Simulation of Electromagnetic Materials

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Graduate Student Seminar
April 8, 2026

Projects

1. Electromagnetics
   - Dispersive Media
   - Numerical Analysis
   - Inverse Problems
   - Uncertainty Quantification
   - Homogenization
2. Hydropower Reservoir Networks
   - River system and modeling equations
   - Optimal Control
   - Robust Optimization
3. Magnetohydrodynamics
   - Modeling
   - Numerical Methods
   - Inverse Problems

Acknowledgments

Students

https://internal.math.oregonstate.edu/alumni?keyword=gibson

- Duncan McGregor (PhD 2016)
- Parnian Hosseini (PhD 2016 Civil Eng.)
- Huanqun Jiang (PhD 2019)
- Evan Rajbhandari (PhD 2022)
- Wei Boo (PhD 2025)
- Kamrul Chowdury (PhD 2026^E)
- Emmanuel Oguadimma (PhD 2028^E)
- Christopher Cericola (PhD 2029^E)

Outline

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1. **Electromagnetics**
   - Motivation
   - Maxwell's Equations
   - Polarization Models
2. **Heterogeneous Multiscale Method**

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Motivation

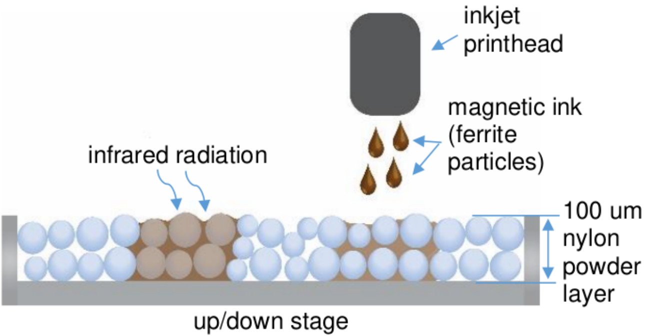
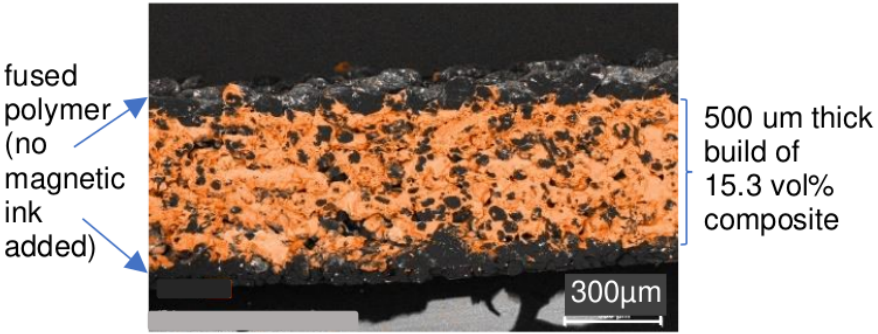


Illustration of the printing process to create a heterogeneous electromagnetic composite.

Motivation



The resulting composite with magnetic nanoparticles.

We want to design and analyze numerical methods for simulating the interaction of light and the nano particles with Maxwell's equations.

Maxwell's Equations

$$\frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} = \nabla \times \mathbf{H} \qquad \text{(Ampere)}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \qquad \text{(Faraday)}$$

$$\nabla \cdot \mathbf{D} = \rho \qquad \text{(Poisson)}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \text{(Gauss)}$$

$\mathbf{E} =$	Electric field vector	$\mathbf{D} =$	Electric flux density
$\mathbf{H} =$	Magnetic field vector	$\mathbf{B} =$	Magnetic flux density
$\rho =$	Electric charge density	$\mathbf{J} =$	Current density

With appropriate initial conditions and boundary conditions.

Constitutive Laws

Maxwell's equations are completed by constitutive laws that describe the response of the medium to the electromagnetic field.

$$\mathbf{D} = \epsilon \mathbf{E} + \mathbf{P}$$
$$\mathbf{B} = \mu \mathbf{H} + \mathbf{M}$$
$$\mathbf{J} = \sigma \mathbf{E} + \mathbf{J}_s$$

$\mathbf{P} =$	Polarization	$\epsilon =$	Electric permittivity
$\mathbf{M} =$	Magnetization	$\mu =$	Magnetic permeability
$\mathbf{J}_s =$	Source Current	$\sigma =$	Electric Conductivity

