Flexible Decision Variables in Reservoir Operation Using Dimension Reduction Approach

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Outline

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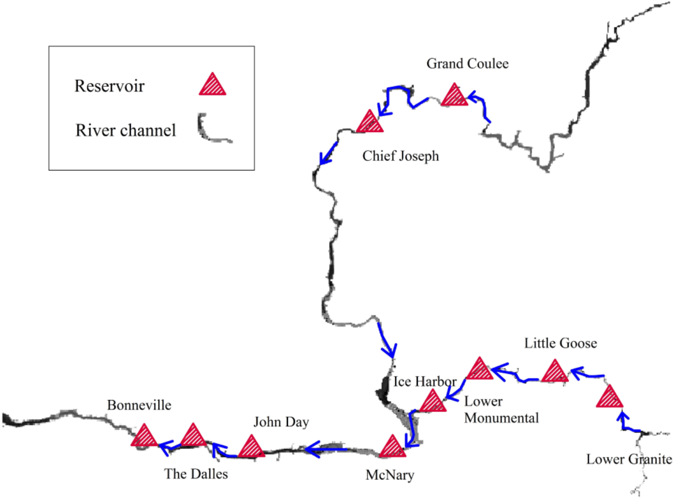
Big 10 Columbia River System



Reservoir Operations

The broad context of the problem of interest is a PDE-constrained optimal control problem with uncertainty. In particular, one must

- meet electrical demand with hydro-power production
- mitigate flooding
- preserve ecological conditions
- possibly maximize revenue
- etc.

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all without perfect knowledge of the system, the inflows, the demand, or prices.

Additionally, decisions made upstream (eventually) affect conditions downstream.

Simulation of Unsteady Flows

- Most free surface flows are unsteady and nonuniform.
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Saint-Venant equations: PDEs representing conservation of mass and momentum:

$$B\frac{\partial y}{\partial t} + \frac{\partial Q}{\partial x} = 0, \quad (1)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\left(\frac{\partial y}{\partial x} + S_f - S_0\right) = 0, \quad (2)$$

where x is a distance along the channel in the longitudinal direction, t is time,
y is a water depth, Q is a flow discharge,
B is a width of the channel, g is an acceleration due to gravity,
A is a cross-sectional area of the flow, S_f is a friction slope, S_0 is a river bed slope.

Suitable initial and boundary conditions are required to close the system.

Objectives

There exist multiple objectives in reservoir operations.

Here we focus on “flexible hydro” which means surplus power generation capacity after obligations have been met.

In this case we can reduce our objectives to simply

- f_1: minimizing deviation from fore-bay (FB) target elevation at the end time
- f_2: maximize revenue (minimize cost)

As the objectives are inherently conflicting, an optimal trade-off of solutions (Pareto optimal) must be found.

Constraints

While there are numerous constraints on the actual operation of a reservoir system, including bound and ramping constraints on the control and the states, we focus here on

- bound and ramping constraints on control
- maximum allowable deviation from fore-bay (FB) target elevation at the end time

Uncertainty

- There are many sources of uncertainty for the forward model, including price, inflow, and power demand.
- We explicitly account for inflow uncertainty in terms of random variables, determined by statistical analysis.
- We formulate a risk-adverse (mean-variance) version of each objective in terms of the probability distribution of the random inflows, as well as probabilistic (chance) constraints.

Parametrization of the Stream Inflow

- $L_i(t_j) = \ln I_i(t_j)$ is the value of the logarithm of the ith inflow at t_j.
- Expectation of the log stream inflow $\bar{L}$ and its covariance $C(t_j, t_k)$,

$$\bar{L}(t_j) = \frac{1}{M}\sum_{i=1}^{M} L_i(t_j), \quad j = 1, \dots, n,$$

$$C(t_j, t_k) = \frac{1}{M-1}\sum_{i=1}^{M}(L(t_j) - \bar{L}(t_j))(L(t_k) - \bar{L}(t_k)).$$

- I can be represented as

$$I(t_j, \vec{\zeta}) = exp\left(\bar{L}(t_j) + \sum_{k=1}^{M}\sqrt{\lambda_k}\psi_k(t_j)\zeta_k\right).$$

- (λ_k, ψ_k) solves: $\lambda\psi = C\psi$.
- $\{\zeta\}_{k=1}^{M}$ is a sequence of standard normal random variables.

