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Key Points:

- Spectral dimensionality-reduction method couples within optimization routine
- High-dimension decision variable is transformed to fewer coefficients in frequency domain
- Superior performance for optimizing multireservoir operation due to reduced dimension

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Dimension reduction of decision variables for multireservoir operation: A spectral optimization model

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Abstract Optimizing the operation of a multireservoir system is challenging due to the high dimension of the decision variables that lead to a large and complex search space. A spectral optimization model (SOM), which transforms the decision variables from time domain to frequency domain, is proposed to reduce the dimensionality. The SOM couples a spectral dimensionality-reduction method called Karhunen-Loeve (KL) expansion within the routine of Nondominated Sorting Genetic Algorithm (NSGA-II). The KL expansion is used to represent the decision variables as a series of terms that are deterministic orthogonal functions with undetermined coefficients. The KL expansion can be truncated into fewer significant terms, and consequently, fewer coefficients by a predetermined number. During optimization, operators of the NSGA-II (e.g., crossover) are conducted only on the coefficients of the KL expansion rather than the large number of decision variables, significantly reducing the search space. The SOM is applied to the short-term operation of a 10-reservoir system in the Columbia River of the United States. Two scenarios are considered herein, the first with 140 decision variables and the second with 3360 decision variables. The hypervolume index is used to evaluate the optimization performance in terms of convergence and diversity. The evaluation of optimization performance is conducted for both conventional optimization model (i.e., NSGA-II without KL) and the SOM with different number of KL terms. The results show that the number of decision variables can be greatly reduced in the SOM to achieve a similar or better performance compared to the conventional optimization model. For the scenario with 140 decision variables, the optimal performance of the SOM model is found with six KL terms. For the scenario with 3360 decision variables, the optimal performance of the SOM model is obtained with 11 KL terms.

1. Introduction

The operation of a multireservoir system generally requires simultaneous operational decisions (e.g., a time discretization of outflows) for each reservoir in the system. For a system with a large number of reservoirs, hundreds or thousands of decision variables may be introduced in optimization of multireservoir operation. This high dimensionality of decision variables greatly increases the complexity of the optimization since the search space grows exponentially with the dimension of the decision variables [*Parsons et al*., 2004; *Houle et al*., 2010]. Finding global optimal solutions for multireservoir operation is challenging and time consuming within such a large search space that is often nondifferentiable, nonconvex, and discontinuous [*Yeh*, 1985; *Wurbs*, 1993; *Cai et al*., 2001; *Labadie*, 2004]. Therefore, reducing dimensionality of the problem is critical to improve the performance of optimization when involving multiple reservoirs [*Archibald et al*., 1999; *Lee and Labadie*, 2007].

System decomposition is a widely used method to reduce dimensionality of multireservoir operation [*Turgeon*, 1981; *Archibald et al*., 2006]. System decomposition consists in dividing a reservoir system into smaller subsystems. In this way, the large-scale optimization is converted to many small-scale optimization problems, which are solved separately. A model based on stochastic programming and Benders decomposition was proposed and applied to a 37-reservoir system [*Pereira and Pinto*, 1985]. *Finardi and Silva* [2006] combined sequential quadratic programming with a decomposition method to solve a large-scale optimization problem with 18 hydro plants. Despite advantages of reducing a complex problem into a series of small tractable tasks, the decomposition-based optimization generally finds local optima rather than global optima [*Nandalal and Bogardi*, 2007].

Aggregation of a reservoir system [*Saad et al*., 1994] is another way to solve the high-dimensional optimization problem by developing an auxiliary model which normally projects the whole system into a hypothetical single reservoir. Subsequently, disaggregation of the composite operational strategy is needed for deriving control policies for an individual reservoir. A good review of this approach can be found in *Rogers et al*. [1991]. Although aggregation/disaggregation methods are conceptually straightforward for reducing the dimensionality of large-scale problems, careful selection of the principles and intensive efforts are required at each aggregation and disaggregation step [*Rogers et al*., 1991]. In addition, there might be errors that are introduced by aggregating/disaggregating the problem representation [*Nandalal and Bogardi*, 2007].

In addition to decomposition and aggregation, other dimension-reduction methods were applied to the optimization of multireservoir operation. *Saad and Turgeon* [1988] applied Principal Component Analysis (PCA) for reducing the number of state variables in the stochastic long-term multireservoir operating problem. In their research, some significant state variables were observed based on the correlation of the inflows and reservoir trajectories thus the original problem of 10 state variables were reduced to a problem of 4 state variables. *Fu et al*. [2011] used Global Sensitivity Analysis (GSA) to calculate the sensitivity indices of all decision variables and define a simplified problem that considers only the most sensitive decision variables. These studies aim to solve a simplified optimization problem with less state or decision variables. The reduction of the number of decision variables reduces the complexity of the problem. However, it may also reduce the accuracy of the optimization as some decision variables are explicitly ignored.

The present study proposes a new optimization model which aim to reduce the dimensionality of multireservoir operation by transforming the decision variables from time domain to frequency domain. The proposed model does not decompose or aggregate the system, hence avoiding local optima. The model builds a connection of the decision variables between time domain and frequency domain by coupling a spectral dimensionality-reduction method with an evolutionary optimization algorithm (i.e., NSGA-II). This connection allows a transformation between discrete decision variables in time domain and undetermined coefficients, i.e., decision variables in frequency domain. In the optimization algorithm, the large number of decision variables is first transformed to fewer coefficients, the number of which is predetermined. Operators of the optimization algorithm (e.g., crossover) are only applied to the coefficients rather than the large number of decision variables in time domain. This approach greatly reduces the search space. Once the frequency-domain coefficients are determined at a given generation, they are transformed back to the original decision variables in time domain in order to check the violation of constraints and to evaluate the objectives. In this way, the formulation of the problem (i.e., decision variables, objectives and constraints) is fully preserved during the optimization process, and hence, does not simplify the representation of the problem. The proposed optimization model is compared to a conventional optimization model (without dimension reduction) using a 10-reservoir system in the Columbia River of the United States as test case. The study also includes a sensitivity analysis on the number of coefficients that needs to be predetermined. Finally, the limitations of this approach and future work are discussed.

2. Methodology

2.1. Karhunen-Loeve (KL) Expansion

The Karhunen-Loeve (KL) expansion [*Kosambi*, 1943; *Karhunen*, 1947; *Williams*, 2015] is a representation of a random process as a series expansion involving a complete set of deterministic functions with corresponding random coefficients. Consider a random process of $Q(t)$ and let $\bar{Q}(t)$ be its mean and $C(s,t) = cov(Q(s), Q(t))$ be its covariance function. The $Q(s)$ and $Q(t)$ are variables at different time step. Then, the KL expansion of $Q(t)$ can be represented by the following function

$$Q(t)=\overline{Q}(t)+\sum_{k=1}^{\infty}\sqrt{\lambda_k}\psi_k(t)\xi_k \tag{1}$$

where $\{\psi_k, \lambda_k\}_{k=1}^{\infty}$ are the orthogonal eigen functions and the corresponding eigen values, respectively, and are solutions of the following integral equation

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$$\lambda\psi(t)=\int C(s,t)\psi(s)ds \tag{2}$$

Equation (2) is a Fredholm integral equation of the second kind. When applied to a discrete and finite process, this equation takes a much simpler form (discrete) and we can use standard algebra to carry out the calculations. In the discrete form, the covariance matrix *C*(*s*,*t*) is represented as an N × N matrix, where N is the time steps of the random process. Then, the above integral form can be rewritten as $\sum_{s,t=1}^{N} C(s,t)\Psi(s)$ to suit the discrete case.