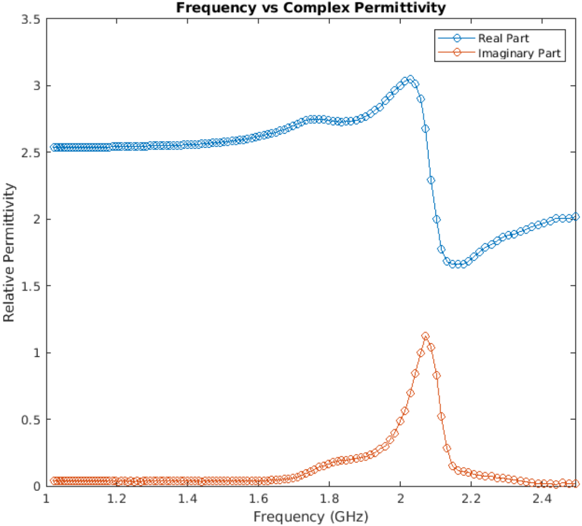
Complex permittivity

- We can usually define $\mathbf{P}$ in terms of a convolution

$$\mathbf{P}(t,\mathbf{x}) = g * \mathbf{E}(t,\mathbf{x}) = \int_0^t g(t-s,\mathbf{x};\mathbf{q})\mathbf{E}(s,\mathbf{x})ds,$$

  where g is the dielectric response function (DRF).
- In the frequency domain $\hat{\mathbf{D}} = \epsilon\hat{\mathbf{E}} + \hat{\mathbf{g}}\hat{\mathbf{E}} = \epsilon_0\epsilon(\omega)\hat{\mathbf{E}}$, where $\epsilon(\omega)$ is called the complex permittivity.
- $\epsilon(\omega)$ described by the polarization model
- We are interested in ultra-wide bandwidth electromagnetic pulse interrogation of dispersive dielectrics, therefore we want an accurate representation of $\epsilon(\omega)$ over a broad range of frequencies.

Dispersive Media: Motivation



2D TE Maxwell-Lorentz System

The 2D TE Maxwell-Lorentz system is stated as

$$\frac{\partial H}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right) \tag{1a}$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon_0 \varepsilon_\infty}\left(\frac{\partial H}{\partial y} - J_x\right) \tag{1b}$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon_0 \varepsilon_\infty}\left(-\frac{\partial H}{\partial x} - J_y\right) \tag{1c}$$

$$\frac{\partial J_x}{\partial t} = -2\nu J_x - \omega_0^2 P_x + \varepsilon_0 \omega_p^2 E_x \tag{1d}$$

$$\frac{\partial J_y}{\partial t} = -2\nu J_y - \omega_0^2 P_y + \varepsilon_0 \omega_p^2 E_y \tag{1e}$$

$$\frac{\partial P_x}{\partial t} = J_x \tag{1f}$$

$$\frac{\partial P_y}{\partial t} = J_y. \tag{1g}$$

Problem

It is computationally expensive to solve the Maxwell-Lorentz system numerically within any material with micro-structures.

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- $$\frac{\text{cost of multiscale model}}{\text{cost of micro-scale model}} \ll 1$$

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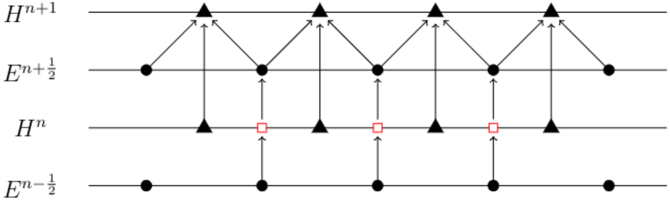
- The step to connect the micro-scale model to the macro-scale model is called upscaling.

HMM on 1D Maxwell's Equation

Macro-scale stencil:



HMM on 1D Maxwell's Equation

Micro-scale stencil:

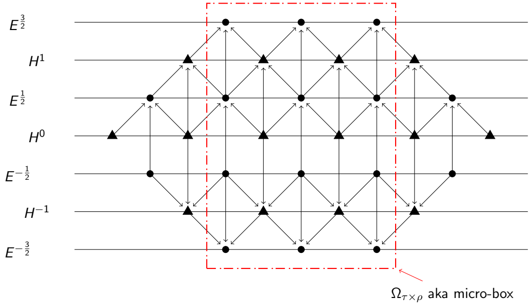


Figure: Instead of prescribing a boundary condition in the micro-scale. We started with more initial data and use the data from the inner square region in our domain.