Risk-adverse Formulation

The deterministic constrained optimization problem can be formulated as

$$\begin{aligned} \text{find} \qquad & \min_q C(q), & (3) \\ \text{subject to} \qquad & y(x,t;q) \leq y_{crit}(x), & (4) \end{aligned}$$

where C is cost, q is a control, and y is a state.

We assume the inflows are random and reformulate our problem as follows

$$\begin{aligned} \text{find} \qquad & \min_q \Big(E[C(q)] + r\mathrm{Var}[C(q)] \Big), & (5) \\ \text{subject to} \qquad & P[y(x,t;q) > y_{crit}(x)] \leq \alpha, & (6) \end{aligned}$$

where r is a risk tolerance coefficient, α is a reliability level the decision maker wishes to achieve.

Outline

1. Introduction
   - Motivation
   - Ingredients
2. **Flexibility**
   - Deterministic
   - Stochastic
3. Conclusions

Flexibility

- Given the above multi-objective, constrained optimization under uncertainty problem, we now consider the potential implications of additional uncertainties which are so far accounted for, or even impossible to account for.

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- We model a range of options using random variables, and then maximize the variance of the distributions in order to provide the largest flexibility possible.
- The amount of flexibility becomes an additional objective, and constraints become probabilistic with respect to the randomness due to the flexibility.

Flexibilty Formulation Example

Consider

$$\max_{x \in \mathcal{X}} \quad f_1(x) \text{ and } f_2(x) \tag{7}$$

$$\text{subject to} \quad a(t) \leq x(t) \leq b(t), \quad \forall t \in [0, T] \tag{8}$$

where $\mathcal{X}$ is a suitable space of functions, a and b are lower and upper bounds, respectively, for x, and f_1 and f_2 are competing objectives.
In order to incorporate flexibility in decision making, we model x as a range of options, i.e., a stochastic process $\xi(t, \omega)$, with $\omega \in \Omega$.
The problem becomes

$$\max_{F \in \mathcal{F}} \quad \mathbb{E}_F[f_1(\xi(\omega))] \text{ and } \mathbb{E}_F[f_2(\xi(\omega))] \text{ and } \|\sigma\| \tag{9}$$

where $\mathcal{F}$ represents all probability distributions with support wholly within the feasible set. Here $\|\sigma\|$ is a measure of the standard deviation of ξ.

Deterministic Problem Statement (one dam)

Neglecting inflow uncertainty, we seek $\vec{Q} = [Q_n]_{n=1}^{N_t}$, in order to

$$\text{Minimize} \quad f_1(\vec{Q}, \vec{I}) := \left| FB_{end}(\vec{Q}, \vec{I}) - FB_{target} \right|, \text{ and} \tag{10}$$

$$\text{Minimize} \quad f_2(\vec{Q}) := -\left(\sum_{n=1}^{N_t} (PG_n(\vec{Q}) - PL_n) * Pr_n \right) \tag{11}$$

$$\text{subject to} \quad Q^{min} \leq Q_n \leq Q^{max} \quad \text{for all } n, \tag{12}$$

$$\text{subject to} \quad |Q_n - Q_{n+1}| \leq Q^{ramp} \quad \text{for all } n, \tag{13}$$

$$\text{subject to} \quad f_1(\vec{Q}, \vec{I}) \leq \delta FB \tag{14}$$

where $\vec{Q}$ is the set of turbine outflows from the reservoir for N_t time-steps, and $\vec{I}$ are the (deterministic) inflows.

Deterministic-Flexible Problem Statement (one dam)

Neglecting inflow uncertainty, we seek F, which is parameterized by means and standard deviations, $\vec{\mu}$ and $\vec{\sigma}$, and where $\vec{\xi} = [\xi_n]_{n=1}^{N_t} \in F$, in order to