$\{\xi_k\}_{k=1}^{\infty}$ in equation (1) is a sequence of uncorrelated random variables (coefficients) with mean 0 and variance 1 and are defined as:

$$\xi_k=\frac{1}{\sqrt{\lambda_k}}\int\left[Q(t)-\overline{Q}(t)\right]\psi_k(t)dt \tag{3}$$

The form of the KL expansion in equation (1) is often approximated by a finite number of discrete terms (e.g., M), for practical implementation. The truncated KL expansion is then written as:

$$Q(t)\approx\overline{Q}(t)+\sum_{k=1}^{M}\sqrt{\lambda_k}\psi_k(t)\xi_k \tag{4}$$

The number of terms *M* is determined by the desired accuracy of approximation and strongly depends on the correlation of the random process. The higher the correlation of the random process, the fewer the terms that are required for the approximation [*Xiu*, 2010]. One approach to roughly determine *M* is to compare the magnitude of the eigen values (descending order) with respect to the first eigen value and consider the terms with the most significant eigen values. With the truncated KL expansion, the large number of variables in time domain is reduced to fewer coefficients in the transformed space (i.e., frequency domain). The KL expansion has found many applications in science and engineering and is recognized as one of the most widely used methods for reducing dimension of random processes [*Narayanan et al*., 1999; *Phoon et al*., 2002; *Grigoriu*, 2006; *Leon et al*., 2012; *Gibson et al*., 2014]. Since the KL expansion method transforms variables from time domain to frequency domain, this method is referred as a spectral method for dimensionality reduction [*Le Maître and Knio*, 2010].

2.2. NSGA-II

NSGA-II [*Deb et al*., 2002] is one of the most popular methods for optimization of multiobjective problem (MOP) and increasingly receives attention for practical applications [*Prasad and Park*, 2004; *Atiquzzaman et al*., 2006; *Yandamuri et al*., 2006; *Sindhya et al*., 2011; *Chen et al*., 2014; *Leon et al*., 2014]. The NSGA-II follows the primary principles of the classical Genetic Algorithm by mimicking evolution process of genes using selection, crossover, and mutation operators. For a MOP, a set of nondominated solutions is obtained according to the concept of nondominance, rather than a single solution. The final set of nondominated solutions that satisfies the stopping criteria is referred to as Pareto-optimal solutions or Pareto front. The steps of the NSGA-II applied to reservoir operation are illustrated in Figure 1. A parent-centric crossover (PCX) [*Deb and Jain*, 2011] is adopted in the NSGA-II instead of the original simulated binary crossover (SBX). The PCX was found to be superior in the rotated/epistasis optimization problem [*Hadka and Reed*, 2013; *Woodruff et al*., 2013], where the decision variables have strong interaction between each other.

2.3. Coupling the KL Expansion and the NSGA-II

Incorporating the KL expansion into an optimization framework is not intuitive because the KL expansion is mainly used for random processes. The fundamental novel contribution of this work is to introduce the perspective that the optimal decision variables can be treated as a realization of a random process, which itself can be described by a collection of previous realizations. The random coefficients in the KL expansion are then understood to be the unknown coefficients of the desired realization to be determined, and therefore to be the new decision variables.

One of the major uncertainties in reservoir operation is the inflow discharge, which can be treated as a random process with a predefined probability distribution, such as Log-Pearson type III [*Durrans et al*., 2003]. Historical inflows can be thought as different realizations of this distribution. On the other hand, the purpose of the optimization is to find optimal operational policies, i.e., decision variables through a

