Team Control Number

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Problem Chosen

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2010 Mathematical Contest in Modeling (MCM) Summary Sheet

(Attach a copy of this page to each copy of your solution paper.)

Type a summary of your results on this page. Do not include the name of your school, advisor, or team members on this page.

Our goal was to create a geographical profile that could predict the location of a serial offender's next crime. We defined `geographic profile' as the criminal's individual spatial priorities in selecting crime sites. We generated this profile based on observed patterns in the times and locations of previous crimes, which we considered to be the most important factors.

We made the following assumptions in our models: the criminal returns to a single stationary anchor point between crimes; all crimes occur within a feasible region; a minimum of two crimes must have been documented, all of which were known to be committed by the same criminal; the occurrence of a crime is a reflection of crime probabilities in a surrounding circular region; and the criminal's target is selected purely on its location, i.e. not physical properties of the target.

Our variables included the coordinates of the past crime sites, days elapsed between the first and each consecutive crime, and the half-lives of the time-decay functions.

Since criminals have some motivation for their choice of crime location, these locations cannot be truly random. The offender's reasoning may include several spatial and temporal factors, including the anchor point where the criminal returns after committing the crime; the buffer zone of low crime probability immediately surrounding the anchor point; the change in location or angle between crimes to avoid previous location; and the preference for regions where crime has highest profit-risk ratio.

We designed five independent schemes to reflect the above factors, then weighted and combined them into a cumulative model which would best reflect offender spatial preferences. The system of weighting makes the model highly adaptable to varying offender spatial patterns and crime types, for which a single scheme might be insufficient.

We tested our model using the data from the Sutcliffe case. Our results confirm the accuracy of our model in estimating the next crime location. The error in the model must be defined in terms of the smallest region that would contain the next crime (since the model ranks potential sites rather than eliminating them). This error will change based on how well the case fits the model's assumptions.

The model does not account for behavioral, psychological, or intellectual aspects of offenders' choice of location. Some of the schemes do not restrict search area to any appreciable degree and are therefore limited in scope on their own. Our assumptions are not necessarily valid in a real-life scenario, weakening the model's applicability. To be useful, the model should produce an estimate of the timeframe for the next crime, as well as its location, which could merit further research.

However, our model is highly flexible and can easily be adjusted to accommodate other scenarios, either by weighing the schemes we used or by incorporating additional probability grids. It is also relatively simple and easy for non-technical users to understand.

COMAP 2010

GEOGRAPHICAL PROFILING AND CRIME FORECASTING

TEAM #8596

February 21, 2010

Contents

Part 1. **Executive Summary**

Geographical Profiling is an emerging method which can make locating serial criminals more cost and time efficient. Geographical Profiling is used to predict serial criminals' behavior based on previous crime times and locations.

We have developed a model to geographically profile a serial offender's next target location. This model is based on a combination of five individual schemes:

- Time-weighted estimate of high crime regions
- Mean angles between consecutive crimes
- Mean distance from the center of the crimes
- Distance from the most recent crime
- Distance from the criminal's probable anchor point, as determined by Rossmo's formula [6]
- Final cumulative model

Time-weighted estimate of high crime regions. This method was intended to model a situation in which previous crimes are indicative of a pattern of repeated crime occurrence in the surrounding area. It is representative of a landscape in which crime activity may change over time. For example, a city's low-security region may obtain a police station, decreasing the probability that the offender will return. This model could also reflect changes in the offender's spatial pattern. The rate at which these probabilities decline with respect to time can be adjusted according to the specific region of interest. This model is not suited for a situation in which offender activity may occur away from previous activity.

Mean angles between consecutive crimes. This method was intended to model a situation in which an offender purposefully avoids the locations of previous crimes. This could be the case in high profile crimes such as murder and bank robbery. It has been observed that offenders may create a pattern in the angles of crimes by trying to avoid locations of previous crimes in order to attempt to simulate randomness [2]. This model fails to restrict the search area to any practical degree but may be used in conjunction with other models to narrow the field of search.

Mean distance between consecutive crimes. This method was intended to model a situation in which the offender is based out of a stationary anchor point that they return to between crimes. We first estimate the anchor as the mean center of all past crimes. By applying a time-weighted distance we are able to determine a higher probability circular region about the center of past criminal activity. However, if the crime locations are skewed or clustered this method may not be appropriate.

Distance from most recent crime. This method was intended to model a situation in which a offender tends to avoid previous crime locations (similar to the angle model). This method predicts that likelihood of next crime location increases with distance from most recent crime. This method does not account for patterns in previous crime locations. However, as one of many methods of narrowing the search field, it is useful.