$$\text{Minimize} \quad E_F[f_1(\vec{\xi}, \vec{I})], \text{ and} \tag{15}$$

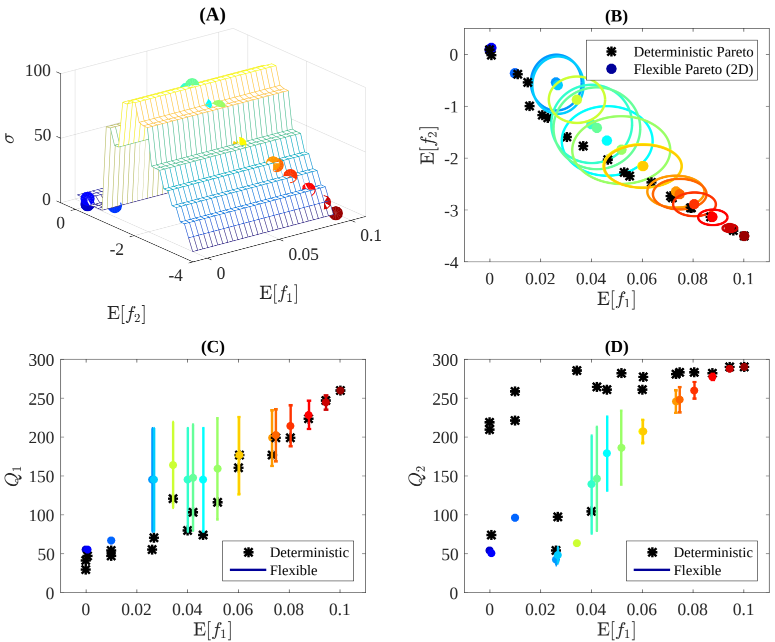
$$\text{Minimize} \quad E_F[f_2(\vec{\xi})], \text{ and} \tag{16}$$

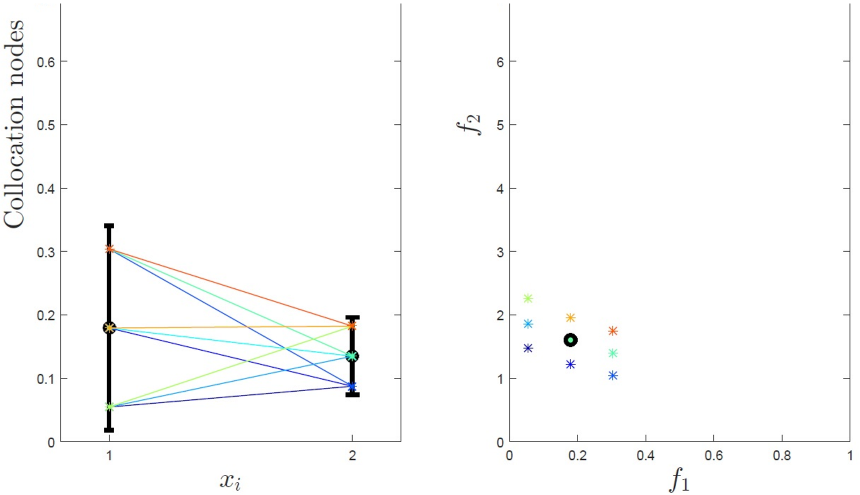
$$\text{Maximize} \quad f_3(F) := \|\vec{\sigma}\|, \tag{17}$$

$$\text{subject to} \quad P_F[Q^{min} \leq \xi_n \leq Q^{max}] = 1 \quad \text{for all } n, \tag{18}$$

$$\text{subject to}^* \quad P_F[|\xi_n - \xi_{n+1}| \leq Q^{ramp}] = 1 \quad \text{for all } n, \tag{19}$$

$$\text{subject to} \quad P_F[f_1(\vec{\xi}, \vec{I}) \geq \delta FB] < \alpha. \tag{20}$$


(A)
100
50
0
σ
0
-2
-4
E[f2]
0
0.05
0.1
E[f1]
(B)
Deterministic Pareto
Flexible Pareto (2D)
0
-1
-2
-3
-4
E[f2]
0
0.02
0.04
0.06
0.08
0.1
E[f1]
(C)
300
250
200
150
100
50
0
Q1
Deterministic
Flexible
0
0.02
0.04
0.06
0.08
0.1
E[f1]
(D)
300
250
200
150
100
50
0
Q2
Deterministic
Flexible
0
0.02
0.04
0.06
0.08
0.1
E[f1]


Collocation nodes
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
x_i
f_2
6
5
4
3
2
1
0
0
0.2
0.4
0.6
0.8
1
f_1

Dimension Reduction

- Solving for decision variables on each time step is computationally impractical.
- We construct a reduced dimension random space within which to determine optimal flexible decisions.
- We use the deterministic Pareto solutions to inform our random space.
- Specifically, we apply a Karhunen-Loeve (KL) expansion (or PCA) to the deterministic solutions

$$Q(t_j, \vec{\xi}) = \bar{Q}(t_j) + \sum_{k=1}^{M} \sqrt{\lambda_k} \psi_k(t_j) \xi_k.$$

- However we do not require that the random coefficients have mean 0 and variance 1. In fact, the joint distribution of these coefficients is the flexible probability distribution F to be optimized.