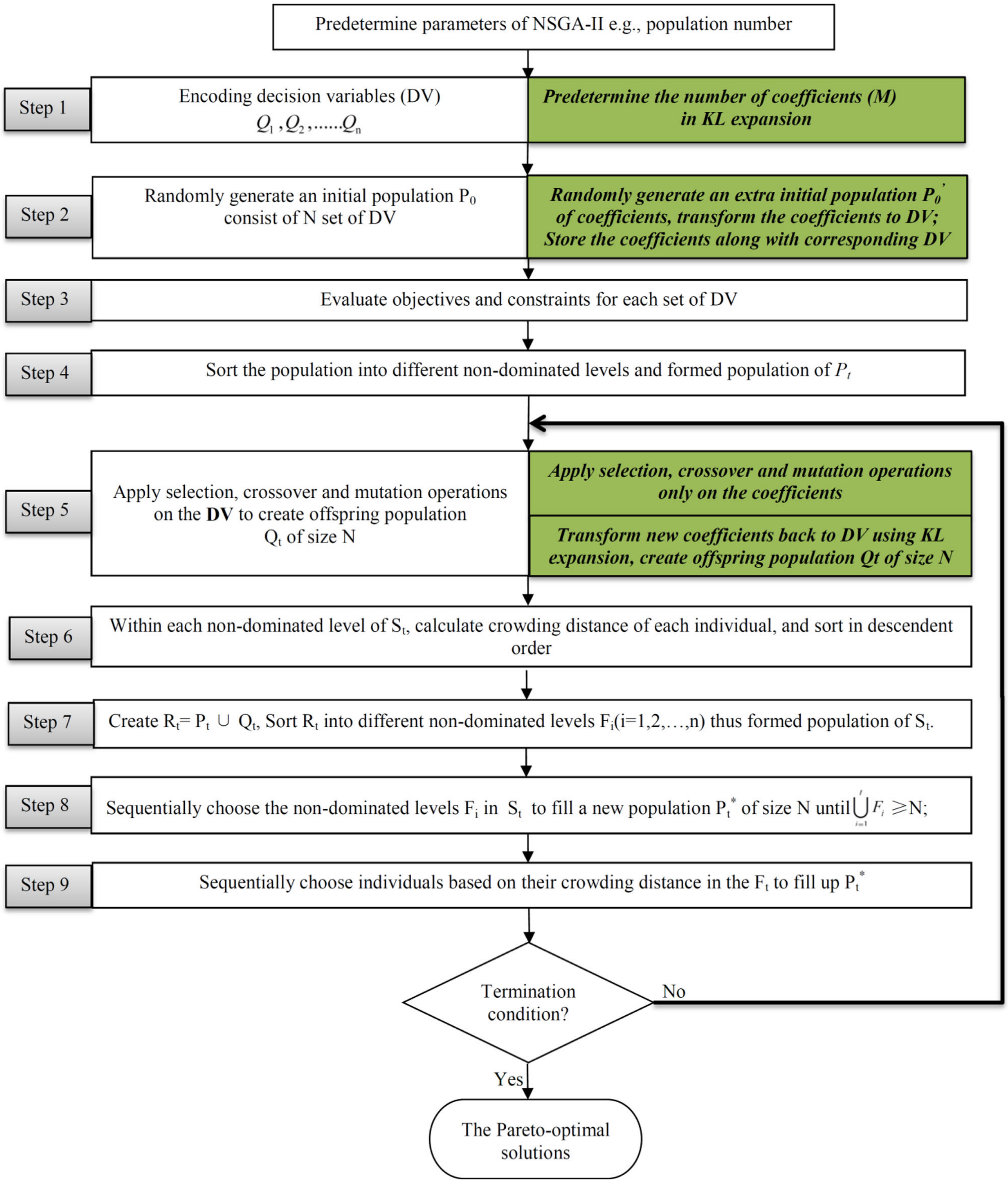

Figure 1. Steps of the NSGA-II and spectral optimization model (in bold and italic).

deterministic optimization model given an inflow realization. In this context, the optimal decision variables can be thought of as another random process associated to the inflow random process. Each optimal operational policy is a realization of that random process for a given inflow scenario. Therefore, we can apply the KL expansion to construct a representation of the decision variables.

To obtain a KL representation of the decision variables, multiple sets of decision variables are needed in order to calculate the covariance function for the random process. The construction of KL expansion for the decision variables and the incorporation of the KL expansion into the NSGA-II algorithm are described below.

2.3.1. Construction of KL Expansion for the Decision Variables

First, a conventional optimization model is set up (i.e., with no incorporation of the KL expansion). This optimization model is repeatedly run for various historical inflow schemes. Then, the optimal decision variables for each inflow scenario are collected and treated as a sample space for the decision variables. By using this collection of decision variables, a representation of the KL expansion can be constructed using the steps described in section 2.1. The KL expansion requires the mean and covariance of the decision variables, which are calculated from the aforementioned collection. The eigen values and eigen functions are computed using equation (2), where a predetermined number of terms in this equation are used (e.g., 50 terms). After selecting the distribution of random variables, e.g., uniform or Gaussian, the KL expansion is constructed using equation (1). Then, the KL expansion can be truncated by comparing the ratios of the eigen values or more rigorously through sensitivity tests on the number of truncated terms [*Leon et al*., 2012; *Gibson et al*., 2014].

It is clear that the construction of the KL expansion for the decision variables needs a collection of optimal solutions. This requires multiple runs of the conventional optimization model under different inflow schemes, which demands intensive computational burden. However, this computation is only required prior to the construction of the KL expansion and is a once-for-all task.

2.3.2. Coupling the KL Expansion With NSGA-II

After the (truncated) KL expansion is constructed, it is incorporated into the NSGA-II algorithm using the following procedure (also illustrated in Figure 1):

1. Predetermine parameters for starting the NSGA-II and then predetermine the number of truncation terms for the KL expansion.
2. Randomly generate multiple sets (e.g., populations) of realizations for the coefficients ξ_k in the KL expansion. Since the eigen functions and the corresponding eigen values are determined and remain unchanged in the optimization, the decision variables can be obtained by simply substituting the coefficients ξ_k in equation (4). Then, store the values of the coefficients along with the obtained decision variables.
3. Implement steps 3 and 4 in the NSGA-II procedure (Figure 1), evaluating the objective and constraints for the decision variables. Then, sort the population according to their dominance relations.
4. Implement step 5 in the NSGA-II procedure, i.e., creating offspring by the operators. Note that the variables to be optimized are KL expansion coefficients. Store the values of the coefficients that have been changed by the operators. Generate a new set of decision variables as offspring population by using the KL expansion with the changed coefficients.
5. Implement steps 6–9 in the NSGA-II procedure.

The steps mentioned above require few changes in the NSGA-II algorithm and its implementation is straightforward. Once the KL expansion is constructed, no extra effort is required during the optimization. The number of KL terms, i.e., the number of random coefficients, is the only parameter that needs to be specified. The distribution of the random coefficients is often assumed to be uniform or Gaussian but other distributions can also be used [*Phoon et al*., 2005].

2.4. Evaluation Metric

The performance of multiobjective optimization is measured based on mainly two aspects: convergence and diversity of the Pareto front [*Deb et al*., 2002]. Various metrics have been proposed in the past decades (e.g., generational distance for convergence and spread metric for diversity). Recently, a hypervolume index was found to be a good metric for evaluating the performance of multiobjective optimization [*Zitzler et al*., 2000, 2003; *Reed et al*., 2013] due to its property of combining the convergence and diversity metrics into a single index. The hypervolume index basically measures the volume of objective space covered by a set of nondominated solutions. A higher hypervolume index denotes better quality of the solutions in terms of convergence and diversity. Generally, a true Pareto front or the best known Pareto approximation set (i.e., reference set) is ideal or preferred for performance evaluation. However, the hypervolume index can be

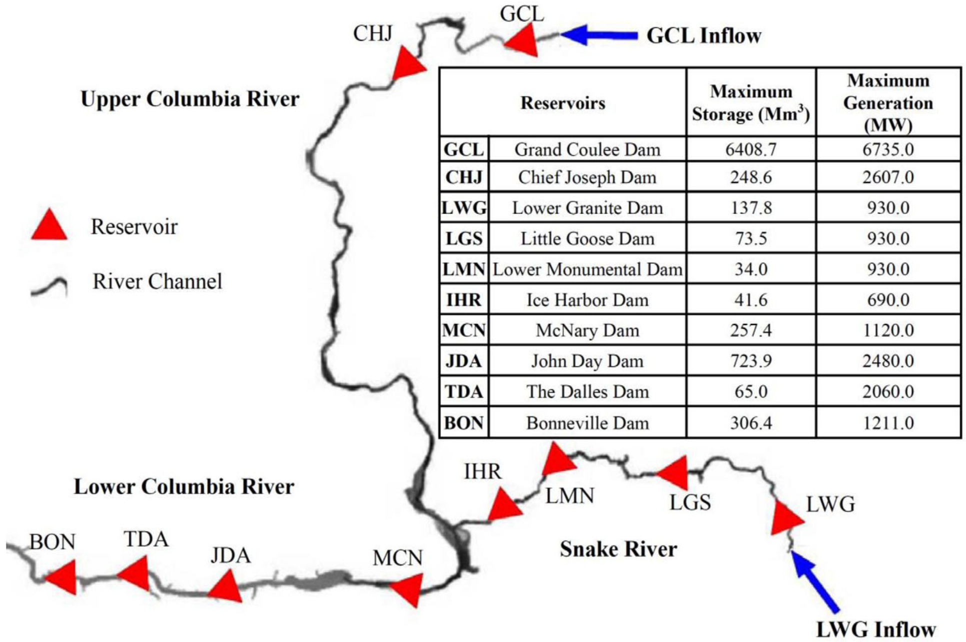

Figure 2. Sketch of the 10 reservoir system in the Columbia River.

used to compare two solution sets based on its properties [*Knowles and Corne*, 2002]. The hypervolume index (I_h) [*Zitzler et al*., 2000, 2003] is defined as:

$$I_h(A)=\int_{(0,0)}^{(1,1)} \alpha_A(Z)dz \tag{5}$$

where A is an objective vector set, Z is the space $(0,1)^n$ for the normalized objectives ($n=2$ in our test case), and $\alpha_A(Z)$ is the attainment function which will have a value of 1 if A is a weakly dominated solution set in Z. The hypervolume index calculates the volume of the objective space enclosed by the attainment function and the axes. In this study, the hypervolume index is adopted to compare performances of optimization solutions at various generations.

