robust optimization and why it is necessary in hydropower generation problems. As noted in the previous section, there are multiple sources of uncertainty in hydropower generation, and the optimal operational decisions should provide consistent results, even when there are variations of the uncertain parameters from the expected value. Robust optimization is the best approach for these types of problems, where the decision can be made based on the risk attitude of the decision maker. Much literature is available which presents a stochastic model for solving problems in the energy market to deal with uncertainties, and to provide an optimal result with minimal risk [9]–[11]. Robustness has also been studied extensively in the engineering literature as a means to account for uncertainty. Robustness is defined as the ability of a given system configuration to perform well over a wide range of conditions over the product lifecycle, such as the occurrence of faults and resulting functional losses. Taguchi-based approaches [12]–[15] utilize an optimization framework in which the system design is optimized based on an objective which considers both the mean and variance of the system performance. Variance in the system performance can result from multiple sources of random noise, both external and internal to the system. Robustness has also been investigated in the biological network literature, with principles for achieving robustness defined as system control, redundancy, diversity, modularity, and decoupling [16].

McIntire et al. applied a Robust Design Optimization framework to the Columbia River System to provide an optimal outflow for maximizing expected revenue considering the inflow uncertainties. The probabilistic framework results in lower risk solution than the deterministic approach, when uncertainties are accounted for [17]. However, the concept of allocating flexibility and the Real Option analysis was not attempted in this framework. Finally, it is obvious to mention that design under uncertainties following the RO framework is not only limited to hydro energy problems and has been attempted throughout the years in solving many complex engineering systems facing various uncertainty to build a robust design [18], [19].

2.3 An overview on Bi-Level Optimization

In real world complex design problems for large scale systems, multidisciplinary optimization has been utilized and plays a key role in design research. In large-scale complex systems, it is often difficult to optimize for the whole system in a single level consisting of large numbers of design variables, objective functions and constraints. Therefore, it is desirable to break down the system into several components or subsystems. It is easier to optimize each of the subsystems which guarantee an optimal solution to the main system. This idea arises in the multidisciplinary optimization framework (MDO), where the optimization of a large-scale system is done by optimizing each of the subsystems, which are coupled with each other. Analytical Target Cascading (ATC) and Collaborative Optimization (CO) are the two methods of MDO. Several studies have been done in recent years in MDO [20], [21]. McAllister and Simpson [22] presented a CO framework for an Internal Combustion Engine. Another emerging approach in the area of Multi-disciplinary optimization is the Bi-Level Optimization method. Bi-level optimization is a certain type of optimization where one problem is embedded (nested) within another. The outer optimization task is commonly referred to as the upper-level optimization task, and the inner optimization task is commonly referred to as the lower-level optimization task. The lower level optimization acts as a constraint in the upper level. These problems involve two kinds of variables, referred to as the upper-level variables and the lower-level variables [23]. The lower level optimization, also referred to as follower's optimization problem, is solved first. The upper level optimization, also referred as leader's optimization problem, considers the optimal solution of the follower. Therefore, the optimal solution of the upper level optimization problem will guarantee optimality also in the lower level optimization problem. Bi-level optimization was first realized in the field of game theory by a German economist Heinrich Freiherr von Stackelberg in 1934 that described this hierarchical problem.

A simple formulation of Bi-Level Optimization can be written below as:

$\min_{x} F(x, y)$ (<u>Upper Level</u>)

s.t (<u>Upper Level Constraints</u>)

$$G_i(x, y) \leq 0 \; for \; i = \{1,2 \ldots N\}$$

$$\min_{y} f(x, y) \; (\underline{Lower\ Level})$$

s.t (<u>Lower Level Constraints</u>)

$$g_j(x, y) \leq 0 \; for \; j = \{1,2 \ldots n\} \tag{1}$$

Where x and y are upper and lower level decision variables, respectively; G_i and g_j are the i^{th} and $j^{\,th}$ inequality constraints in upper and lower level, respectively.

Papers have been published attempting Bi-Level programming in various design problems. This approach has been extensively applied in the field of transportation and defense strategy. Labbe, Marcotte and Savard in 1998 proposed a bilevel model of taxation and its application in toll-setting problem in highways. In this bi-level model the leader wants to maximize revenue from taxation schemes, while the follower rationally reacts to those tax levels [24]. Chen and Subprasom [25] formulated a stochastic bi-level programming model for a Build-Operate-Transfer (BOT) road pricing problem under demand uncertainty. Braken and McGill [4] proposed a bi-level optimization model in defense applications which includes strategic force planning problems and two general purpose force planning problems. In recent years, this approach has been accepted and is being widely used in strategic bomber force structure, and allocation of tactical aircraft to missions. Roghanian, Sadjadi and Aryanezhad [26] presented a bi-level multi-objective programming model in enterprise wide supply chain planning problem considering uncertainties on market demand, production capacity and resource availability. Lv et al. [27] and Xu [28] attempted a bi-level optimization technique for a water allocation problem; however, the uncertainties were handled in a fuzzy random environment in both works. Also, the operational constraints of the reservoirs were not considered in the model, which leads to a complex design. RO analysis has been used previously in bi-level programming in the FACTS investment problem [29]. However, the model was not integrated within a robust optimization framework. In this paper, we will show the efficiency of the integration of robust optimization and real options analysis in a bi-level design framework by providing better operational decisions in hydropower scheduling problems

3. PROBLEM DESCRIPTION

In this section, we will discuss the framework of our model and provide the detailed mathematical formulation as an approach to optimize a general Hydropower Scheduling problem of large scale systems. In this section, mathematical equations of the proposed optimization problem will be presented. The solution methods used for the problem will be discussed later in Section 4 of the paper. In this paper, we have considered two major sources of uncertainty in the inputs of our problem: Water inflows and Market price of electricity. The output or the decision variables of our model will be deterministic, which will be the robust decision considering the uncertainty on the inputs variables. Unlike our model, an attempt has been done previously where uncertainty is also incorporated into the decision variables [30]. However, in our model, the main focus is to provide a robust deterministic decision to the operators, causing minimal negative effect on the revenue due to the future stated uncertainties. The mathematical formulation provided in this section will work for *N* number of reservoirs in the system. However, in this paper, we have provided a case study of 3 reservoirs in Lower Columbia River system. In addition to being bi-level, the model is also broken down into a two-stage optimization problem as shown in Fig. 2. Previously this type of multi-stage model has been presented [31] in optimization of Real-time Hydrothermal system operations where 3 sub-models (hourly, daily and monthly) have been coupled with different time-steps (hourly, daily and monthly) and optimization period (daily, monthly and yearly) with an objective to maximize revenue.

Figure 2 provides an overview of conceptual framework of the proposed model. Stage 1 has the robust optimization which gives the maximum possible energy generation during the optimization period. The output from the Stage 1 will be an input to the Stage 2 optimization problem and will act as a boundary constraint. Given the maximum total energy generation during the optimization period, the main purpose of Stage 2 is to optimally allocate the flexibility after meeting the demand at each time period in order to get better revenue by avoiding any failures of meeting demand and also making sure of safe operations. The bi-level optimization framework has been implemented in the Stage 2 of our model. The overall purpose of Stage 2 has been divided into two levels in the bi-level programming. The upper level includes the flexible-robust objective for maximizing revenue and to address any negative shocks. The lower level focuses on making sure the decisions provided in the upper level are environmentally safe and physically achievable. We will continue the detail description of designing the model step by step in the below subsections. As this is a continuation of our previous work with simplified model which is restricted only for single reservoir. In this mathematical formulation, we have extended the problem for multi-reservoir system where M reservoirs can be considered, and therefore to help the reader, we keep the nomenclature of the variables as much consistent as possible with our previous work.

3.1 Stage 1

We will provide a brief overview of Stage 1 problem as it is the pre-requisite of Stage 2 optimization problem. In this paper, we will primarily focus on Stage 2 since bi-level flexible robust optimization framework is introduced in Stage 2. Stage 1 problem involves the maximization of energy generation capacity considering physical and operational constraints of reservoirs. The maximum total energy generation is the final output of Stage 1 and will be passed as input to Stage 2. Enforcing the mentioned constraints will make sure that the solution of Stage 1 will not be infeasible or unachievable. Being infeasible will lead to an infeasible solution from Stage 2 as well since Stage 2 will try to allocate that infeasible total energy. The decision variable for Stage 1 model is total water *outflow* as each time-step.
Below is the mathematical formulation of the Stage 1 optimization problem:

Note:
In the mathematical formulation,

- Variables **with** the overscript ~ are uncertain variables.
- Variables **without** the overscript ~ are deterministic variables.
- Variables having superscript * are optimal solutions.
- 1 *kcfs= 28.316847 m*^{3}*/s* (S.I unit)
- 1 *ft= 0.3048 m* (S.I unit)

Model Input:
(Water) Inflows, $\tilde{\boldsymbol{Q}}_{\boldsymbol{in}}$

Model Decision Variables:
The total outflows in *kcfs* at each time step and each reservoir are defined as decision variables in the optimization and is denoted by matrix, $\boldsymbol{Q}_{\boldsymbol{out,Stage1}}$. Each row represents daily timesteps of 14 days (t =14) and each column represents number of reservoirs (N = 3 in this case study).

$$\begin{bmatrix} Q_{out,1,1}, Q_{out,1,2}, \dots\dots, Q_{out,1,t} \\ \vdots \\ Q_{out,N,1}, Q_{out,N,2}, \dots\dots, Q_{out,N,t} \end{bmatrix}$$

Model Objective:
Maximize Energy generation capacity,

$$\max_{Q_{out,i,t}} \sum_{t=1}^{14} \sum_{i=1}^{N} \tilde{e}_{i,t}(Q_{out,i,t}, \widetilde{hd}_{i,t}(\widetilde{FB}_{i,t}, TW_{i,t}), \xi_t) \quad (2)$$

Where,

$$\tilde{e}_{i,t} = \eta \times 9.81 \times \widetilde{hd}_{i,t} \times Q_{out,i,t} \times 8.6310 \times 10^{-3} \times \xi_t \quad (3)$$

In this equation, $\tilde{e}_{i,t}$ is the energy generation of reservoir i at each time step t in MWh, where η is the efficiency of the reservoir, taken as 0.75; ξ_t is assumed as 1 hour; $\widetilde{hd}_{i,t}$ is the head in ft and is calculated as below:

$$\widetilde{hd}_{i,t} = \widetilde{FB}_{i,t}(\tilde{S}_{i,t}(\tilde{Q}_{in,i,t}, \tilde{Q}_{in,i,t-1}, Q_{out,i,t}, Q_{out,i,t-1}, \text{delt}_t)) - TW_{i,t} \quad (4)$$

$$TW_{i,t} = A_i + B_i \times Q_{out,i,t} + C_i \times \widetilde{FB}_{i+1,t-1} \quad (5)$$

$$TW_{i,t} = A_i + B_i \times TW_{i,t-1} + C_i \times (Q_{out,i,t} - Q_{out,i,t-1}) \quad (6)$$

$\widetilde{FB}_{i,t}$ is the reservoir water level of reservoir i at time t; $\widetilde{FB}_{i+1,t-1}$ is the water level elevation at the next downstream reservoir at the previous timestep (we assume *i+1* is the next downstream reservoir of reservoir i and *i-1* is the previous upstream reservoir of reservoir i); $\tilde{S}_{i,t}$ is reservoir storage of reservoir i; $TW_{i,t}$ is the tailwater of reservoir i and is evaluated from Eq. (5) and (6). The tailwater elevation-discharge relationship has been found to be well approximated using simple linear regression relationships. Equation 6 is used to calculate the tailwater for the most downstream reservoirs in the system whereas Eq. (5) is used to calculate the tailwater for all the other reservoirs in the system. A_i, B_i and $C_i$are the coefficients for reservoir i. Table 1 shows the value of the coefficients obtained from the existing model of Bonneville Power Administrator (BPA), used in our case study of the 3-reservoir system.