Distance from the offender's probable anchor point This method was intended to model a "buffer zone" [6] behavioral pattern. A buffer zone is an area around the anchor point in which an offender will avoid committing crimes. This type of pattern is reasonable to assume because the offender will be more likely to be known close to their anchor point, and because target options increase with distance from anchor point. They also tend to not commit crimes very far from their anchor point, because of time and cost associated with transportation. The distance from anchor point model is a two part model. The formula for locating the offender's anchor point (Rossmo's formula) is applied and a distance decay function should be determined based on the type of crime and geographical characteristics of the region. This method is useful because it provides a feasible search region for both the anchor point and the next crime location. This model is harder to apply due to the need to determine an accurate distance decay function, which relies on having more data on possible transportation methods.

Final Cumulative Model The final model is a cumulative probability grid based on each of the previous models. The models are individually weighted to reduce their weaknesses and increase their strengths. This is based on the idea that some models may more accurately portray certain types of crimes than others.

Our approach is flexible and intuitive, separating a complex probability grid into grids generated by distinct theories of spatial behavior. However, since serial criminal targets are not determined completely geographically, this method should be used cautiously, and its results are not guaranteed nor rigorously tested. While we are confident that this model could be useful in narrowing a search field, in practice, the results of alternative methods should be considered.

Part 2. Introduction

With population booming and justice department budget cuts all over the United States it has become increasingly important for criminals to be caught quickly and efficiently. In order to increase efficiency and decrease cost it is useful to develop a model for locating serial criminals before they are able to strike again. We developed a model that, based on the times and locations of previous crimes, will generate predictions of the next crime's location.

Our model uses the Probability Grid Method (PGM) described by Heimann and Hill [5]. We used five separate schemes to generate probability grids, which we then weighted and combined into a final model of the most likely regions for the next crime to occur. These models were based on:

- Time-weighted kernel density estimate
- Mean angles between consecutive crimes
- Mean distance from the center of the crimes
- Distance from the most recent crime
- Distance from the criminal's probable anchor point, as determined by Rossmo's formula [6]

Part 3. Methods

1. Assumptions

We made the following assumptions in our models:

- **Single stationary anchor point.** The criminal is assumed to return to a fixed point between crimes.
- **All crimes occur within a feasible region.** We do not consider the probabilities of crimes outside this region.
- **Minimum of two crimes.** With only one crime, there would be no pattern between crime locations. All crimes must be known to have been committed by the same criminal.
- **Sufficiently uniform geography.** We assumed that geographical formations and population characteristics have minimal effect on crime probabilities.
- **Target determined geographically.** The criminal has no target criteria besides geographical location.

2. Variables and Parameters

Table 1. Parameters

n	Size of grid containing the feasible crime region
H_{krn}	Half-life of probabilities in kernel density model (days)
B_{krn}	Bin size used in kernel density model (relative to grid units)
H_{ctr}	Half-life of distances in distance from center model (days)

Table 2. Variables

$\begin{pmatrix} x_1 & y_1 \\ \vdots & \vdots \\ x_n & y_n \end{pmatrix}$	Crime coordinates
$\begin{pmatrix} t_1 \\ \vdots \\ t_n \end{pmatrix}$	Days elapsed since first crime ($t_1 = 0$)

3. Time Weighted Kernel Density Model

Description. In Paulsen's spatial prediction method evaluation, the time-weighted kernel density model was found to be most accurate (it reduced search area while maintaining high correctness) [8]. This model assumes that previous crimes occurred in areas of high crime potential. This potential is modelled with a kernel density approximation, as described by Chainey and Ratcliffe [3].

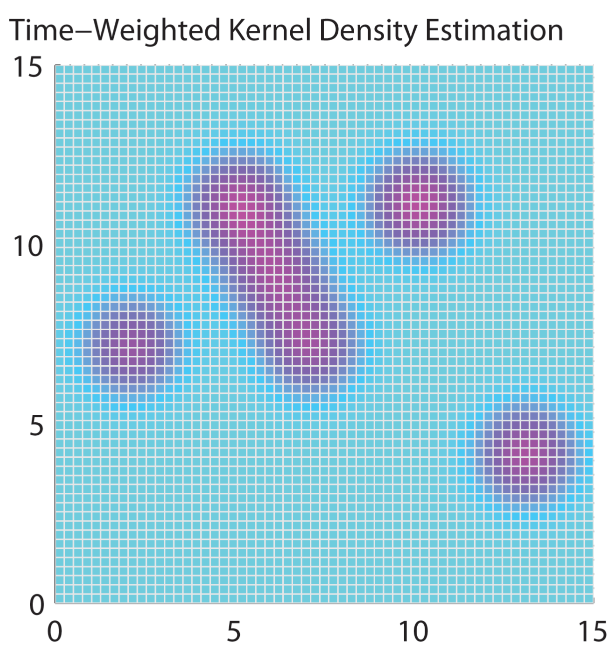
At each crime site, we simulated a bivariate normal distribution of probabilities with covariance matrix

$$B_{krn}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

where the parameter B_{krn} adjusts the bin size and should be chosen to reflect the degree of uniformity of the region. These probabilities were scaled by the time-weighing factor: scale $= 2^{(-t_n/H_{krn})}$.