3. Test Case

Ten reservoirs, which are the core part of the Federal Columbia River Power System (FCRPS) in the United States, are used as a test case. A sketch of the 10-reservoir system is shown in Figure 2. The Grand Coulee (GCL) reservoir and other five reservoirs are located on the main stem of the Columbia River (Upper and Lower Columbia in Figure 2). The Lower Granite (LWG) reservoir and three other reservoirs are located on the Snake River, the largest branch of the Columbia River. Five small private dams, which are located downstream of the Chief Joseph reservoir (CHJ), are not part of the FCRPS and hence are not considered in the present study.

The FCRPS system serves multiple purposes, e.g., power generation, ecological and environmental objectives [*Schwanenberg et al*., 2014]. During spring and summer, the reservoir system is operated to help migration of juvenile anadromous fish by maintaining a minimum operation pool level (MOP) and spilling certain amount of flow (called fish flow) through nonturbine structures. For LWG, LMN, IHR, and BON reservoirs, the fish flow requirements are set as different various rates of flow. For LGS, MCN, JDA, and TDA reservoirs, the fish flow requirements are expressed as percentages of the reservoir outflows. During autumn and winter, the reservoir system normally has no longer fish flow requirements.

The optimization period was selected as 2 weeks, a week before and after the date of 1 September, at which the reservoir system usually shifts its objectives from maximizing power generation and minimizing fish flow violation to only maximizing power generation [*Chen et al*., 2014]. Two optimization scenarios are considered in this study. These two scenarios have the same objectives, constraints, and optimization horizon but with different time step. The decision variables are the outflows of each reservoir at each time interval during the optimization horizon, which is 14 days. The first scenario, with daily time step, consists of 140 decision variables. The second scenario, with hourly time step, comprises 3360 decision variables.

3.1. Objectives

3.1.1. Maximizing Power Revenue

An important objective of the reservoir system is to meet power load in the region and gain maximum revenue from electricity generation. Power generated that exceeds the load can be sold in a power market. On the other hand, electricity needs to be purchased if a deficit to the load occurs. Net electricity is defined as hydropower generated minus the load. The revenue is then quantified by multiplying the net electricity by real-time prices from the power market. The revenue objective is expressed as:

$$\max \sum_{t=1}^{T} \left(\left(\sum_{i=1}^{Nr} PG_t^i \right) - PL_t \right) * PR_t \quad (6)$$

where *PG* is hydropower generated in the system (MW) and *PL* is power load in the region (MW). The variable *t* is time, e.g., in days(first scenario) or hours (second scenario); *T* denotes the optimization period, e.g., 14 days, the index *i* represent reservoirs in the system, N_r is total number of reservoirs, and *PR* is the market price for hydropower (MW/dollar). The prices of hydropower for the 2 week period were predetermined by an economic model [*Chen et al*., 2014] and were treated as deterministic parameters in the study.

3.1.2. Minimizing Fish Flow Violation

Most of the reservoirs in the system are required to spill certain amount of flow through nonturbine structures such as sluices or gates. These flow requirements are expressed as either a fixed flow rate or a percentage of the total outflow of a reservoir. The objective for minimizing violation on the fish flow requirements is expressed as

$$\min \sum_{t=1}^{T/2} \left(\sum_{i=3}^{Nr} |QS_t^i - QF_t^i| / QF_t^i \right) \quad (7)$$

where *QS* is the spill flow and *QF* is the fish flow requirement. According to the Columbia River operational scheme, the Grand Coulee ($i = 1$) and Chief Joseph ($i = 2$) reservoirs are not required to satisfy any fish flow requirements. In addition, all the fish flow requirements are only specified for the first week of the period under consideration.

In the optimization model, the two objectives are converted into a minimization problem and are normalized using a dimensionless index between zero and one. The power revenue and fish flow violation objectives are denoted as f1 and f2, respectively. Because our optimization problem is a minimization, a better result is achieved when the value is closer to zero for each objective. Other purposes of reservoir operation such as flood control or MOP (minimum operation level) requirements are expressed as constraints on either reservoir water surface elevations or storage limits, as described below.

3.2. Constraints

The constraints considered in the model include:

1. Water Balance Constraints

$$V_i^{t+1} - V_i^t = \left((Q_{in,i}^t + Q_{in,i}^{t+1})/2 - (Q_{out,i}^t + Q_{out,i}^{t+1})/2 \right) \cdot \Delta t \quad (8)$$

   where *V* is reservoir storage; Q_{in} and Q_{out} are inflow to and outflow from reservoirs, respectively; Δt is unit time within a time interval, i.e., time step. Water losses such as evaporation are not considered in the model.

2. Reservoir Forebay elevation Constraints

$$H_{rmin,i} \leq H_{r,i}^t \leq H_{rmax,i} \quad (9)$$

   where H_r is Forebay elevation or reservoir water surface elevation; H_{rmin} and H_{rmax} are allowed minimum and maximum Forebay elevations, respectively.

3. Reservoir MOP Constraints

   In the present test case, the MOP requirements are only necessary during the first week for helping fish migration. The MOP requirements are expressed as follows:

$$MOP^{i}_{low} \leq H^{t}_{r,i} \leq MOP^{i}_{up} \tag{10}$$

   where H_r is Forebay elevation, and MOP_{low} and MOP_{up} are lower and upper boundary for the MOP requirement, respectively.

4. Turbine Flow Constraints

   The turbine flow constraints are expressed as follows:

$$Q_{tb\_\min,i} \leq Q^{t}_{tb,i} \leq Q_{tb\_\max,i} \tag{11}$$

   where Q_{tb} is turbine flow, Q_{tb_min} and Q_{tb_max} are allowed minimum and maximum turbine flows, respectively;

5. Ramping Limits for Outflow

$$|Q^{t}_{out,i} - Q^{t+1}_{out,i}| \leq Q^{t}_{out\_ramp\_allow,i} \tag{12}$$

   where Q_{out} is outflow from reservoir, $Q_{out\_ramp\_allow}$ is allowed ramping rate for the outflow between any two consecutive time steps.