Subject to:
Model Constraints (Operational Constraints):

a. *Water Balance Constraints*

$$0 \le \tilde{S}_{i,t}(\tilde{Q}_{in,i,t}, \tilde{Q}_{in,i,t-1}, Q_{out,i,t}, Q_{out,i,t-1}, \text{delt}_t) \le S_{i,max} \quad (7)$$

where,

$$\tilde{S}_{i,t+1} = ((\tilde{Q}_{in,i,t} + \tilde{Q}_{in,i,t+1})/2 - (Q_{out,i,t} + Q_{out,i,t+1})/2).\text{delt}_t + \tilde{S}_{i,t} \quad (8)$$

In these Equations, $\tilde{S}_{i,t}$ is reservoir storage in *kcfs-day* for reservoir i. $S_{i,max}$ is the maximum storage capacity of reservoir i, $\tilde{Q}_{in,i}$ and $Q_{out,i}$ are inflow and outflow to reservoir in, respectively of reservoir i, and delt_t is time (day) between each time step. At this stage, the water leakage and natural water loss is not considered. We consider Equation (7) as a *Reliability Constraint*. Therefore, Equation (7) becomes:

$$Pr\{0 \le \tilde{S}_{i,t}(\tilde{Q}_{in,i,t}, \tilde{Q}_{in,i,t-1}, Q_{out,i,t}, Q_{out,i,t-1}, \text{delt}_t) \le S_{i,max}\} \ge R, \quad 0 \le R \le 1 \quad (9)$$

where R is the reliability factor.

b. *Reservoir Water Surface Elevation (WSE) Constraints*

$$FB_{i,min} \le \widetilde{FB}_{i,t}(\tilde{S}_{i,t}) \le FB_{i,max} \quad (10)$$

where,

$$\widetilde{FB}_{i,t} = A_i \times (\tilde{S}_{i,t})^2 + B_i \times (\tilde{S}_{i,t}) + C_i \quad (11)$$

where $\widetilde{FB}_{i,t}$ is the reservoir water level in ft at time t of reservoir i, $FB_{i,min}$ and $FB_{i,max}$ are the allowable minimum and maximum reservoir water elevation respectively. A_i, B_i and $C_i$are the coefficients for reservoir i and are determined by fitting actual forebay elevation observations with a polynomial regression model. Table 2 shows the value of the coefficients obtained from the existing model of Bonneville Power Administrator (BPA), used in our case study of 3-reservoir system. We consider Equation (10) as a *Reliability Constraint*. Therefore, Equation (10) becomes:

$$Pr\{FB_{i,min} \le \widetilde{FB}_{i,t}(\tilde{S}_{i,t}) \le FB_{i,max}\} \ge R, 0 \le R \le 1 \quad (12)$$

where R is the reliability factor.

c. *Turbine Flow Constraints*

$$Q_{i,tb-min} \le Q_{tb,i,t} \le Q_{i,tb-max} \quad (13)$$

In this constraint, $Q_{tb,i,t}$ is turbine flow for power generation in *kcfs* at each time step of reservoir i and $Q_{i,tb-min}$ and $Q_{i,tb-max}$ are allowed minimum and maximum discharge respectively. Since we are ignoring spill flow, Turbine flow and Outflow will be same. Therefore, we can re-write Equation (13) as below:

$$Q_{tb-min} \le Q_{out,i,t} \le Q_{tb-max} \quad (14)$$

d. *Output Constraints*

$$N_{i,d-min} \le \widetilde{N}_{d,i,t}(Q_{out,i,t}, \widetilde{hd}_{i,t}) \le N_{i,d-max} \quad (15)$$

where,

$$\widetilde{N}_{d,i,t} = \eta \times 9.81 \times \widetilde{hd}_{i,t}(\widetilde{FB}_{i,t}, \mathrm{TW}_{i,t}) \times Q_{out,i,t} \times 8.6310 \times 10^{-3} \quad (16)$$

In the output constraint, $\widetilde{N}_{d,i,t}$ is power output in MW at time t of reservoir i, and $N_{i,d-min}$ and $N_{i,d-max}$ are the minimum and maximum output capacity, respectively. We consider Equation (15) as a *Reliability Constraint*. Therefore, Equation (15) becomes:

$$Pr\{N_{i,d-min} \le \widetilde{N}_{d,i,t}(Q_{out,i,t}, \widetilde{hd}_{i,t}) \le N_{i,d-max}\} \ge R, \quad 0 \le R \le 1 \quad (17)$$

where R is the reliability factor.

e. *Reservoir Water Surface Elevation (WSE) Constraints on the end-of-period*

The optimization is conducted over a 14-day time span, which is a relatively short-term for reservoir operations. To be consistent with middle-term or long-term operation, the water surface elevation (WSE) in the reservoir at the end of optimization period is expected to stay within a target WSE to fulfil future requirements. In the example problem we have formulated, the historical data from the actual operation scheme is used as the target WSE for the optimization model. To avoid equality constraints, a small range on the target WSE is used to restrain the WSE on the end-of-period to be close to the target WSE:

$$FB_{i,tar,end} - \Delta \le \widetilde{FB}_{i,t} \le FB_{i,tar,end} + \Delta \quad (18)$$

where $FB_{i,tar,end}$ is the target WSE on the end-of-period of reservoir i, and Δ is the deviation from the target WSE. The Δ is set as 1% in the model and $FB_{tar,end}$ is taken as 1280 ft. We consider Equation (18) as a *Reliability Constraint*. Therefore, Equation (18) becomes:

$$Pr\{FB_{i,tar,end} - \Delta \le \widetilde{FB}_{i,t} \le FB_{i,tar,end} + \Delta\} \ge R, \quad 0 \le R \le 1 \quad (19)$$

where R is the reliability factor.

Model Output:

- Maximum energy generation capacity $\tilde{E}^{max,1}$

3.2 Stage 2 (Bi-level Optimization)

Once the output from Stage 1 is obtained, Stage 2 optimization starts. The Bi-level optimization framework has been implemented in Stage 2. Therefore, the stage 2 problem is divided into two sublevel problems: Upper Level and Lower Level Optimization problems.

Upper Level: The upper level optimization sets a target allocation of energy in each iteration which is to be passed to the lower level problem to determine the feasibility of the target decision. Details of the lower level optimization problem will be discussed later in this section. The upper level only focuses on the economic strategy and therefore the Real Option (RO) model is integrated with the robust optimization framework. Real Option analysis is an economic tool which helps to value the multiple courses of actions in a decision: that is to either sell the flexibility or hold it for future use based upon the future value of flexibility. Since there is a lot of uncertainty from river inflows in future days, it is desirable for energy sectors to have some knowledge if the current allocation of flexibility may cause any shortage of energy in future. This will allow them to address any negative shocks and thereby not to lose revenue. A Real Option model provides this information considering the Inflow uncertainty and is thus integrated with the robust optimization framework for better quantification of uncertainty and making efficient operations. In our case study, we took two sources of uncertainty: Prices and Inflows. Since the RO model quantifies the inflow uncertainty to provide a valuation for flexibility, it will be redundant to quantify the same uncertainty source again in the robust objective formulation: lower uncertainty is preferred to higher uncertainty in both the RO model and the robust objective and therefore moves the solution in the same direction. However, Price uncertainty seems to have much less variation than inflows; therefore, Real Option analysis on Price uncertainty is redundant as this will increase the computational cost of the model significantly without much improvement in the solution. Thus, we will formulate the integrated Flexible-Robust objective such that Price and Inflows uncertainty will be handled separately by the robust objective and the real option model, respectively. As the RO model is key to incorporating flexibility in the overall framework, it will be described in the next subsection.

The Real Option Model to compute Option value, $\tilde{V}_t(h, \tilde{F}_t)$

Given the possible future economic values of flexibility (one value for each future time point), we need to figure out the current value of flexibility so that it is directly comparable to the current sales revenue. This is accomplished using *option theory*. To facilitate illustration, we start with a simplified, discretized model.

As shown in Fig. 3, the realization of past uncertainties leads us to the current state, denoted by the red dot in the figure. We can sell h amount of flexibility now and get current revenue or we can hold it to the future. Depending on the realization of future uncertainties, we may evolve to different states in period $t+1$ along different future paths as denoted by the broken lines. First, consider the case in which the distribution of future flexibility is given. The different realizations of uncertainties generate different future scenarios with different flexibilities. In Fig. 3, the probability distribution is given and is denoted by the blue line in the figure. The probability distribution also stipulates the probability of energy shortage ($\tilde{F}_{t+\tau} < 0$) as denoted by the shaded area. If provided with the information about the market supply function, we can derive $\widetilde{W}_{t+\tau}(h, \tilde{F}_{t+\tau})$, the future value of flexibility for period $t+\tau$, which will later be explained in detail.

