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72317

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2017
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Summary Sheet

When traffic in the Seattle area gets congested, it is generally because cars are moving very slowly and human reaction time is very slow so it takes a while for traffic to clear up when cars are allowed to start flowing again. Human beings have a slow reaction time, which adds up when there is a long chain of cars that need to go.

When you add self-driving cars into the traffic, these automated cars are going to have a light-speed reaction and be able to receive real-time data from other such cars in the area, and so they will be allowed to follow behind other vehicles much more closely. As a result, the traffic volume capacity increases as we introduce more self-driving, cooperating vehicles.

Utilizing this model, we are most interested in predicting the effects of increasing the percentage of self-driving, cooperating cars, and we use this to seek answers to our most important questions. In doing so, we also examine other parameters such as the speed and reaction times.

Our most interesting find of this model analysis is that if we made only half of the cars driverless and cooperative, we could increase the traffic flow in congested traffic by about 10%, and reduce the traveling time by about 15%.

Modeling Traffic Flow of Self-driving, Cooperating Vehicles

Team # 72317

January 24, 2017

Abstract

We are interested in modeling how the flow of traffic is improved when we start increasing the rate of driver-less cars on the road. We are most interested in examining the case where traffic is relatively congested and the average traffic speed is lower than the speed limit. In this case, replacing human drivers with automated drivers tends to increase the flow and make traffic less congested. We quantify this with a descriptive model which depends on number of lanes, number of vehicles, and ratio of self-driving vehicles on the road.

Contents

1 Letter to the Governor

Governor:

We are a team of mathematicians researching the effects of automated vehicles on our nation's roadways. We have examined data from the largest roads in the Seattle-Tacoma area to assess the impacts of the increased presence of automatic vehicles on traffic. We are writing to you today to share our results and insight.

We have developed a model that predicts the improvement of traffic-congestion when automated vehicles become prevalent on major highways. The model takes several, intuitive concepts of traffic into consideration. In situations of high congestion, vehicles are close together and move slowly as a consequence. The largest benefit of the automated vehicle is that it can react to its environment much more quickly than a human driver. Thus, such a vehicle can follow more closely the vehicle in front of it, while retaining a higher speed and the necessary degree of safety. This means that as the share of automated vehicles increases, the major roadways will be able to handle more traffic during rush hours, and motorist's commute will be shorter.

We applied this model to some of the busiest roads in the Seattle-Tacoma area, including Interstate-5, Interstate-90, Highway-520, among others. We found that the largest alleviation occurs in the busiest parts of these roads. For example, in the peak hours of a day, roughly 18,000 vehicles pass through the intersection of I-5 and Route-101. However, if 50% of vehicles were autonomous, then 2100 more vehicles could pass through this segment of road in the same time period. Equivalently, the amount of time for 18,000 vehicles to traverse the road is reduced by 15%.

With these insights in mind, we advise that extra infrastructure spending is unnecessary. The benefit of automated driving occurs in every lane, so constructing additional lanes will have the same impact regardless of the presence of automated vehicles. Additionally, designating a automated-only lane similar to carpool lanes would have no additional impact on the flow of traffic. However, to hasten the deployment of driver-assisted vehicles, we recommend a tax subsidy to Washington citizens who purchase a vehicle with such features.

We hope you keep these findings in mind as you work to improve Washington's growing infrastructure needs.

2 Introduction

In this problem, we are most interested in analyzing the effects that self-driving cars have on the flow of traffic. We have a set of traffic flow data of the major highway arteries in the counties of King, Pierce, Thurston, and Snohomish. We analyze how adding self-driving cars improves and levels out the traffic flow. We take into account the percentage of self-driving cars, the reaction time, speed, and other parameters. We also take into account the fact that humans are going to have much slower reaction times than their software counterparts. By implementing this model, we are able to make an educated guess as to how to answer these big questions.