6. Ramping Limits for Forebay Elevation

   The ramping limits for the Forebay elevation are expressed as follows:

$$H^{t}_{r,i} - H^{t+1}_{r,i} \leq H^{t}_{ramp\_down,i} \; (if \; H^{t}_{r,i} - H^{t+1}_{r,i} > 0) \tag{13}$$

$$H^{t+1}_{r,i} - H^{t}_{r,i} \leq H^{t}_{ramp\_up,i} \; (if \; H^{t}_{r,i} - H^{t+1}_{r,i} < 0) \tag{14}$$

   where H_{ramp_up} is allowed ramping rate when reservoir water level is increasing and H_{ramp_down} is allowed ramping rate when reservoir water level is decreasing.

7. Ramping Limits for Tail Water Elevation

$$TW^{t}_{r,i} - TW^{t+1}_{r,i} \leq TW^{t}_{ramp\_down,i} \; (\text{if } TW^{t}_{r,i} - TW^{t+1}_{r,i} > 0) \tag{15}$$

   where TW_{ramp_down} is allowed ramping rate for tail water. This ramping rate is only applied when tail water elevation is decreasing.

8. Output Constraints

$$N_{d\_\min,i} \leq N^{t}_{d,i} \leq N_{d\_\max,i} \tag{16}$$

   where N_d is power output, N_{d_min} is minimum output requirement, and N_{d_max} is maximum output capacity.

9. Constraints on end-of-optimization Forebay Elevation

   The Forebay elevation of the 10 reservoirs at the end of optimization is expected to stay within certain elevations in order to fulfill their future obligations. These targets are often determined by middle-term or long-term optimization models [*Lund*, 1996] which are not part of this study. In the present test case, historical Forebay elevations were used as the target elevations at the end of the optimization. These constraints are expressed as:

$$H^{end}_{r,i} \geq H_{tar,i} \tag{17}$$

   where $H^{end}_{r,i}$ is Forebay elevation at the end of optimization; H_{tar} is the target Forebay elevation at the end-of-optimization.

3.3. Spectral Optimization Model

Initially, a conventional optimization model is set up (i.e., with no KL expansion) using the NSAG-II as the optimization method. It is expected that the number of inflow schemes used in the construction of the KL has some influence on the optimization results. It is recommended that the user include as many inflow schemes as possible in order to cover all possible realizations of the inflow. These can be from historical

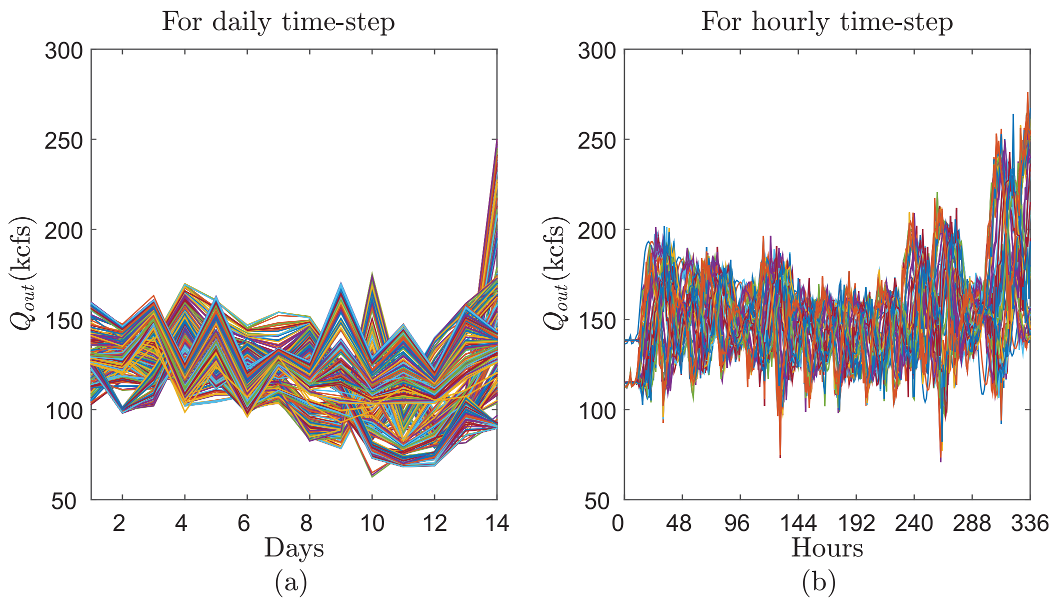

Figure 3. Collection of daily decision variables and hourly decision variables for Grand Coulee Reservoir.

records or from synthetic inflows. It is also recommended to exclude abnormal inflow schemes due to, for instance, dam reconstruction.

In our case study, the Mica dam, which is one of the large reservoirs situated upstream of the 10-reservoir system, was completed in 1973 and expanded its power house in 1977. The Grand Coulee dam, the upstream reservoir in our case study, was constructed between 1933 and 1942 but its third power station was completed in 1974. Therefore, we considered historical inflow schemes from the year of 1977 (the most recent change) to the year of 2011 (the most recent available inflow). Those 35 historical inflow schemes are used as deterministic inflows and the conventional optimization model is solved for each inflow scenario. Each run of the optimization provided multiple sets of optimal decision variables (number of sets is equal to the population size), the collection of which constituted a sample space for the decision variables.

The population size of each run in our study is 50 which then result in 1750 ($=35 \times 50$) set of decision variables. Figure 3 shows a collection of decision variables (daily and hourly time step) for the Grand Coulee reservoir as an example. The oscillation of the hourly decision variables are expected due to the variation of the power demand during the day, which is normally high during certain hours of the day (so-called Heavy Load Hours) and low in other hours of the day (so-called Light Load Hours). By using this collection of decision variables, a representation of the KL expansion can be constructed by following the procedures in sections 2.1 and 2.3. In this study, we chose uniform random variables to represent the random coefficients in equation (3), although other probability distributions could be used, for example, a Beta distribution, in order to give more preference to realizations near the mean. We emphasize that the probability distribution only affects the initial population as remaining aspects of the model do not utilize the distribution of the random variables. It is worth mentioning that the KL construction of the decision variables is expected to be more accurate if more inflow schemes are available to be included. The relation between the inflow information and the results of the spectral optimization model will be investigated in a follow-up paper.