Inclusions

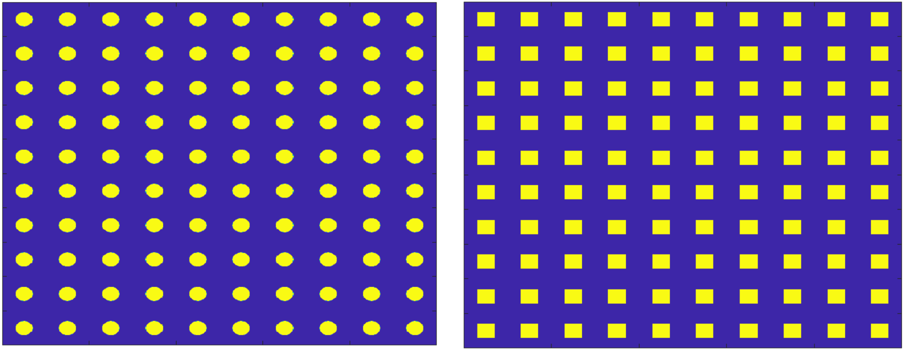


Figure: The material might have inclusions of different shapes, size, and periodicity.

Results

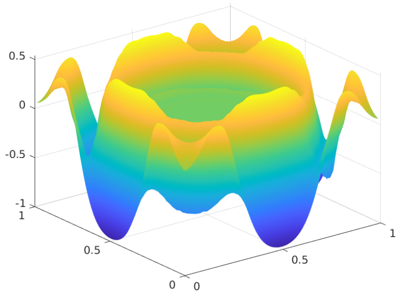
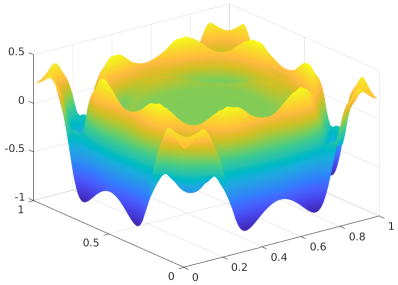


Figure: Plot of fine scale solution final snap-shot



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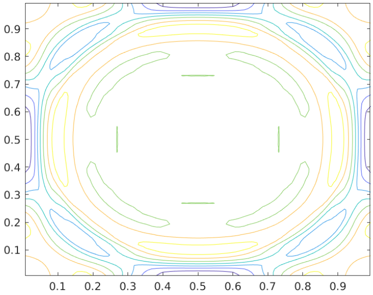

Figure: Contour plot of fine scale solution final snap-shot

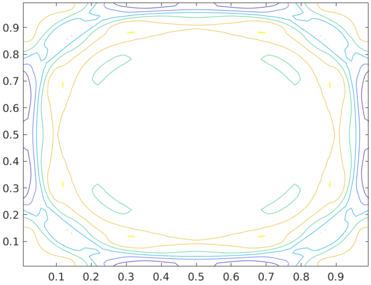

Figure: Contour plot of HMM final snap-shot

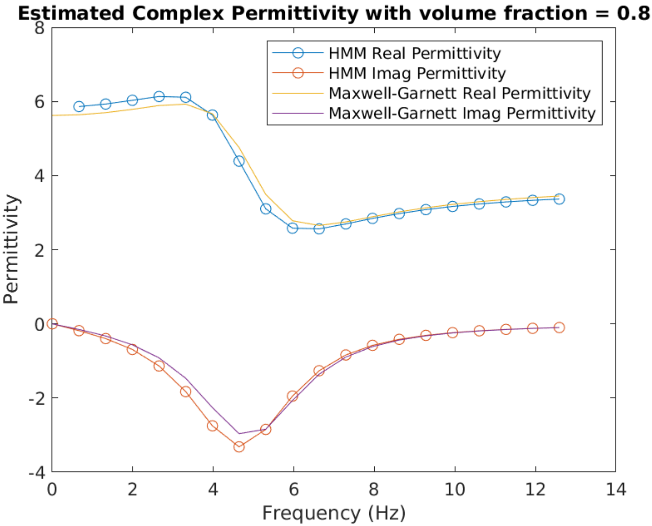

Figure: Estimated complex permittivity from HMM simulation and Maxwell-Garnett effective permittivity with different volume fraction.

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Conclusion

- We have designed and analyzed four operator splitting schemes for 2D Maxwell-Lorentz system.
- We have combined the HMM with operator splitting method to create a multiscale method for Maxwell-Lorentz system.
- We have analyzed the HMM-operator splitting scheme for Maxwell-Lorentz system.
- We want to extend our current numerical method for Maxwell-Lorentz-LLG system.