3 Parameters and Variables

In deriving a model for traffic flow, we use the following parameters:

- n = Number of vehicles over a given interval
- N = Number of lanes
- l = Length of given interval (in feet)
- t = Time (in seconds)
- $k = \frac{n}{l}$ = Traffic density
- $q = \frac{n}{t}$ = Traffic flow
- r = Percentage of self-driving vehicles on the road
- t_{HU} = Time interval in front of a vehicle with a driver
- t_{DL} = Time interval in front of a driverless or cooperative vehicle.
- A = The total daily traffic volume
- $p = 0.08$ = Percentage of daily traffic volume in peak congestion.

4 Model Development

The advantage of driverless and cooperative vehicles is two-fold: first, driverless and cooperative vehicles can safely trail the vehicle in front of them at a consistently shorter distance than driven vehicles; second, driverless and cooperative vehicles have the potential to reduce traffic accidents. We use averages to model the effect of former advantage on the average flow rate q of the highway as a function of the ratio of driverless and cooperative vehicles to driven vehicles.

First, we note that the advantages of driverless and cooperative vehicles are only useful during high density traffic, i.e. when a vehicle cannot safely decrease the distance between it and the vehicle ahead of it without reducing speed. In the case of low-density traffic, there is no need to minimize this distance. During high density traffic, the average distance between vehicles is given by

$$d = u(t_{hu}(1-r) + t_{DL}r).$$

The time constants t_{hu} and t_{DL} represent the average time interval between a vehicle's back bumper crossing a marker and the following vehicle's front bumper crossing the same marker, i.e. the minimal time it takes the vehicle to come to a complete stop. We choose the relatively safe time interval for humans $t_{hu} = 2s$ [3]. As for driverless and cooperative vehicles, we set $t_{DL} = 0.1s$. We remark that though a driverless or cooperative vehicle could begin breaking instantaneously, such a vehicle would still need time to come to a complete stop while preserving the passengers inside. This time constant is to be regarded as an lower bound; in reality, driverless cars would likely need more time to stop.

From this we can determine traffic density. The number of vehicles that can fit into a road at length l is given by $n = Nl/(d + l_v)$ (cf. [2]). Hence,

$$k = \frac{n}{l} = \frac{1}{u(t_{hu}(1-r) + t_{DL}r)}.$$

Moreover, if n vehicles are distributed throughout a segment of road with length l, and these vehicles are moving at a speed u, then after a time l/u, all vehicles will have left the road segment. Hence $q = n/(l/u) = q = uk$ [1]. We can conclude that the maximal flow rate is given by

$$q = \frac{N}{t_{hu}(1-r) + t_{DL}r}.$$

We interpret "peak hours" to be the time of day in which there is high density traffic. Letting the length of this time τ, and letting p be the ratio of daily traffic that traverses a given road segment during peak hours, we have that the average flow rate during peak hours is Ap/τ. Hence, as q increases with r, we can conclude that as r increases, either the length of time in which there is high density traffic decreases, or the percentage of traffic that traverses a given road segment during peak hours increases.

5 Assumptions and Limitations

- The traffic volume is linearly dependent on the number of lanes.
- The self-driving, cooperating vehicles do not cause any accidents to occur, and are able to perfectly and rapidly respond to any irregular behavior of human drivers.
- The traffic volume during peak hours is 8% of the daily average volume.
- The self-driving, cooperating vehicles have light-speed reaction times.
- Crashes are not considered in this model. Though they do happen occasionally, they tend to occur infrequently thanks to high quality driver's education.
- The average speed of traffic can be modeled as an inverse linear function of the traffic density. This assumption is well-documented and justified by the cited literature.
- The effects of lane-merging are relatively limited given that the data shows that all of these major roads have at least three lanes. Additionally, self-driving cooperative cars can merge very efficiently and thus this will be beyond the scope of our model.
- The speed limit is 60 mph, or 88 feet per second.

6 Conclusions

The model was applied to the given road segments in the Seattle-Tacoma area. The volume of additional cars that can pass through a road segment in peak hours is proportional to the peak volume with no driver-assisted vehicles. As the ratio of automated vehicles increases, the performance of the roads increases smoothly, and there are no equilibrium points. The optimal ratio is one, which is the case where no humans are driving their vehicles.