The covariance structure $C(s,t)$ of the decision variables for Grand Coulee reservoir is shown in Figure 4. The large values in the covariance map indicate a strong correlation between the decision variables. The first 50 eigen values in the KL expansion are presented in Figure 5. This figure shows that only the first few eigen values are significant for both, daily decision variables and hourly decision variables. Some of the first eigen functions are also shown in Figure 6 for reference. The eigen functions for hourly decision variables show oscillations in a similar way to the decision variables (Figure 3a). As mentioned earlier, these oscillations are driven by the variation of the power demand during the day. After the KL expansion is constructed, the optimization model, coupling the KL expansion and the NSGA-II, is assembled by following the steps in Figure 1. The resulting model is referred as spectral optimization model because it incorporates a spectral

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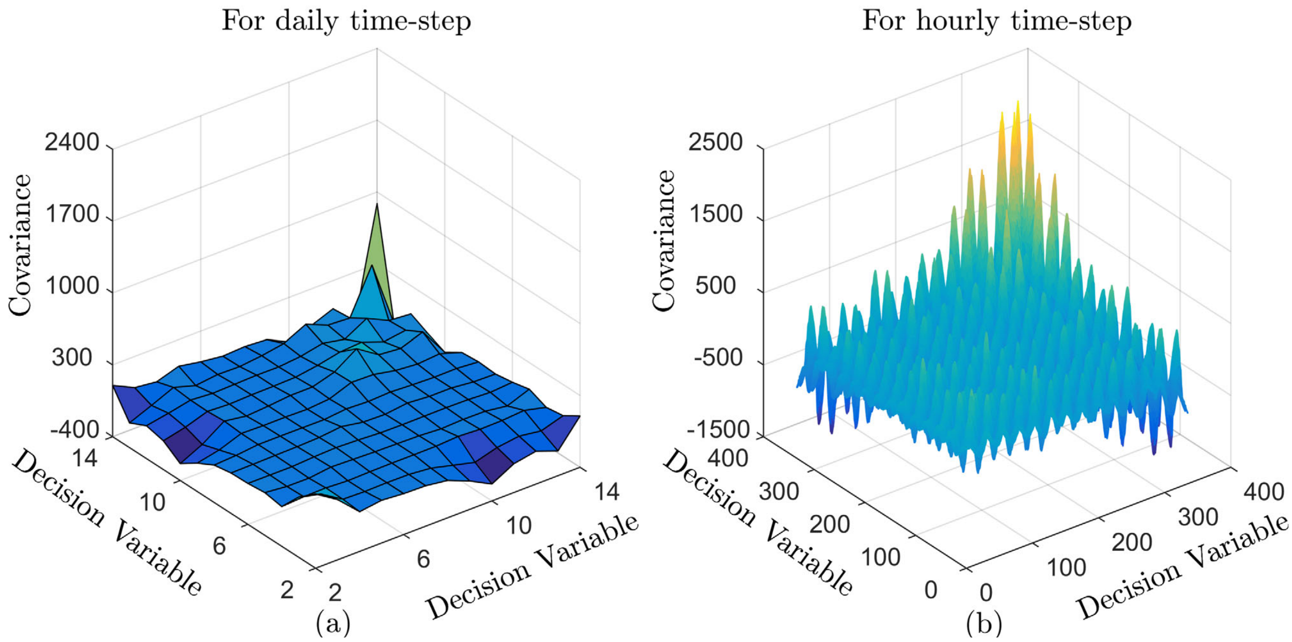

Figure 4. Covariance structure of daily decision variables and hourly decision variables.

method for dimension reduction into an optimization routine. It is pointed out that the KL representation of the decision variables is specific to a reservoir system (e.g., if one or more reservoirs are added to the system, the KL needs to be reconstructed). The KL construction of the decision variables will also depend on the choice of time horizon, time step and other features of the model which affect the structure of the decision variables. From a practical point of view, the construction of the KL may be viewed as an "off-line" computation which can be done before any optimization for actual reservoir operation. After the KL has been constructed, the spectral optimization model is available to be used as a conventional optimization model, e.g., NSGA-II for any "on-line" optimization.

3.4. Experiments

The most critical parameter of the spectral optimization model is the number of terms in the KL expansion, i.e., M in equation (4). Normally, fewer terms result in a higher computational efficiency due to dimension

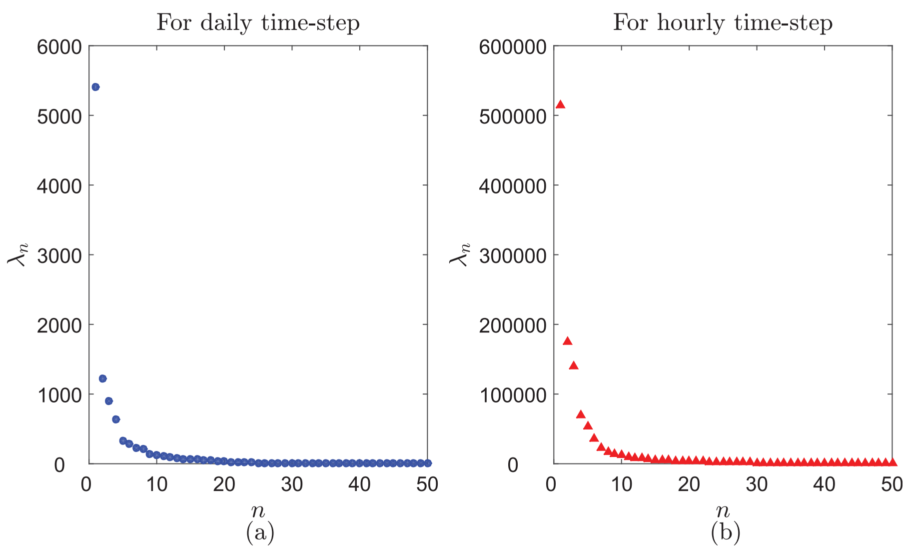

Figure 5. The first 50 eigen values of daily decision variables and hourly decision variables.

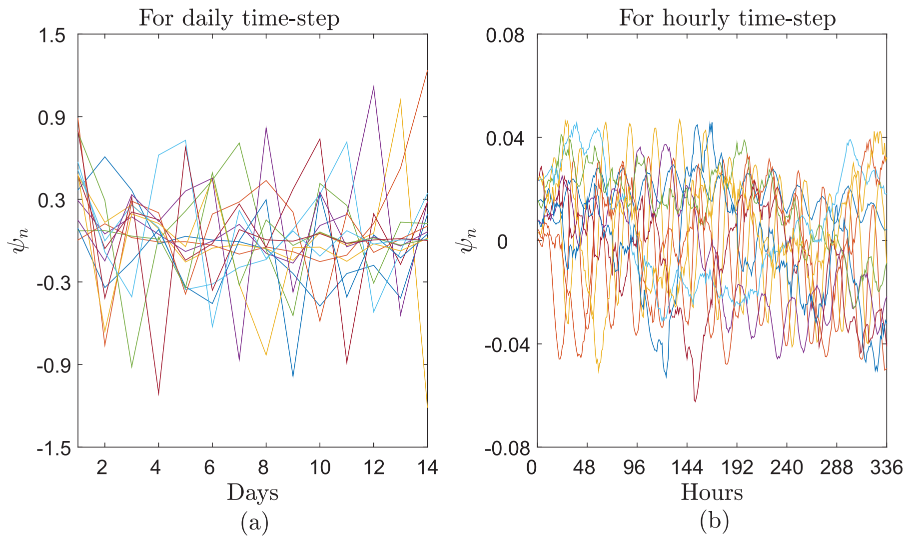

Figure 6. Some of the eigen functions of daily decision variables and hourly decision variables for constructing the KL expansion.

reduction but may lead to a lower accuracy. For each scenario of the decision variables (i.e., daily decision variables and hourly decision variables), we tested 1–12 KL terms with an increment of 1, and 12–40 KL terms with an increment of 4 to investigate the effect of *M* in the optimization. This resulted in a total of 38 experiments for the two scenarios.