Next, we consider the uncertainty in the distribution of future flexibility. This allows us to investigate the uncertainty from specific sources, such as inflows, that may shift the distribution of future flexibility and consequently affect the opportunity cost of selling h in the current period t. This can be accomplished using the multi-period multinomial option price model [32], [33] in general. The example shown in Fig. 4 generates a classical multi-period binomial option model [34]. Fig. 4 has a binomial decision tree structure for a three-period option. Given the flexibility in the current period t, there are two possible scenarios associated with different levels of flexibility in period t+1: *shortage* and *no shortage*. Due to the uncertainties in the system, the incidence of shortage is governed by the probabilistic event $\tilde{F}_{t+1} < 0$, where $\tilde{F}_{t+1}$ is the total flexibility of the system at t+1 period after the allocation at t period, which occurs with the probability given by the blue-shaded area in the Fig. 4.

In the current period t, we can either sell the h amount of flexibility for sales revenue or hold on the flexibility as an option to use it in future periods. Depending on the realization of uncertainties, the option value of holding h amount of flexibility to period $t+1$ may equal either $\widetilde{W}_{t+1,1}(h, \tilde{F}_{t+1})$ or $\widetilde{W}_{t+1,2}(h, \tilde{F}_{t+1})$. Then conditional on the amount of flexibility in period $t+1$, we may have two additional possible scenarios corresponding to the incidence of shortage in period $t+2$. Depending on the uncertainties in period $t+2$, the value of holding h amount of flexibility to period $t+2$ may have different values denoted as $\widetilde{W}_{t+2,1}(h, \tilde{F}_{t+2})$, $\widetilde{W}_{t+2,2}(h, \tilde{F}_{t+2})$, $\widetilde{W}_{t+2,3}(h, \tilde{F}_{t+2})$, and $\widetilde{W}_{t+2,4}(h, \tilde{F}_{t+2})$. This structure can be extended to multiple periods. The cost of doing so is an increase in computing time.

The determination of the option value for each period $\tilde{V}_{t+\tau}(h, \tilde{F}_{t+\tau})$ $(\tau = 1, 2, \ldots, T)$ uses a backward induction scheme. Starting from the last period *T* (*T*=*t*+*2* in case 3-day optimization period decision tree structure as shown in Fig. 4), if the flexibility h is held to the last period, its value equals the purchase cost saved if shortage occurs:

$$\widetilde{W}_T(h, \tilde{F}_T) = -\min(0, \max(-h_t, \tilde{F}_T)) \times \Delta_P = \begin{cases} -\max(-h_t, \tilde{F}_T) \times \Delta_P, & if\ \tilde{F}_T < 0 \\ 0 & if\ \tilde{F}_T \geq 0 \end{cases} \quad (20)$$

Note that a shortage occurs when $\tilde{F}_T < 0$, and that Δ_P= 2 where Δ_P is the multiplicative difference between the purchase and sell price of electricity. In a large hydro-energy sector like Bonneville Power Administration, when they have shortages, that is an indication of insufficient hydro-power and a need for power generation from other sources such as fossil, steam or gas turbines whose cost is 3 times the cost of hydro-electricity [35]. Therefore, we have assumed that the purchase price is 3 times the sell price of hydro-electricity (and $\Delta_P = 3 - 1 = 2$). It is important to emphasize that the incidence of $\tilde{F}_T < 0$ follows a random distribution conditional on the realized values of past flexibility like $\tilde{F}_{T-1}$.

Similarly, we can calculate the value of using *h* to avoid additional purchase cost if a shortage occurs in period *T*-1, denote the value as: $\widetilde{W}_{T-1}(h, \tilde{F}_{T-1})$:

$$\widetilde{W}_{T-1}(h, \tilde{F}_{T-1}) = -\min(0, \max(-h_t, \tilde{F}_{T-1})) \times \Delta_P = \begin{cases} -\max(-h_t, \tilde{F}_{T-1}) \times \Delta_P, & if\ \tilde{F}_{T-1} < 0 \\ \mu_{T-1}\tilde{V}_T(h, \tilde{F}_T)/r & if\ \tilde{F}_{T-1} \geq 0 \end{cases} \quad (21)$$

where r is the interest rate. Given that the time step in our example is daily, $r \approx 1$. However, when $\tilde{F}_{T-1} \geq 0$, instead of have zero value, holding onto the flexibility generates an expected future payoff of $\delta\mu_{T-1}\widetilde{W}_T(h, \tilde{F}_T | \tilde{F}_{T-1} > 0)$. This is done for each of the possible scenarios in period *T*-1. Then, taking the period *T*-1 as the final period, we use this procedure iteratively until we are at the first time period.

As flexibility can only be used once, the option value of h in time period t is denoted as $\tilde{V}_t(h, F_t)$, which can be calculated as:

$$\tilde{V}_t(h, \tilde{F}_t) = \max_{\tau} r^{-k}\mu_t\widetilde{W}_{t+\tau}(h, \tilde{F}_{t+\tau}) \quad \text{for } \tau = 1, \ldots, T-t, \quad (22)$$

where $\mu_t(.)$ is the expectation operator based on information about $\tilde{F}_t$. The mean value of $\tilde{V}_t(h, \tilde{F}_t)$ is reported to the robust optimization framework to evaluate the flexible-robust objective as in Eq (24).

Computing Expected Net Revenue at the end period of optimization, r_{14}

In our problem, although the optimization period is 14 days, the decision tree includes a 13-day period. That is T=t+12. This is because on the 14th day (last day of optimization), we will allocate any remaining flexibility such that the water level or the forebay elevation of the reservoirs will be within the desired level as per the operation constraints. Thus, it is not valid to calculate the future value of flexibility at the end optimization period as there are no future days. Instead, based on the remaining flexibility on the end period after allocating flexibility on previous 13 days, we will calculate the expected shortages and the probability of the shortages. Then the Expected Net Revenue at the end period of optimization, r_{14} can be calculated as below:

$$r_{14} = \begin{cases} (pb_{shortage,14} * 24 * \tilde{p}_{14} * (h_{14} - \mu(\widetilde{Sh}_{14}))) + \\ ((1 - pb_{shortage,14}) * 24 * \tilde{p}_{14} * h_{14}) \ \ if\ \tilde{F}_{14} \geq \mu(\widetilde{Sh}_{14}) \\ (pb_{shortage,14} * 24 * (1 + \Delta_P) * \tilde{p}_{14} * (h_{14} - \mu(\widetilde{Sh}_{14}))) + \\ ((1 - pb_{shortage,14}) * 24 * \tilde{p}_{14} * h_{14}) \ \ if\ \tilde{F}_{14} \leq \mu(\widetilde{Sh}_{14}) \end{cases} \quad (23)$$

Where $pb_{shortage,14}$ is the probability of shortage at day 14, $\tilde{p}_{14}$ is the price at day 14, h_t is the total allocated flexibility at day 14 in MWh for *N* reservoirs, $\mu(\widetilde{Sh}_{14})$ is the expected shortage at day 14, $\tilde{F}_{14}$ is the remaining flexibility at day 14 before allocation, Δ_P= 2 where Δ_P is the multiplicative difference between the purchase and sell price of electricity in the case when the expected shortage is greater than the available flexibility to allocate at day 14. As we say, at the end period we allocate all the remaining flexibility, therefore $h_{14} = \mu(\tilde{F}_{14})$. With the RO model formulated, we can now write out the upper level optimization problem.

3.2.1 Upper Level Optimization Problem:

Below is the mathematical formulation of the Stage 2 Upper level optimization problem:

Model Input:

- Demand, $\boldsymbol{D} = [d_1, d_2, \ldots, d_{14}]$
- Maximum energy generation each day for each reservoir $\tilde{\boldsymbol{E}}^{\boldsymbol{max,1}}$ obtained from the output of previously mentioned Stage 1 problem.
- Price, $\tilde{\boldsymbol{P}}$

Model Decision Variables:

We propose the decision variable to be the allocated energy for each reservoir at each time-steps.

$$\begin{bmatrix} e_{1,1}, e_{1,2}, \ldots \ldots, e_{1,t} \\ \vdots \\ e_{N,1}, e_{N,2}, \ldots \ldots, e_{N,t} \end{bmatrix}$$

Each row represents daily timesteps of 14 days (t=14) and each column represents number of reservoirs (N=3 in this case study)

The Flexible-Robust Objective:

$$\max_{e_{i,t}}[\sum_{t=1}^{13} 24 * \tilde{p}_t * ((h_t - \mu(\tilde{V}_t(h, \tilde{F}_t))) - (Pen * \sum_{i=1}^{N} dev_{i,t}))] + [r_{14} - (Pen * \sum_{i=1}^{N} dev_{i,14})] \quad (24)$$

where

$$h_t = \sum_{i=1}^{N} e_{i,t} - d_t \quad (25)$$

$$dev_{i,t} = |(e_{i,t} - \mu(\tilde{e}_{i,t})|; i = 1, .., N; t = 1, .., 14 \quad (26)$$

$\sum_{i=1}^{N} h_{i,t}$ or h_t is the total allocated flexibility in period *t* in MWh for N reservoirs; $\tilde{p}_t$ is the price/MWh to sell electricity in each day over 14 days period; $e_{i,t}$ is the decision of allocation of energy in MWh on Day t for reservoir *i*; $\mu(\tilde{V}_t(h, \tilde{F}_t))$ is the expectation of the Option Value of total allocated flexibility h_t of N reservoirs at time t which depends on amount of cumulative flexibility left in the system at time t, $\tilde{F}_t$; r_{14} is the Expected Net Revenue at the end period of optimization. We have already discussed the computation of the Real Option Model to evaluate $\mu(\tilde{V}_t(h, \tilde{F}_t))$ and r_{14}. For the Multi-Reservoir we only know the total Demand d_t each day. From which reservoir the demand energy should be generated is an additional optimization problem that is not considered in this example (it can be considered a Stage 3 or integrated in Stage 2). Therefore, information of the amount of energy generated from individual reservoirs to meet the demands $d_{i,t}$ is unknown in this problem. Thus, in multi-reservoir system, $\sum_{i=1}^{N}[d_{i,t}]; t = 1, 2, \ldots 14$ or d_t is known but $[d_{i,1}, d_{i,2}, \ldots, d_{i,14}]$ for reservoir *i* is unknown and we only know total flexibility in each time step. $dev_{i,t}$ is the deviation between the target allocated energy (Upper Level) and the mean of the achievable allocated energy obtained in Lower Level Optimization problem. We will define it later in this section in the computation of $\mu(\tilde{e}_{i,t})$ in the Lower Level optimization problem. *Pen* is the Penalty factor and is considered as 0.5, 1, 1.5. We first attempted our formulation of the flexible-robust objective (Equation 24) without taking Penalty term. Instead, our approaches were to consider a certain target allocation of energy $e_{i,t}$ a feasible decision only if $\text{dev}_{i,t} \leq \delta$. δ is very small value (1 MWh) which was defined as the maximum deviation of the achievable solution (Lower level) from the target solution (Upper level) such that we can ignore the effect on the net revenue. With the complexity of the problem considering a multi-reservoir system, we fail to find feasible optimal solutions using the allowable deviation approach, i.e. ($\text{dev}_{i,t} \geq \delta$). To get sufficient feasible decision points, we need to put a larger δ which then will not guarantee optimality in terms of revenue (Upper level objective) since the achievable solution may be too far away from the target. To mitigate this issue, we used an approach introducing a Penalty term as shown in Equation 24. In this way, any deviation of the achievable solution from the target solution will be penalized and thus we will not discard target decision points set in the Upper level which are unachievable but will be penalized in the calculation of the objective function. The penalty will increase with increase of the deviation of the achievable solution, $\tilde{e}_{i,t}$ from the target solution, $e_{i,t}$.