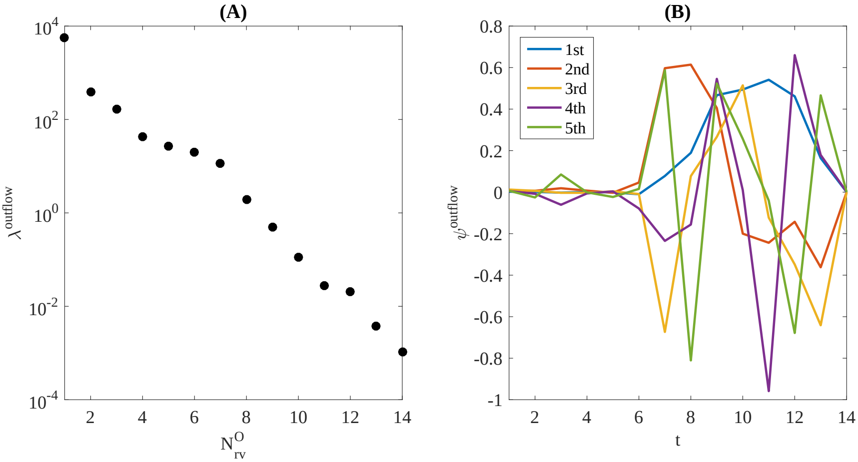
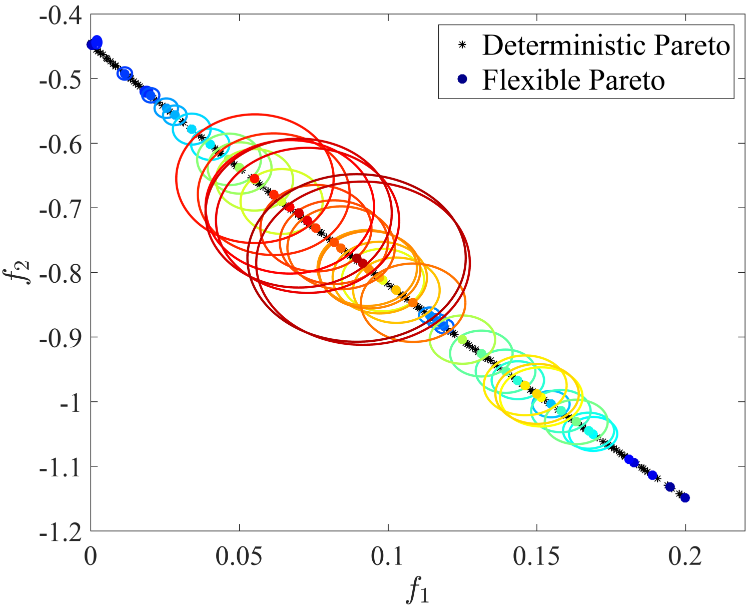
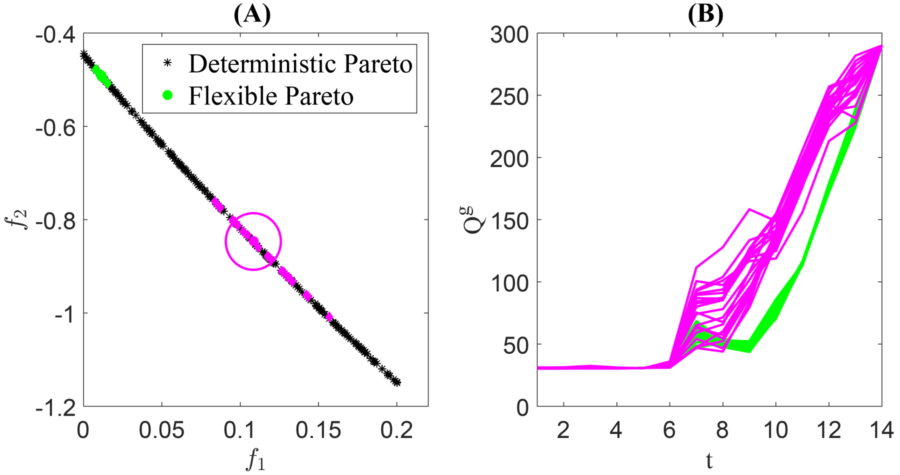
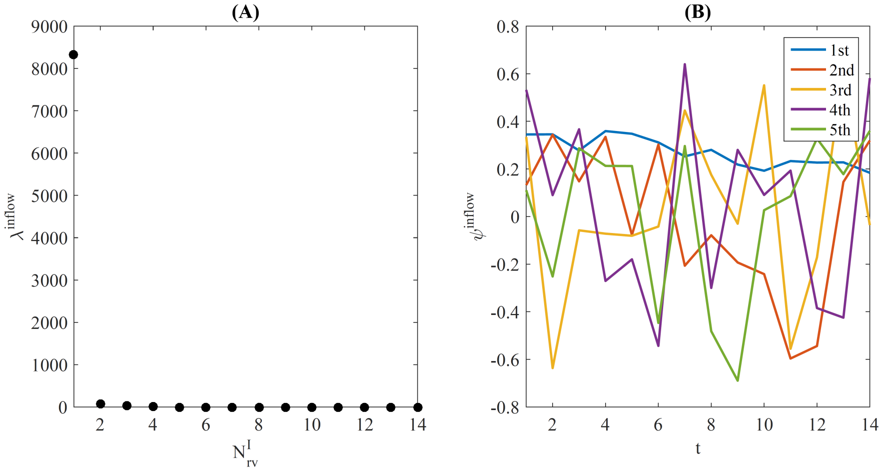