We conducted another two experiments to compare the spectral optimization model (SOM) against a conventional optimization model (COM) which also uses the NSGA-II as the optimization routine. For a fair comparison between the COM and SOM, the collection of decision variables is included in the COM. This collection was used in the COM as the so-called "preconditioning" technique [*Nicklow et al*., 2009; *Fu et al*., 2011], which employ some known good solutions into the first generation (starting points) to improve the search process of optimization problems. However, those good solutions are not directly taken as the first generation in the conventional optimization model. Instead, the initial population is obtained using the constructed KL representation. This way of "preconditioning" utilizes the range and covariance (e.g., distribution) calculated from good solutions. The initial population for the spectral optimization model is obtained in the same way (step 2 in Figure 2). The population and generation used for the COM and the SOM are also the same. The population is set as 50 and the number of generations is set as large as 10,000 in order to ensure solution convergence.

The performance of both models (SOM and COM) for the 2 week period was tested using a new inflow scheme, i.e., the historical inflow record of year 2012, the optimal decision variables of which were not used for constructing the KL expansion. The results of the two models for the historical inflow of year 2012 are compared and discussed in the next section.

4. Results and Discussion

Because of the random nature of Genetic Algorithms, optimization results may have some differences for different runs, like other random-based search algorithms. For each experiment, a 50 random-seed replicate runs are used and the average values are reported. For the daily decision variable scenario, the average computational time for the COM is 233 s in the environment of CPU Intel 3.40 GHz/64 bit. The average CPU time for the SOM ranges between 227 and 241 s for all experiments under the same computational environment. As the decision variables change into hourly time step, the average computational time increases to 787 s for the COM and 754–832 s for the SOM.

The hypervolume index of the last generation (i.e., the ten thousandth generation) for all the SOM experiments are shown in Figure 7. For the COM, the hypervolume index of the last generation for the daily and hourly decision variable scenarios is 0.45 and 0.46, respectively. Since the hypervolume index represents the quality of the solution, the higher is the value of this index, the better is the quality of the solution in

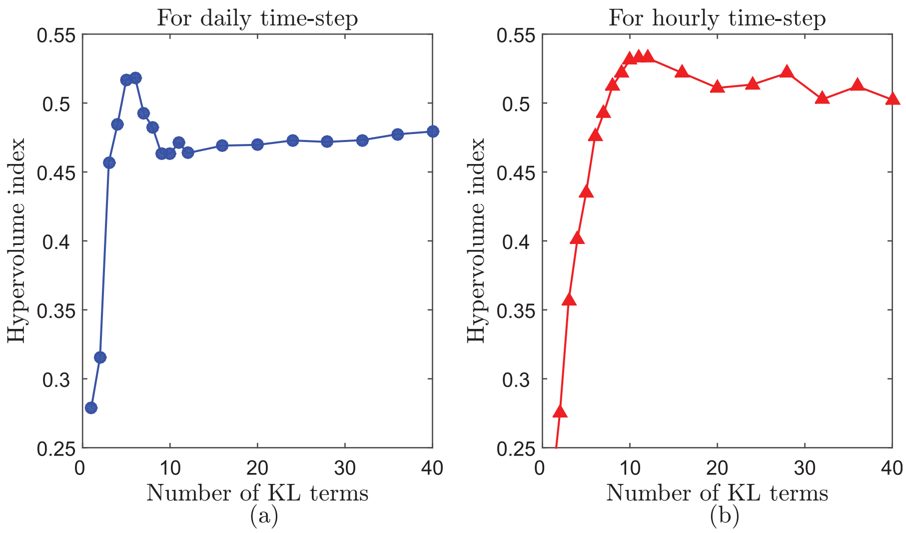

Figure 7. Hypervolume index of last generation for the SOM with different KL terms.

terms of convergence and diversity. Compared to that of the COM, most of the SOM experiments resulted in higher hypervolume index except when the SOM has one to two KL terms for the daily decision variable scenario, and one to four KL terms for the hourly decision variable scenario. For the daily decision variable scenario, the SOM with three KL terms has a hypervolume index of 0.459, which is slightly larger than that of the COM (0.45). An increase in the number of KL terms results in a larger hypervolume index, i.e., better performance. However, the performance of the SOM optimization does not improve monotonically. As can be observed in Figure 7a, the hypervolume index of the SOM increases with the number of KL terms and reaches its highest value (0.52) for six KL terms. As the number of KL terms increases to 12, the hypervolume index decreases to around 0.47 and this value is maintained almost constant when the number of KL terms is further increased up to 40. Figure 7b shows similar results for the hourly decision variable scenario. The optimal number of KL terms can be identified as 11. However, there is no obvious "peak" for the hypervolume index compared to that of the daily decision variable scenario. For the hourly decision variable scenario, the SOM hypervolume index drastically increases for the first few KL terms. After five KL terms, the SOM hypervolume index exceeds the COM hypervolume index (0.46). This means that the 3360 decision variables can be reduced to only five coefficients for achieving the same optimization performance. In a similar way to the daily decision variable scenario, the hypervolume index for the hourly decision variable scenario tends to stabilize after the number of KL terms reach to a certain point (11 KL terms in this case).

For better visualization, we compare the nondominated solutions for the last generation (i.e., Pareto front) for the COM and the SOM with different KL terms (Figure 8a). Due to space limitations and because of the similarity of the two scenarios, only the results for the daily decision variable scenario are presented herein. To avoid cluttering the figure as well, only the results of the SOM for 3, 6, and 40 KL terms are presented. Unlike test functions with theoretical Pareto-optimal solutions, there is no "known" true Pareto-optimal front for a real-world reservoir operation. Alternatively, a reference set of solutions could be used to approximate the true Pareto-optimal front. In this study, in a similar way to *Kollat et al*. [2008] and *Reed et al*. [2013], the reference set was generated by combining the best solutions from all the experiments and performing nondominated sorting of the obtained results. This reference set is included in Figure 8a.