Subject to:

Model Constraints (for Multi-reservoir system):

a. Maximum Allocation of Energy (does not require lower level optimization):

This constraint is applied to validate that allocated energy is less than or equal to the actual availability of energy each reservoir each day. It is unrealistic to allocate more energy than actually will be available on a given day:

$$e_{i,t} - \tilde{E}_{A,i,t}(\tilde{E}_{A,i,t-1}, e_{i,t-1}, \tilde{e}_{i,t}^{max,1}) \leq \delta \quad (27)$$

where,

$$\tilde{E}_{A,i,t} = \tilde{E}_{A,i,t-1} - e_{i,t-1} + \tilde{e}_{i,t}^{max,1} \quad (28)$$

$\tilde{E}_{A,i,t}$ is the actual availability of energy in MWh at day t for reservoir I, $e_{i,t}$ is the decision of allocation of energy in MWh on Day t for reservoir i, $\tilde{e}_{i,t}^{max,1}$ is the maximum energy that can be generated in MWh on day t for reservoir i, δ is the maximum tolerance for violation and is set as 1 MWh. We treat Equation (27) as a *Reliability Constraint*. Therefore, Equation (27) becomes:

$$Pr\{ e_{i,t} - \tilde{E}_{A,i,t}(\tilde{E}_{A,i,t-1}, e_{i,t-1}, \tilde{e}_{i,t}^{max,1}) \leq \delta \} \geq R, \; 0 \leq R \leq 1 \quad (29)$$

where R is the reliability factor (i.e., the probability of meeting the constraint).

b. Total Energy (does not require lower level optimization):

This constraint is applied to validate that the total allocation of flexibility during the optimization period is equal to the maximum total flexibility we get from Stage 1 optimization results. It is unrealistic to allocate more flexibility than is actually available during the optimization period. In addition, it is not beneficial economically to BPA if we keep some flexibility without allocating at the end of optimization period. Storing extra water will increase water level in storage and forebay elevation which needs to be at a fixed range at the end of optimization period, as given by:

$$0 \leq \sum_{t=1}^{14} \text{e}_{\text{i,t}} - \sum_{t=1}^{14} \tilde{e}_{i,t}^{max,1} \leq tol; i = 1,..,N \quad (30)$$

where tol is maximum allowable deviation and is set as 0.2% of total energy $\sum_{t=1}^{14} \tilde{e}_{i,t}^{max,1}$. In the future, we can further tighten the allowance. We consider Equation (30) as a *Reliability Constraint*. Therefore, Equation (30) becomes:

$$Pr\{ 0 \leq \sum_{t=1}^{14} \text{e}_{\text{i,t}} - \sum_{t=1}^{14} \tilde{e}_{i,t}^{max,1} \leq tol \} \geq R, \; 0 \leq R \leq 1 \quad (31)$$

where R is the reliability factor.

c. Minimum Allocation of Energy (does not require lower level optimization):

We assume that there will be sufficient energy generation each period to meet the demands at a minimum: the energy sectors must meet contracted demand. They cannot hold the water for future use without meeting current demand. Therefore, there will not be instances of negative flexibility, as quantified by:

$$\sum_{i=1}^{N} e_{i,t} \geq d_t + \delta \quad (32)$$

where $\sum_{i=1}^{N} e_{i,t}$ is the total allocated energy in MWh which is the sum of the energy allocation from N reservoirs, d_t is the total demand on Day t in MWh, and δ is a very small value set as 1 MWh.

d. Deviation from target allocation (**requires result of lower-level optimization**):

This constraint is given by:

$$\text{dev}_{i,t} = |\left(e_{i,t} - \mu(\tilde{e}_{\text{i,t}})\right| = 0 \quad (33)$$

where $\tilde{e}_{\text{i,t}}$ is the achievable allocated energy in MWh for reservoir i and day t, closest to the target energy allocation $e_{i,t}$ satisfying all the constraints (Physical, Operational and other Probabilistic constraints) in the Nested Optimization with reliability factor R. This constraint $\text{dev}_{i,t}$ is handled by penalizing the violation in the Upper level objective function (Equation 24).

Below we describe the **Lower Level Optimization** mathematical formulation to achieve the mean of achievable energy $\mu(\tilde{e}_{\text{i,t}})$

3.2.2 Lower Level Optimization Problem

While the upper level problem set a target decision of allocation of energy, the lower level problem focuses on safe operational strategy by making sure the target meet the physical and operational constraints of the reservoirs. The objective function is to minimize the deviation of allocation of energy (achievable solution from lower level) from the target allocated flexibility set in the upper level. The lower level optimization will be solved first and will provide us an achievable solution of allocated flexibility closest to the target set in each iterations of the upper level flexible-robust optimization problem. Below is the mathematical formulation of the Stage 2 Lower level optimization problem:

Model Inputs:

- Demand, $\boldsymbol{D} = [d_1, d_2, \ldots, d_{14}]$
- Maximum energy generation each day for each reservoir $\tilde{\boldsymbol{E}}^{max,1}$ obtained from the output of previously mentioned Stage 1 problem.
- Allocated energy (target set from previously mentioned Stage 2 Upper level problem)
- Inflows, $\tilde{\boldsymbol{Q}}_{in}$

Model Decision Variables:

The total outflows at each time step and each reservoir are defined as decision variables in the optimization. Each row represents daily time steps of 14 days (t =14) and each column represents number of reservoirs (N = 3 in this case study). $\boldsymbol{Q}^{`*}_{out,Stage2}$ is represented as follows:

$$\begin{bmatrix} Q_{out,1,1}, Q_{out,1,2}, \ldots\ldots, Q_{out,1,t} \\ \vdots \\ Q_{out,N,1}, Q_{out,N,2}, \ldots\ldots, Q_{out,N,t} \end{bmatrix}$$

Model Objective:

Minimize the total deviation from target:

$$\min_{Q_{out,i,t}} \sum_{t=1}^{14} \left(\sum_{i=1}^{N} e_{i,t} - \tilde{e}_{\text{i,t}}(\text{Q}_{\text{out,i,t}}, \widehat{\text{hd}}_{\text{t}}(\widetilde{\text{FB}}_{\text{t}}, \text{TW}_{\text{t}}), \xi_t)\right)^2 \quad (34)$$

where,
$\grave{\tilde{e}}_{i,t} = \eta \times 9.81 \times \widetilde{hd}_{i,t}(\widetilde{FB}_{i,t}, TW_{i,t}) \times Q_{out,i,t} \times 8.6310 \times 10^{-3} \times \xi_t$ (35)

In this Equation, $e_{i,t}$ is the target energy of reservoir *i* in MWh should be generated to meet the target allocation of flexibility on day *t*, $\grave{\tilde{e}}_t$ is the actual energy generated in MWh on day *t*, η is the efficiency of the reservoir, taken as 0.75, $\widetilde{hd}_{i,t}$ is the head in ft, and ξ_t is taken as 1 hour.

Subject to:
Model Constraints:

- Operational Constraints:

We include all the operational constraints implemented in Stage 1 optimization problem as per Equation 7, 10, 14, 15 and 18. Just as in Stage 1, we convert Equation 7, 10, 15 and 18 into Probabilistic constraints as per equation 9, 12, 17 and 19 respectively.

- Other constraints:

Below constraints are included in the lower level to make sure the lower level optimal solution (Achievable allocation of energy) is within the feasible design space of the upper level. In other words, the achievable allocation of energy does not violate the constraints (Equation 29, 31 and 32) mentioned in the upper level. Since the lower level optimization will be solved first and the achievable solution will always be feasible to the upper level design space, this will guarantee the final optimal solution of Stage 2 bi-level optimization problem be feasible in upper and lower level.

a. *Maximum Allocation of Energy*

$$\grave{\tilde{e}}_{i,t}(Q_{out,i,t}, \widetilde{hd}_{i,t}, \xi_t) - \grave{\tilde{E}}_{A,i,t}(\grave{\tilde{E}}_{A,i,t-1}, \grave{\tilde{e}}_{i,t-1}, \tilde{e}_{i,t}^{max,1}) \leq \delta \qquad (36)$$

where,

$$\grave{\tilde{E}}_{A,i,t} = \grave{\tilde{E}}_{A,i,t-1} - \grave{\tilde{e}}_{i,t-1} + \tilde{e}_{i,t}^{max,1} \qquad (37)$$

$\grave{\tilde{E}}_{A,i,t}$ is the actual availability of energy in MWh at day t after allocating achievable flexibility for *t*-1 days obtained from lower level optimization, $\grave{\tilde{e}}_{i,t-1}$ is the energy generated in MWh on Day $(t-1)$, $\tilde{e}_{i,t}^{max,1}$ is the maximum energy that can be generated in MWh on day *t*, δ is the maximum tolerance for violation and is set as 1 MWh. We consider Equation 36 as a *Reliability Constraint*. Therefore, Equation 36 becomes:

$$Pr\{\grave{\tilde{e}}_{i,t}(Q_{out,i,t}, \widetilde{hd}_{i,t}, \xi_t) - \grave{\tilde{E}}_{A,i,t}(\grave{\tilde{E}}_{A,i,t-1}, \grave{\tilde{e}}_{i,t-1}, \tilde{e}_{t}^{max,1}) \leq \delta\} \geq R, \ 0 \leq R \leq 1 \qquad (38)$$

where *R* is the reliability factor.