Figure : (A) Eigenvalues (Semilogarithmic scale) and (B) eigenfunctions of the original (deterministic) decision variables


Deterministic Pareto
Flexible Pareto
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-1.1
-1.2
0
0.05
0.1
0.15
0.2
f_2
f_1


(A)
Deterministic Pareto
Flexible Pareto
f_2
f_1
-0.4
-0.6
-0.8
-1
-1.2
0
0.05
0.1
0.15
0.2
(B)
Q^g
t
300
250
200
150
100
50
0
2
4
6
8
10
12
14

Inflow Uncertainty



Risk Adverse-Flexible Optimization

Let $\vec{\xi} \in F$ and $\vec{\zeta} \in G$, where G is given by inflow data. Find F in order to

$$\text{Minimize} \quad (1-w)E_{F\times G}[f_1(\vec{\xi},\vec{\zeta})] + w\sigma_{f_1}, \text{ and} \tag{21}$$

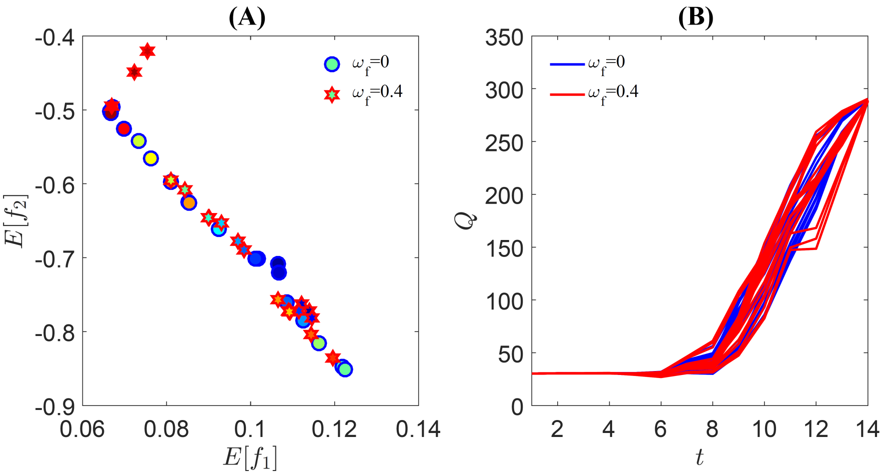
$$\text{Minimize} \quad (1-w)E_F[f_2(\vec{\xi})] + w\sigma_{f_2}, \text{ and} \tag{22}$$