Since this is a minimization problem for both objectives f1 and f2, for convergence, the solutions which are closer to the lower left corner in Figure 6 are the best. For diversity, a Pareto front with large spread of solutions would be desirable. It is clear that the SOM with 3, 6, and 40 KL terms obtained better Pareto fronts than that of the COM, in both convergence and diversity. The SOM with 3 KL terms has similar Pareto front with the SOM with 40 KL terms, in accordance with the results of the hypervolume index (0.459 versus 0.471). Most of the solutions obtained using 40 KL terms show better convergence than those using 3 KL terms; however, the solutions with 3 KL terms show a better performance in diversity. Compared to the COM and the SOM with 3 and 40 KL terms, the SOM with 6 KL terms display better optimization

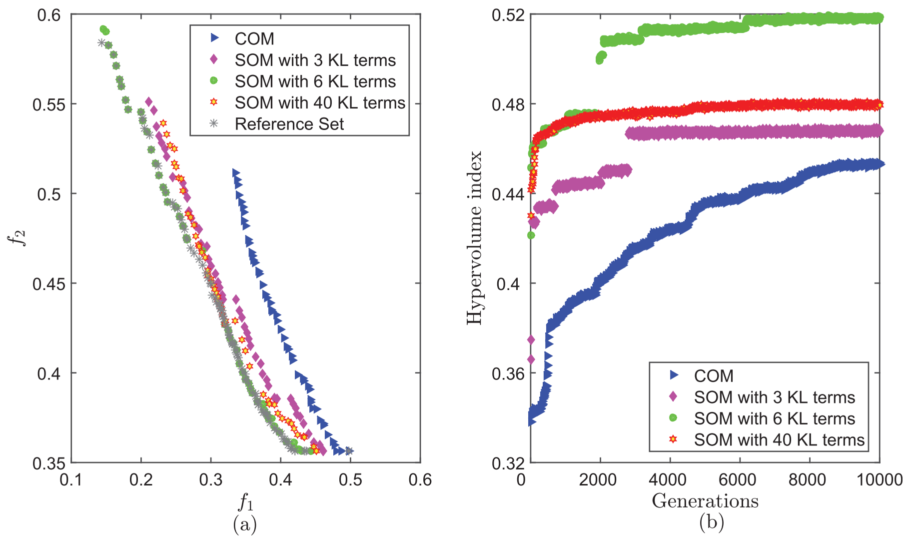

Figure 8. (a) Pareto fronts and (b) Hypervolume index at every 10 generations for the COM and SOM with different KL terms.

performance. Most of the nondominated solutions for the SOM with six KL terms are closer to the lower left corner in Figure 5 indicating a better convergence. Moreover, its Pareto front is more spread indicating a better diversity than the other experiments. In addition, most of the solutions for the SOM with six KL terms are found to overlap with the reference set, which is the best approximation of the "true" Pareto front. These results indicate that performance of the SOM is sensitive to the number of KL terms and that an increase in the number of KL terms does not necessarily improve the solution. A large number of KL terms (e.g., 40) may deteriorate the solution due to a large search space. On the other hand, few KL terms (e.g., one) in the transformed space may not be able to represent all the decision variables in the time domain thus failing to achieve the global optima. An optimal number of KL terms (six in this case) can be identified through a sensitivity analysis.

To better understand the improvement of the solutions during the optimization, the hypervolume index at every 10 generations for the COM and SOM (with 3, 6, and 40 KL terms) is shown in Figure 7. An increase of the hypervolume index denotes an improvement of the solutions. If the hypervolume index remains constant, it can be assumed that the solution has converged. Overall, the results in Figure 7 show that the hypervolume index for the SOM increases at a faster rate compared to the COM. This rate was particularly faster during the first 3000 generations, at the end of which the solutions for the SOM have practically converged. Contrastingly, the solution for the COM did not converge until about 9500 generations. The results also show that all the solutions for the SOM after 1000 generations are superior to the COM even after 10,000 generations. This would imply that the convergence rate for the SOM is at least 10 times faster than the COM. In other words, the SOM would need only one tenth of the number of iterations of the COM to achieve a similar accuracy.

Overall, as expected, the hypervolume index for the SOM and COM increases with the number of generations. The hypervolume index for the SOM with 40 KL terms increased rapidly during the first 1000 generations, after which the increase slows down converging to a constant value smaller than that of the SOM with 6 KL terms. This behavior may be associated to premature convergence which is typical of GA-based algorithms dealing with complex optimization problems. Premature convergence often occurs when some supergenes dominate the population and hence, converge to a local optima instead of the global [*Leung et al.*, 1997; *Hrstka and Kučerová*, 2004]. It should be noted in Figure 7 that the initial hypervolume index (the tenth generation) for the SOM with 40 KL terms is the largest, although it is exceeded by the SOM with 6 KL terms after about 1500 generations. From these results, it can be inferred that a larger number of KL terms results in a higher hypervolume index and faster convergence rate at the beginning of the optimization. This is reasonable since more information is provided to the coefficients from the decision variables in the time domain. On the other hand, fewer KL terms help to reduce the search space, reducing the time for achieving the optimal solutions. Therefore, a dynamic number of KL terms during the optimization process may be a good alternative, where more KL terms can be used in the early stages of the optimization for the so-called "exploration" and then reduced gradually at later stages for the so-called "exploitation." This alternative will be explored in a follow-up work of this paper.

Overall, this study shows that the SOM achieves better convergence and diversity compared to the COM. The efficiency and accuracy of the optimization are greatly improved due to the largely reduced search space. The better performance of the SOM may be also associated with the interdependences between the decision variables. The search difficulty increases as the decision interdependences between the decision variables increases [*Goldberg*, 2002; *Hadka and Reed*, 2013; *Woodruff et al*., 2013]. The decision variables of a multireservoir system are obviously correlated and dependent in some extent, which makes the optimization (e.g., the COM) difficult to solve in the time domain even when the PCX operator is used. Contrastingly, the decision variables in the frequency domain (i.e., coefficients) are mutually independent and hence the optimization (e.g., the SOM) is not as complex as in time domain.

It should be noted that the transformation of the decision variables from time domain to frequency domain requires some prior information for constructing the KL expansion. The prior information can be obtained from historical inflows or from synthetically generated inflows. Alternatively, historical records of decision variables could also be used for constructing the KL expansion. The quality of the prior information (e.g., number of data sets available) may significantly affect the quality of the results. Future studies may need to address these issues. In addition, the problem structure, e.g., constraints and objectives are also expected to influence the representation of the KL expansion since changes of the constraints or objectives certainly change the decision variables. It is worth mentioning that the representation of the KL expansion is specific to a problem with a given structure, inflow distribution, and optimization horizon. The KL representation needs to be done before any optimization for actual reservoir operation and may be thought as an "off-line" computation. This "off-line" preparation can be computational expensive but it is done only once.

5. Conclusion

This paper presents a spectral optimization model for multireservoir operation which can transform a large number of decision variables in time domain to fewer undetermined coefficients in frequency domain, therefore largely reducing the dimensionality of the problem. To assess the benefits of the spectral optimization model (SOM), the SOM with various numbers of coefficients (from 1 to 40) was compared to a conventional optimization model (COM) using a 10-reservoir system in the Columbia River as test case. Overall, the results show that the proposed SOM achieves better convergence and diversity compared to the COM. For the scenario with 140 decision variables, the SOM with only three coefficients (i.e., KL terms) can achieve a similar optimization performance as the COM. The SOM with six KL terms was found to achieve the overall best performance. For the scenario with 3360 decision variables, the SOM with five KL terms exhibit a similar optimization performance as the COM. For this scenario, the SOM with 11 KL terms achieved the overall best performance. Future work needs to be conducted to investigate relations between variability of inflows, correlation degree of reservoir system, and reduction of decision variables.

Although the NSGA-II algorithm is used in this study, the concept of spectral optimization is general and can be easily implemented in other evolutionary algorithms or random search based optimization routines.

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