b. *Total Energy during the Optimization period*

$$0 \leq \sum_{t=1}^{14} e_{i,t} - \sum_{t=1}^{14} \grave{\tilde{e}}_{i,t}(Q_{out,i,t}, \widetilde{hd}_{i,t}, \xi_t) \leq tol \qquad (39)$$

tol is maximum allowable deviation and is set as 0.2% of total energy $\sum_{t=1}^{14} e_{i,t}$. We consider Equation (39) as a *Reliability Constraint*. Therefore, Equation (39) becomes:

$$Pr\{0 \leq \sum_{t=1}^{14} e_{i,t} - \sum_{t=1}^{14} \grave{\tilde{e}}_{i,t}(Q_{out,i,t}, \widetilde{hd}_{i,t}, \xi_t) \leq tol\} \geq R, \ 0 \leq R \leq 1 \qquad (40)$$

where *R* is the reliability factor.

c. *Deviation from target:*

We have included this constraint to reduce the design space of the lower level problem since we already know that we want to get as close as the target allocation of flexibility $e_{i,t}$ as possible. Thus, we know the global optimal solution for the lower level objective function (Eq 34) is 0. This constraint will set bounds around the global optimal solution so that the optimizer will search for best solution in the neighbourhood of the global optimum point. This will at least guarantee a local convergence in the vicinity of the global convergence instead of converging to local minima which is far away from the true optimal solution

$$| e_{i,t} - \grave{\tilde{e}}_{i,t}(Q_{out,i,t}, \widetilde{hd}_{i,t}, \xi_t)| \leq \delta \qquad (41)$$

where $e_{i,t}$ is the target energy in MWh of reservoir *i* should be generated to meet the target allocation of flexibility on day *t*, $\grave{\tilde{e}}_{i,t}$ is the actual energy generated in MWh on day *t* of reservoir i, δ is the maximum deviation from the target and is set as 1 MWh. We consider Equation 41 as a *Reliability Constraint*. Therefore, Equation (41) becomes:

$$Pr\{| e_{i,t} - \grave{\tilde{e}}_{i,t}(Q_{out,i,t}, \widetilde{hd}_{i,t}, \xi_t)| \leq \delta\} \geq R, \ 0 \leq R \leq 1 \qquad (42)$$

where *R* is the reliability factor.

d. *Minimum Allocation of Energy to meet daily demands:*

$$\sum_{i=1}^{N} \grave{\tilde{e}}_{i,t} \geq d_t + \delta \qquad (43)$$

where $\sum_{i=1}^{N} \grave{\tilde{e}}_{i,t}$ is the total achievable energy in MWh which is the sum of the energy generation from N reservoirs, d_t is the total demand on Day *t* in MWh, and δ is a very small value set as 1 MWh. We consider Equation 43 as a *Reliability Constraint*. Therefore, Equation (43) becomes:

$$Pr\{\sum_{i=1}^{N} \grave{\tilde{e}}_{i,t} \geq d_t + \delta\} \geq R, \ 0 \leq R \leq 1 \qquad (44)$$

where *R* is the reliability factor.

To summarize the model outputs:

***Lower Level Optimization*: Model Output**:

- Optimal Outflow corresponding to achievable allocation of flexibility, $\boldsymbol{\grave{Q}^{*}_{out,Stage2}}$
- Deviation from target allocated flexibility, $\boldsymbol{\varepsilon_h}$
- Achievable allocated energy $\boldsymbol{\grave{\tilde{E}}}$ closest to the target allocation ***E***.

***Upper Level Optimization*: Model Output**:

- Optimal Outflow corresponding to achievable allocation of flexibility, $\boldsymbol{\grave{Q}^{*}_{out,Stage2}}$
- Achievable allocated energy $\boldsymbol{\grave{\tilde{E}}}$ closest to the target allocation **E**.
- Maximum Expected Net Revenue in selling the Flexibility, $\boldsymbol{\tilde{R}^{max}}$

4. MATHMATICAL APPROACH

Figure 5 shows an overall framework of methodologies used in each individual stages and levels of the model (Stage 1, Stage 2 upper level and lower level) to solve the respective optimization problems. Bi-level optimization is known to be computationally very expensive [36] and one of the research interests is to decrease the computational cost of the model since energy sectors would need to make decisions within a specific timeframe. So far with the Bi-level approach and the integration between robust optimization and the Real Option model, we have claimed to have a better and robust decision-making process in allocating flexibility in Hydro-energy sectors. It is evident that with such complexities in the model, the computational cost will be significant. A factor to consider is that if the model takes days to provide decisions, it will not be reasonable or efficient for practical implementation. Therefore, as our focus in this paper is to build a model and also ensuring that can be applicable to real world scenarios, we have used a truncated KL-expansion [37] to reduce the dimension of the problem and thereby obtain faster uncertainty quantification and propagation of uncertainty with minimum loss of accuracy. Previous attempts have been done to reduce the dimension of the decision variables using a truncated KL- expansion [38]. In this paper, we reduced the dimensions of the uncertain input parameters of the multi-reservoir system. We have further extended the work on dimension reduction during simulation or uncertainty propagation of the system and quantification of uncertainty of the quantities of interest using Stochastic Collocation methods [39] with a Sparse Grid technique [40]. Utility Theory [41] and Inverse Reliability method [42] are applied for evaluating robust objective and validating probabilistic constraints. We provide detail descriptions on the methodologies in the below sections.

4.1 Uncertainty Quantification of the Input Parameters:
We considered multiple sources of Uncertainty of Inputs: Inflows and Market Price of selling Electricity. Given an ensemble of forecast at discrete times, we quantified the uncertainty by applying a *Truncated KL- Expansion* of the forecast of the input data and therefore generated $M1$ and $M2$ number of realizations for Inflows and Price, respectively. Below we follow the formulation of the Truncated KL-expansion from [30], applied here to both Inflows and Price Uncertainty:

$$Q_{in,t}\big(t, y_{1J}(\omega)\big) = \mu_{Q_{in,t}}(t) + \textstyle\sum_{i=1}^{N1} \sqrt{\lambda_{1,i}}\psi_{1,i}(t) y_{1J,i}(\omega) \tag{45}$$

$$P_t\big(t, y_{2J}(\omega)\big) = \mu_{P_t}(t) + \textstyle\sum_{i=1}^{N2} \sqrt{\lambda_{2,i}}\psi_{2,i}(t) y_{2J,i}(\omega) \tag{46}$$

$$1J = 1{:}\,M1; \qquad 2J = 1{:}\,M2$$

Where $Q_{in,t}\ (t, y_{1J}(\omega))$ and $P_t\big(t, y_{2J}(\omega)\big)$ are the 1J[th] and 2J[th] stochastic realizations respectively for $\widetilde{Q_{in,t}}$ and $\widetilde{P_t}$ with sample means $\mu_{Q_{in,t}}(t)$ and $\mu_{P_t}(t)$; λ_1, λ_2 are eigenvalues and ψ_1, ψ_2 are eigenfunctions of the sample covariance computed from the ensemble forecast of the Inflows and Price uncertainty respectively; $y_{1J}(\omega)$ and $y_{2J}(\omega)$ are (mean 0, variance 1) random variables for $\widetilde{Q_{in,t}}$ and $\widetilde{P_t}$ respectively. N1 and N2 are the number of terms required to cover ≥ 90% of the variance. Therefore, in this problem, we considered N1 and N2= 3. Therefore, we overcame the curse of dimensionality of the high dimensional random space and reduced the dimensions of each uncertain Input parameters from 14 (daily timestep of 14 days optimization period) to 3 dimensions, covering ≥ 90% of the variance and with reduction of computational expenses to generate the realizations of uncertain input parameters. This will then significantly reduce the overall computation effort of the calculation of the expectation and variance of all the Quantities of Interests (QoI) such as Storage, Forebay, Head, Hydroenergy and Revenue in each iteration of the optimization. Considering a single reservoir (i =1), when we compare this technique with Full Tensor Numerical Integration for 3 nodes in computing the expectation or variance of objective function (2), we see we need to run 3^{14} runs (n^m, where n is number of nodes and m is the number of uncertain variables). However, with the reduced random space to 3 applying KL-expansion, we only need 3^3 runs (for Full Grid) to compute the expectation and variance. It is obvious to mention that considering multiple reservoirs, the reduction in computation cost will only improve. Therefore, with this approach we could reduce the computational expense of the simulation of QoI and function evaluations significantly which is always a challenging task in robust optimization of large-scale problems.

4.2 Uncertainty Propagation:
To propagate the uncertainty of the inputs through the system, we started with applying *Stochastic Collocation* method and generated M1 and M2 KL-realizations of the inputs respectively (Equation 45, 46) on the collocation points considering a Full Grid. In the previous section, we have already explained the idea of applying KL-expansion and how this technique succeeded in computational cost reduction. However, considering a Sparse Grid technique instead of Full Grid, we have further reduced the computational effort of simulations and function evaluations without sacrificing any significant accuracy. We further assumed a uniform distribution for the random variables. Therefore, *Level 3 Smolyak Sparse Grid Clenshaw-Curtis* [43] collocation nodes z_j and weights w_j has been considered in this problem. For each j, 1j = 1:m1; 2j = 1:m2 and m1 < M1; m2 < M2, we evaluated the Input functions $Q_{in,t}\big(t, z_j\big)$ and $P_t\big(t, z_j\big)$ respectively. Thus, equations (44), (45) has been modified as below:

$$Q_{in,t}\big(t, z_{1j}\big) = \mu_{Q_{in,t}}(t) + \textstyle\sum_{i=1}^{N1} \sqrt{\lambda_{1,i}}\psi_{1,i}(t) z_{1j,i} \tag{47}$$

$$P_t\big(t, z_{2j}\big) = \mu_{P_t}(t) + \textstyle\sum_{i=1}^{N2} \sqrt{\lambda_{2,i}}\psi_{2,i}(t) z_{2j,i} \tag{48}$$

$$1j = 1{:}\,m1; \qquad 2j = 1{:}\,m2$$

To propagate the uncertainty through the system, we simulate deterministically the corresponding quantities of interests like Storage, Forebay, Head, Hydroenergy and Revenue at the same collocation points.

4.3 Evaluating Robust Objective and Validation of Probabilistic Constraints:
We have modified our objective function into *Utility Function* [41] to calculate the *maximum/minimum Expected Utility*. Applying the Truncated KL expansion and Stochastic Collocation method as described in the previous sections, we calculate our objective function, Y deterministically and then convert to the Utility Function U(Y). We calculate the Expected Utility, E(U(Y)) using the Clenshaw-Curtis collocation weights.