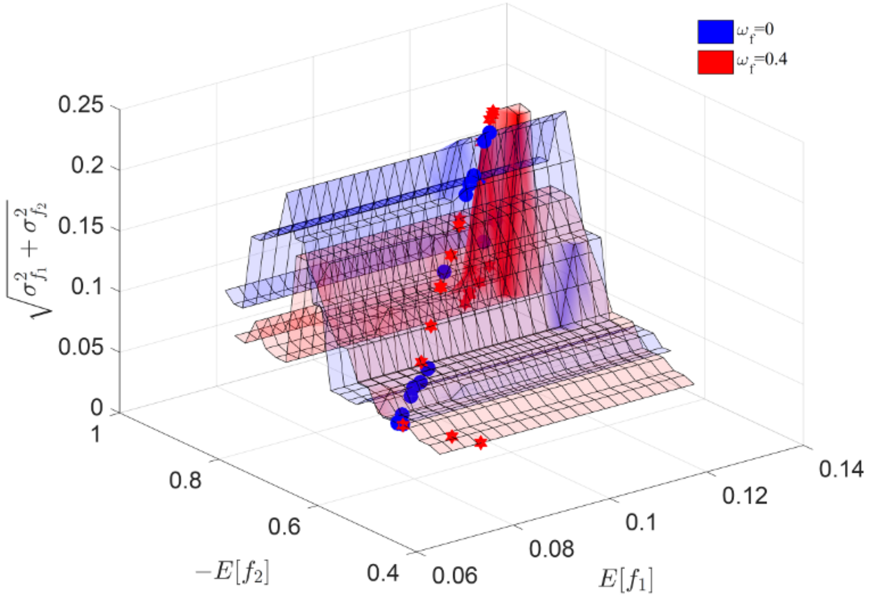
$$\text{Maximize} \quad f_3(F) := \|\Lambda\vec{\sigma}_F\|, \tag{23}$$

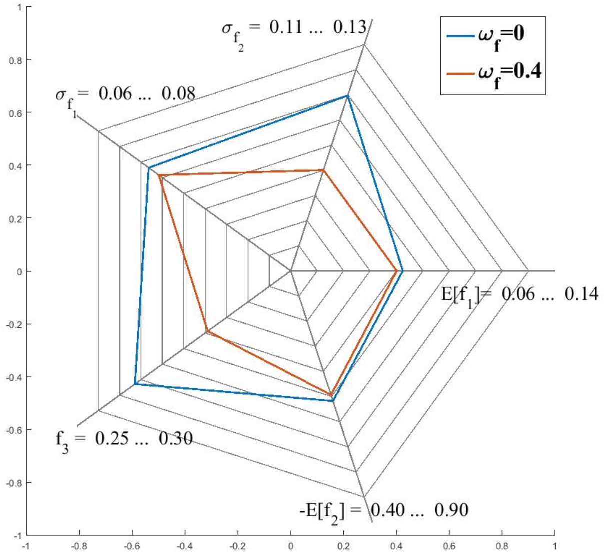
$$\text{subject to} \quad P_F[Q^{min} \leq \xi_n \leq Q^{max}] = 1 \quad \text{for all } n, \tag{24}$$

$$\text{subject to}^* \quad P_F[|\xi_n - \xi_{n+1}| \leq Q^{ramp}] = 1 \quad \text{for all } n, \tag{25}$$

$$\text{subject to} \quad P_{F\times G}[f_1(\vec{\xi},\vec{\zeta}) \geq \delta FB] < \alpha \tag{26}$$


(A)
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
0.06
0.08
0.1
0.12
0.14
$E[f_1]$
$E[f_2]$
ω_f=0
ω_f=0.4
(B)
350
300
250
200
150
100
50
0
2
4
6
8
10
12
14
t
Q
ω_f=0
ω_f=0.4


ω_f=0
ω_f=0.4
0.25
0.2
0.15
0.1
0.05
0
$\sqrt{\sigma_{f_1}^2 + \sigma_{f_2}^2}$
1
0.8
0.6
0.4
$-E[f_2]$
0.06
0.08
0.1
0.12
0.14
$E[f_1]$


σ_{f_2} = 0.11 ... 0.13
σ_{f_1} = 0.06 ... 0.08
E[f_1]= 0.06 ... 0.14
f_3 = 0.25 ... 0.30
-E[f_2] = 0.40 ... 0.90
ω_f=0
ω_f=0.4

Outline

Conclusions and Future Work

Conclusions

- The methodology can find a flexible range of options for each decision variable.
- By using KL expansion the dimension of decision space can be decreased.
- The methodology can be extended to handle inflow uncertainties.

Future Work

- Extend to multiple dams (to really see effect of dynamics)
- Employ hydraulic routing (instead of hydrologic routing)

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