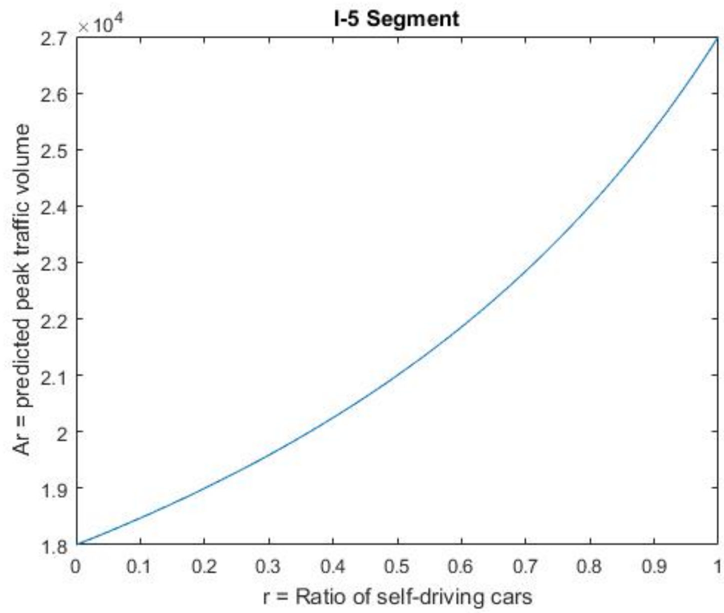

Figure 1: **Increase in peak volume in a road segment along I-5.**

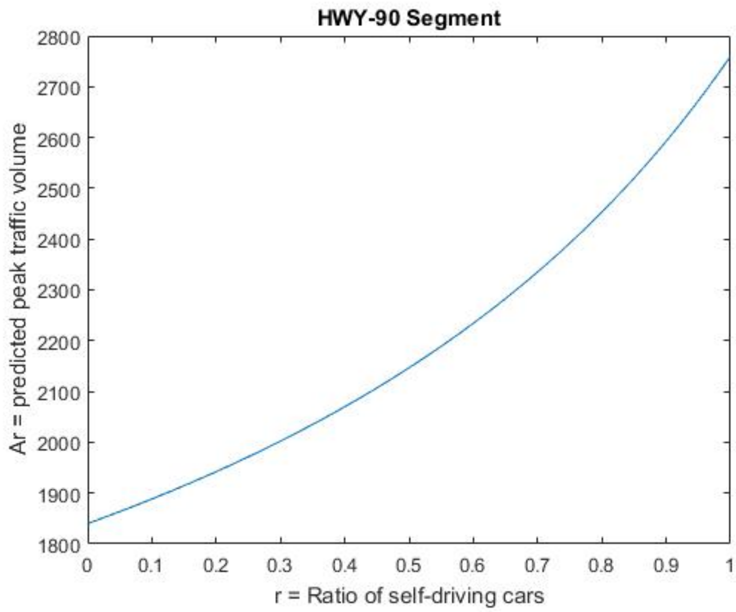

Figure 2: **Increase in peak volume in a road segment along HWY-90.**

The model presented here focuses solely on the change in vehicle flow due to the improved reaction time of automated vehicles. However, there are additional advantages that these vehicles hold over human drivers. These include a lower rate of accidents, more efficient acceleration, and less frequent lane changes. Such advantages may have a place in a similar model to the one presented here.

7 References

1 *Principles of Highway Engineering and Traffic Analysis, Fourth Edition,* By Fred L. Mannering, Scott S. Washburn, and Walter P. Kilareski; John Wiley & Sons, Inc., 2009

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3 Johansson, Gunnar, and Kre Rumar. "Drivers' brake reaction times." *Human Factors: The Journal of the Human Factors and Ergonomics Society 13.1* (1971): 23-27.