Since we are doing robust optimization, we have converted all the constraints into Probabilistic Constraints as shown in Problem description. Once we propagate the input uncertainty through the system using Stochastic Collocation and Sparse Grid methods and evaluate the means and standard deviation of QoI, we validate the Probabilistic Constraints using Inverse Reliability Method [42].

4.4 Optimization Method and Convergence Criteria:
We applied *Sequential Quadratic Programming (SQP) method* to solve the proposed Optimization Problem. We have used the in-built MATLAB function "fmincon" for the SQP algorithm [44] to solve optimization problems in all stages and levels as described in Sections 3. The convergence criteria are step size tolerance, constraint violation tolerance, function tolerance; all set as $\xi = 10^{-5}$ and maximum function evaluations set as 40000.

5. RESULTS
In this section, we will show the efficiency of the Real Option model and finally the results of the **Two-Stage Bi-Level Flexible-Robust Optimization** model having daily time steps for the 14 day optimization period. To illustrate the efficiency of the integration of the RO model with Robust optimization framework in optimal decision of allocation of flexibility, we consider a simple Single Reservoir (Grand Coulee) problem with simplified Two-Stage Single Level optimization model. In this model, the objective of Stage 1 and Stage 2 upper level is similar to our proposed model in this paper. However, unlike our proposed model, only one reservoir has been considered with simplified version of Operational Constraints, considering only Storage constraints and without any implementation of bi-level structure. As we have mentioned, the bi-level structure has been incorporated in the proposed model to integrate the realistic complex operational constraints (Storage, Forebay, Power etc). Reader can further read the detailed mathematical formulation of the simplified model in [45]. The reason we have started with a simplified model with minimum constraints is to make sure we have sufficient design solutions. If the design space is too small, we may not see any benefits of RO analysis. Thus, for understanding the efficiency of integration of RO model in providing better decisions by reducing possible future shortages, we first considered a simplified **Single Level Model** with large design space having many feasible solutions. We show the decisions provided by the model with and without the consideration of Real Option Analysis. We therefore have two model policies to evaluate:

a) **Policy 1** in which the Real Option valuation is *ignored*.
b) **Policy 2** in which the Real Option valuation is *included*.

Table 3 shows the initial conditions of Storage, Inflows and Outflows of GCL used in the models. Uncertainty has been considered only on Inflows. Figures 6, 7 show the results of the 14-day period from the optimization with 27 different inflow scenarios. Figure 6 shows the increase in Net Revenue one will attain by selling flexibility following Policy 2 (i.e., considering the Real Option analysis) instead of Policy 1. Figure 7 shows the increase (percentage) for the respective 27 scenarios. We can see the percentage increases are generally in the range of 2% to 40% which is roughly 10 to 80 thousand dollars for this case study. However, when there are very high inflows and therefore high storage of water, the real option model will not provide significant valuations as the probability of future shortages is negligible, thus the future value of flexibility is dominated by the selling price of electricity. In such case, the energy sectors can neglect the RO model and allocate flexibility when there is higher selling price. Therefore, we can see in the scatter plots (Fig 6, 7), that a few scenarios have no increase in revenue with the RO valuation and therefore we get the same results from Policy 1 and 2, which seems reasonable. Overall, we found an improvement in decision making of allocation of flexibility by integrating the RO model with the robust optimization framework, especially in case of low inflows or storages (dry seasons). Now that we understand the efficiency of RO model, we will move forward and show the results of our **Two-Stage Bi-Level Flexible-Robust Optimization** model as described in Section 3 for a multi-reservoir system. Also, two sources of uncertainty are considered: Price of electricity and Inflows. We have considered 3 dams (Grand Coulee, Lower Granite and McNaire) with two channels of Inflows with uncertainty. In this 3 reservoir system, we have two uncertain input channels of inflows into GCL and LWG. However, the third reservoir, MCN is located at the downstream of the GCL and LWG; therefore, the inflow into MCN is the (deterministic) value of the sum of the deterministic robust decision of total outflows of GCL and LWG. Table 3 shows the initial conditions of Storage, Inflows and Outflows of GCL, LWG and MCN. In this case study, we assumed demand, D = 0.95*μ($\tilde{E}^{max,1}$)(Stage1 output). We have considered the historic data of the Inflows of 3 reservoirs from the NOAA website and generated the predictions with the help of KL expansion. Figure 8, 9 and 10 are the realizations of Inflows of GCL, LWG (both calculated from Eq. 48) and log of Prices (calculated from Eq. 49) respectively generated from truncated KL-expansion covering ≥ 90% variance. The realizations shown are selected from a larger population of realizations (from a Full Tensor Grid) using a Sparse Grid for the sake of increasing computational efficiency of our model without sacrificing significant accuracy. We use log of Price to avoid any negative values in Price realizations which will be unrealistic as Price cannot be negative.

The results obtained from the Bi-Level Flexible-Robust Optimization model (considering Real Option valuation) have been compared with the standalone Robust Optimization model (ignoring Real Option valuation). Figure 11 shows the respective optimal outflows of the 3 reservoirs. Figure 12 represents the optimum allocation of total flexibility (combining GCL, LWG and MCN) among all 3 reservoirs, over 14 days. Figure 13, 14 and 15 represent the optimal decision of allocation of energy from GCL, LWG and MCN, respectively, to attain the allocation of total flexibility as shown in Fig 12. As the uncertainty on Inflows increases in the future (Fig 8, 9), most of the allocation is done in later time periods to minimize a chance of shortage. The Real Option analysis provide the valuation of holding the water for future use. However, some of the allocation of flexibility has been done on the early days due to higher prices (Fig 10) on these days, thereby to increase revenue. The uncertainty on the Prices has been handled by Robust Optimization.

Thus, the proposed integrated Flexible-Robust optimization model helps manage the trade-off between allocations of flexibility now due to higher market prices of electricity (maximizing revenue; handled by Robust optimization) or later due to higher uncertainty on Inflows (minimizing risk of shortage; handled by Real Option model). The expected net revenue obtained is $3935336 and the computational time taken is 1 hr. approx. (Processor I7-4720HQ CPU @2.6GHz, 16GB RAM and used the MATLAB Version 2016a). The Net Revenue is the revenue achieved from selling the flexibility and meeting the required demand. Next, we run the same case study with Two-stage Bi-level Robust Optimization model where Stage 1 is the same as our proposed model, but the Real Option analysis has not been considered in the Stage 2 Upper Level. We observe similar optimal solutions as for Two-Stage Bi-level Flexible Robust Optimization. However, when we evaluate the RO value for this optimal solution, we achieve a significant option value ($\gg 0$). Thus, there is a role in RO analysis, but we still get similar solutions in both cases. This is most likely due to the fact that by including the large number of constraints at both levels of the optimization reduces the design space, which provides limited feasible solutions. Thus, even our optimal solution has significant future value of flexibility which the decision makers could lose and thereby lose revenue, this is the best feasible solution. In this case, although the RO model did not improve the decision, it still plays a significant role in validating that the optimal solution is the best possible decision; we will not get a better solution which is feasible (not violating physical, operational constraints of the reservoirs) and can also increase net revenue by reducing possible future shortages. With more solutions in the design space (large design space), we can see potential improvement in the optimal decision in allocating flexibility due to integration of RO model with Robust optimization framework as shown in Fig 6, 7 (for Single Reservoir System).

Next we compare our proposed model with the Single Stage Single Level Robust Optimization model as described in [17] for our 3 reservoir system case study. Table 4 provides the comparison of solutions. In our proposed model, the first stage gives the maximum energy generation over the optimization period unlike in the Single Stage Single Level Robust Optimization model. Thus in our case study of 14 days optimization period, we see our proposed model provides more total power generation over the 14 day period and thereby more flexibility to allocate, thereby generating higher revenue. Also, since our proposed model has a flexible-robust objective unlike in the Single Stage Single Level Robust optimization model, future value of flexibility is evaluated using RO analysis, reducing future shortages and increasing Net Revenue. Therefore, we can see an improvement [$30764 (1%)] in the Net Revenue from our proposed Two-Stage Bi-level Robust Optimization model for two weeks' optimization period.

6. CONCLUSION

This research has been conducted to resolve the dilemma faced by many energy sectors in their optimal decisions of allocation of energy considering high future uncertainty and consideration of various complex constraints in the multi-reservoir system. To provide an optimal operation in energy allocation problems, we have presented the mathematical formulation of a proposed Bi-Level Flexible Robust optimization model for multi-reservoir system. We demonstrated our approach on 3-reservoirs of the Lower Columbia River system in a case study having two channels of inflows from upstream. Two sources of uncertainty (Inflows, Price) have been considered in this case study to demonstrate the handling of uncertainty in the model. The bi-level programming mitigates the compatibility issues of the integration of RO model with the robust optimization framework by dividing the overall objective into two strategies. The upper level is the economic strategy which solely focuses on maximizing revenue while minimizing future probability of shortage of flexibility due to current allocation, by successfully integrating robust optimization with the real option model (Flexible-Robust Objective). The lower level is the operational strategy which solely focuses on the safe and feasible operations in the allocation of flexibility. The results demonstrate that the Bi-Level Flexible Robust Optimization model provides decisions with higher generation of flexibility and better net revenue. Therefore, the proposed model is more efficient than the existing standalone single level robust optimization model in optimal decision of allocation of flexibility by considering Real Option analysis, especially in dry seasons with lower inflows and/or storages. Even when our proposed model (Bilevel model integrated with RO model) did not show improvement in the decision (compared to the Bilevel model without RO model) due to lack of feasible solutions (limited design space) for the multi-reservoir system, it ensures the decision makers in the energy sectors that the best possible decision has been made which is feasible to the operations and has the minimum risk of future shortages. Thus, the role of Real Option analysis may not always provide a better decision, but its inclusion acts as a validation that we have found the best optimal solution considering the balance of generating revenue in the present while managing water shortage risk in the future. The Real Option model can be implemented with any general energy allocation problem which deals with loss of revenue in the case of future shortage. The computational efficiency of the model has been increased by implementing KL-Expansion and Stochastic collocation with Sparse Grid methods for uncertainty quantification of the inputs and uncertainty propagation to the QoI. For future work, we will work on incorporating uncertainty on decision variables and demand. Further research on improving the accuracy of the model will be done by implementing different classical, heuristic and/or the combination of both algorithms in solving large scale complex bi-level energy allocation problems.

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LIST OF FIGURES

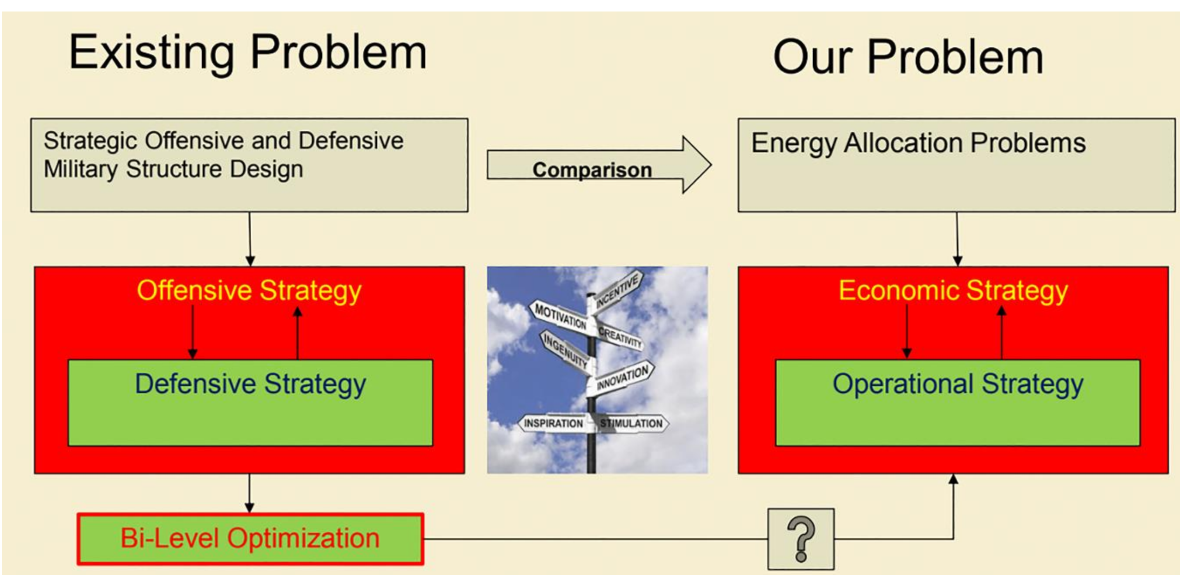


Figure 1. Motivation of Bi-level programming into Energy Allocation Problems

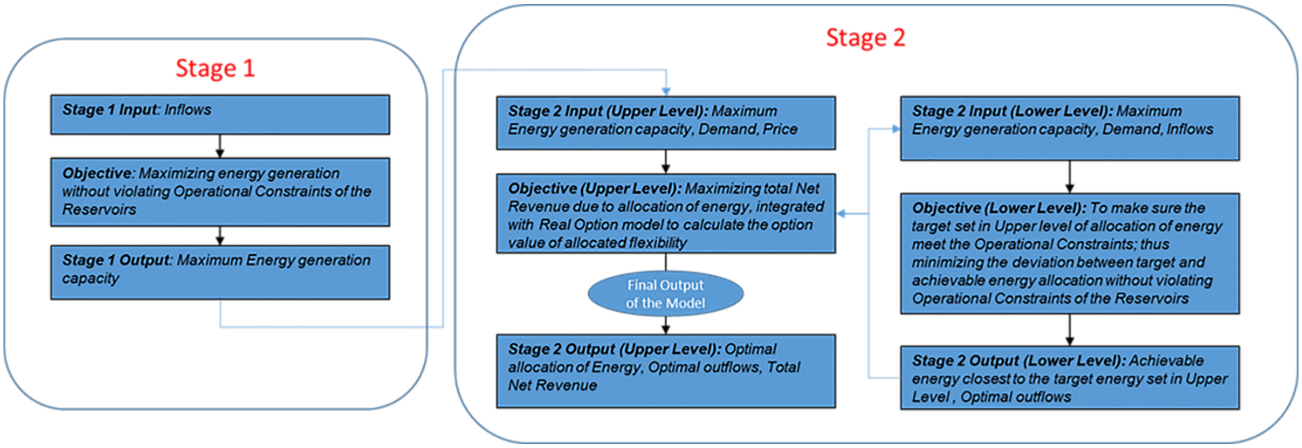


Figure 2. Conceptual Framework

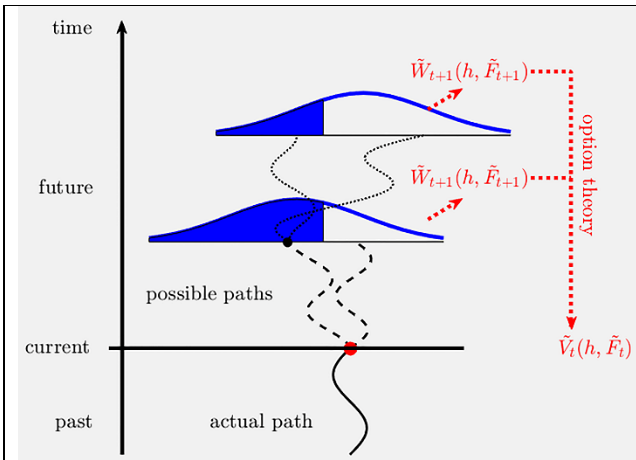
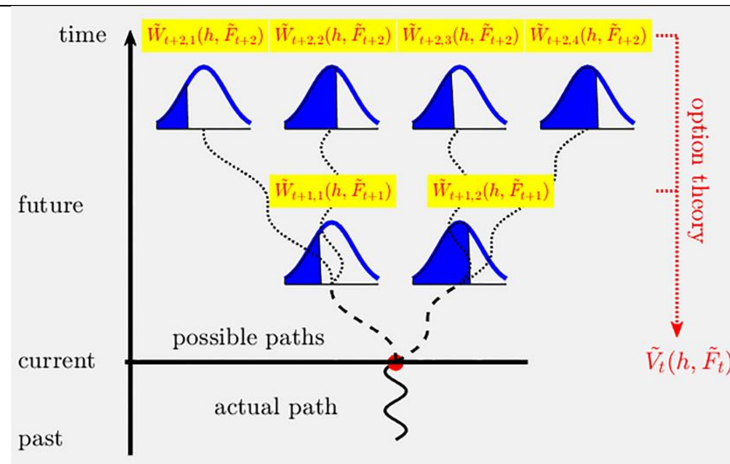


Figure 3. Evolution of States



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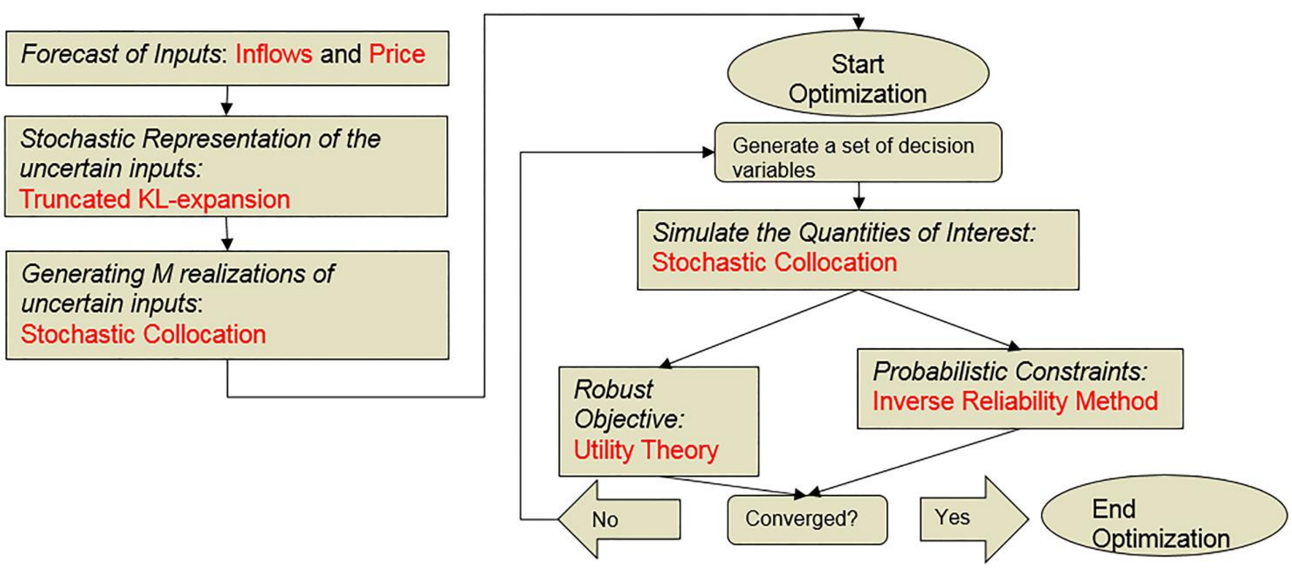


Figure 5. Overall Methodology

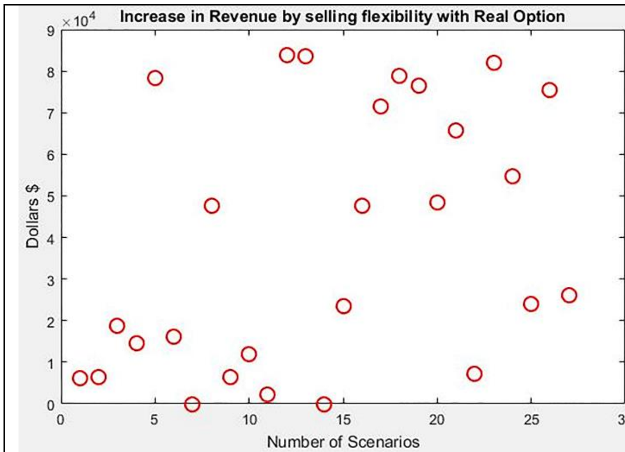


Figure 6. Increase in Revenue ($) by selling only flexibility with Real Option

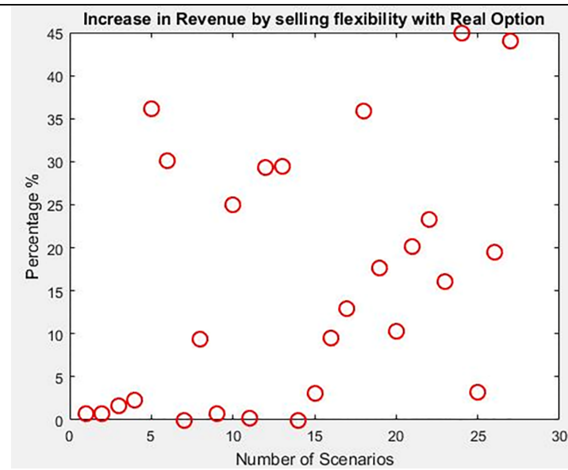


Figure 7. Increase in Revenue (%) by selling only flexibility with Real Option

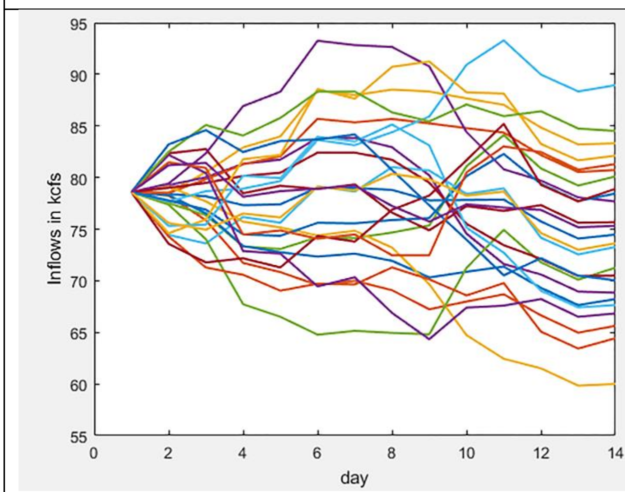


Figure 8: KL Realizations of GCL Inflows

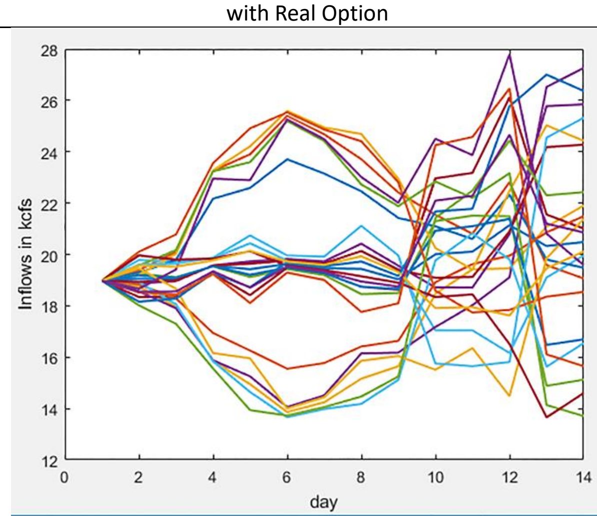


Figure 9: KL Realizations of LWG Inflows

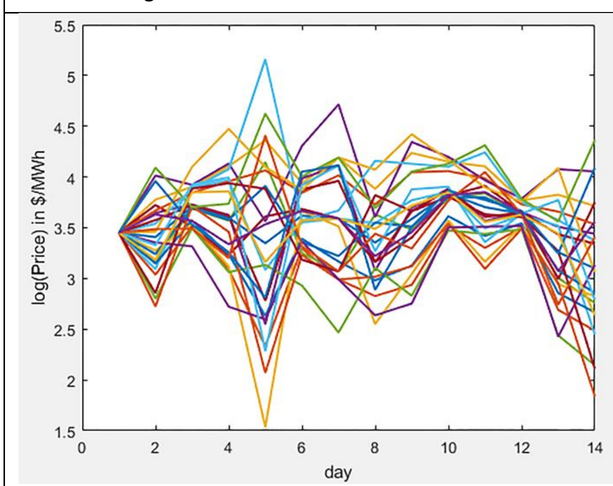
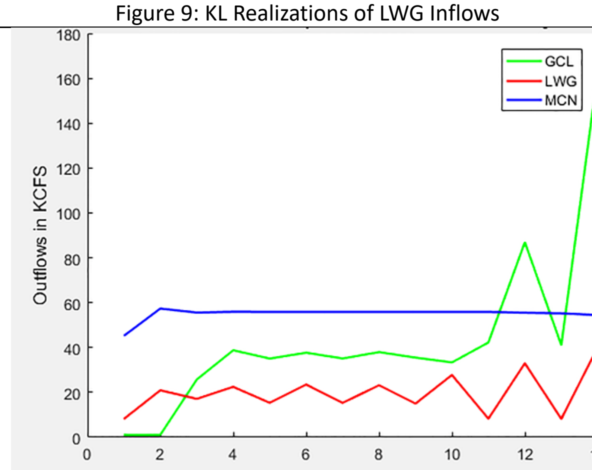


Figure 10: KL Realizations of Log Price



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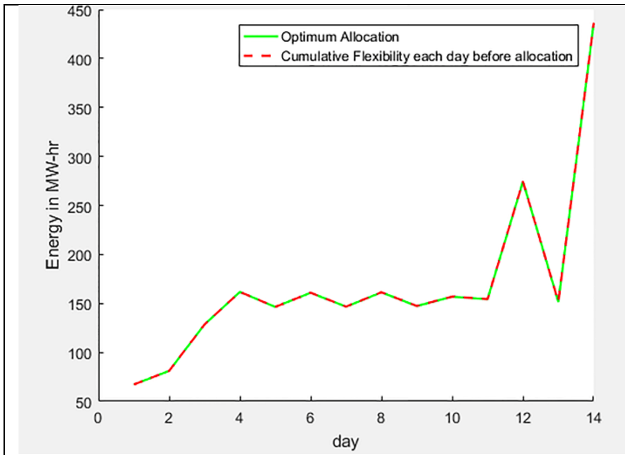

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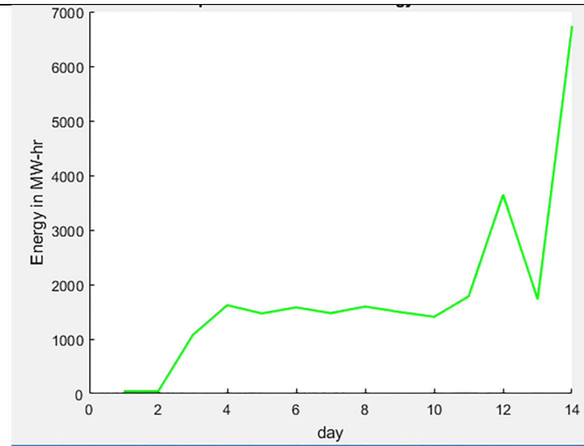

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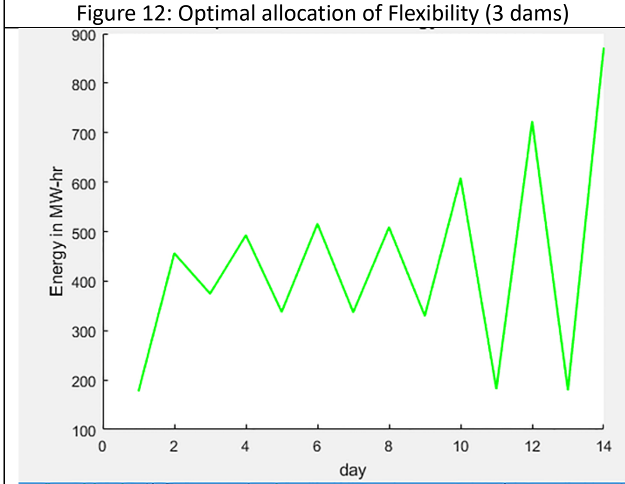

Figure 14: Optimal allocation of Energy of LWG

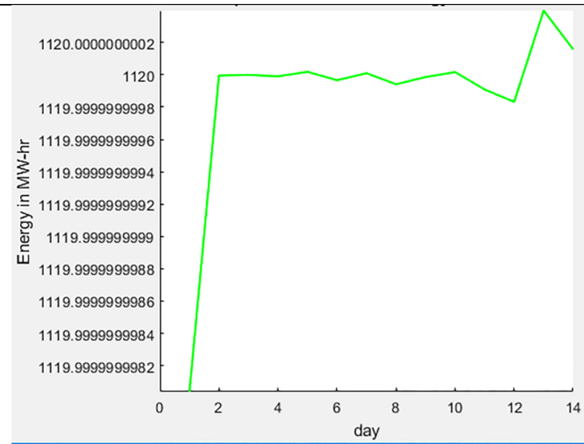

Figure 15: Optimal allocation of Energy of MCN

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Table 1. Coefficients used to calculate Tail-water (Eq. 5, 6)

Coefficients	Grand Coulee Reservoir	Lower Granite Reservoir	McNaire Reservoir
A	447.97	122.81	18.60
B	0.0901	0.0210	0.0202
C	0.5286	0.8060	0.9234

Table 2. Coefficients used to calculate Forebay Elevation (Eq. 11)

Coefficients	Grand Coulee Reservoir	Lower Granite Reservoir	McNaire Reservoir
A	-3.63×10^{-6}	-3.6467×10^{-4}	0
B	0.0406	0.2689	0.0571
C	1208	724	334.5

Table 3. Initial Conditions for Optimization

	Grand Coulee Reservoir	Lower Granite Reservoir	McNaire Reservoir
Storage (kcfs-day)	2260	47.8	79.1
Inflows (kcfs)	81.4519	11.0446	131.2668
Outflows (kcfs)	55.7246	13.5364	127.0271

Table 4. Comparison of the models

	Two Stage Bi-level Flexible-Robust Optimization Model	Single Stage Single Level Robust Optimization Model
Total power generation MW (14 days)	1139976	1138632
Increase in Power generation MW	1344 (0.12 %)	
Net Revenue at Optimal Decision*	$3935336	$3904572
Improvement	$30764 (1%)	

Note*: We run the Single Stage Single Level Robust Optimization Model (ignoring Real Option analysis during optimization) and then calculate the Net revenue at optimal decision provided by the model considering Real Option analysis.

Table 5. Key observations from the results (Fig. 6 -15)

Figure #	Key Observations
6	Increase in Revenue from optimal decision by integrating Real Option Model in Robust optimization framework for most of the Scenarios (test cases).
7	Similar observation as 14 in terms of percentage. We can see mean increase from 27 different inflow scenarios (test cases) is around 15-20%.
8, 9, 10	We provide a sample of 27 truncated KL generated realizations of inflows in Grand Coulee, Lower Granite reservoirs and log of Price respectively, which quantifies >90% of the overall uncertainty, where we see the uncertainty in inflows is greater in future days and the same is roughly constant for Prices.
11	Optimal decision (Outflows) from the proposed model, as we can see higher total allocation (GCL + LWG +MCN) in the future days to account for higher uncertainty in future and minimize the possibility of future shortage
12, 13, 14, 15	Similar observation as 9 in terms of Energy, where figure 10 provides the total optimal allocation in Energy and figure 11, 12 and 13 provides the optimal allocation in GCL, LWG and MCN respectively.