The model fails to account for spatial relationships between crimes. It always predicts that the next crime will occur near a previous crime, and further, that it is most likely to occur at the location of the most recent crime. This is contrary to findings by Catalano, who found that "offenders were tending to pattern their crimes by switching direction away from the last crime" [2].

Probability Grid. An example probability grid created from six randomly-generated crimes.

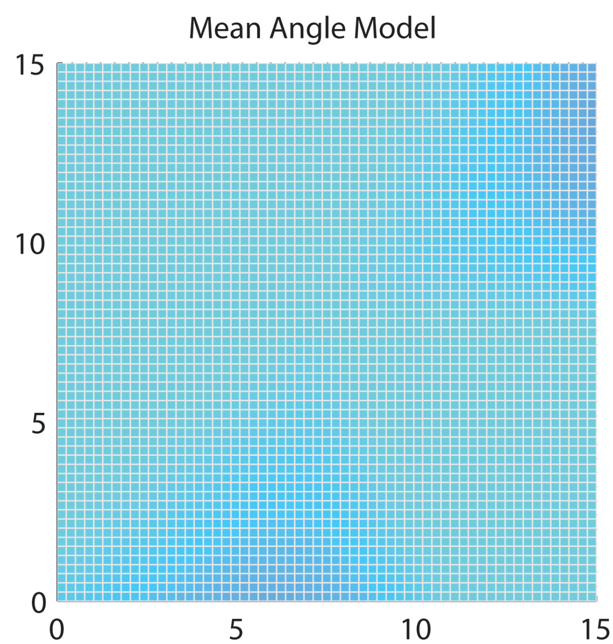


4. Mean Angle Probability Model

Description. This model was based on the argument by Catalano mentioned previously. We determine the mean center of the crime locations and find the angle between each pair of consecutive crimes based on the north point of this center. We then assign probabilities to points in the region based on their angle to the most recent crime. Points within one standard deviation of the mean angle are assigned the highest probabilities.

Although is true that offenders may alternate directions between crimes in order to simulate randomness, this model does not narrow the search field to any appreciable degree. While it does model an observed trend in offenders' spatial reasoning, by itself, it does not account for distance or chronology.

Probability Grid. An example probability grid created from six randomly-generated crimes.



5. Mean Distance from Crime Center Probability Model

Description. The model is based on the idea that criminals operate around a fixed anchor point, and that the mean distance of crimes from this point may change over time. (According to Godwin, in cases of serial murder, "as the number of victims increased, the distances from home decreased" [4].) The model predicts that the next crime will occur in a circular region about the mean center.

We determine the mean center of the crime locations and calculate the time-weighted distance between each crime and this center: distance = distance $\times$ $2^{(-t_n/H_{ctr})}$.

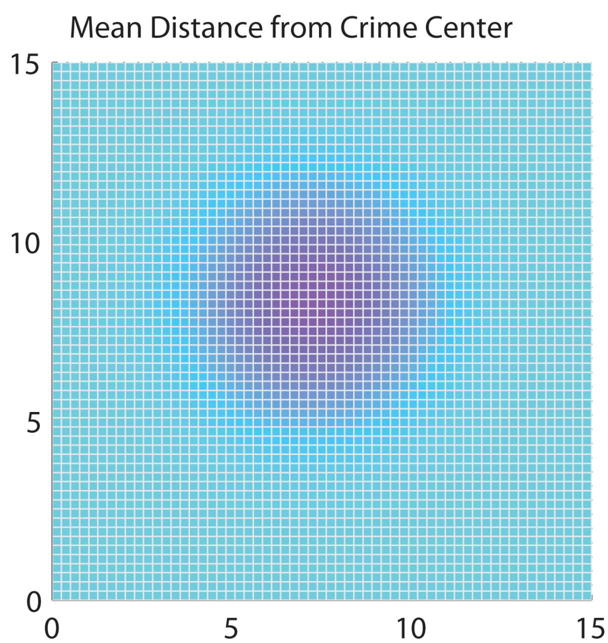
where H_{ctr} is the half-life of the weighting function.

More recent crimes are given higher weights to reflect a potential change in the criminal's target strategy. The model itself gives probabilities based on a bivariate normal probability density function centered at the mean center with covariance matrix

$$\text{(weighted mean distance)} \times \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

This model requires non-skewed crime locations with a circular tendency.

Probability Grid. An example probability grid created from six randomly-generated crimes.

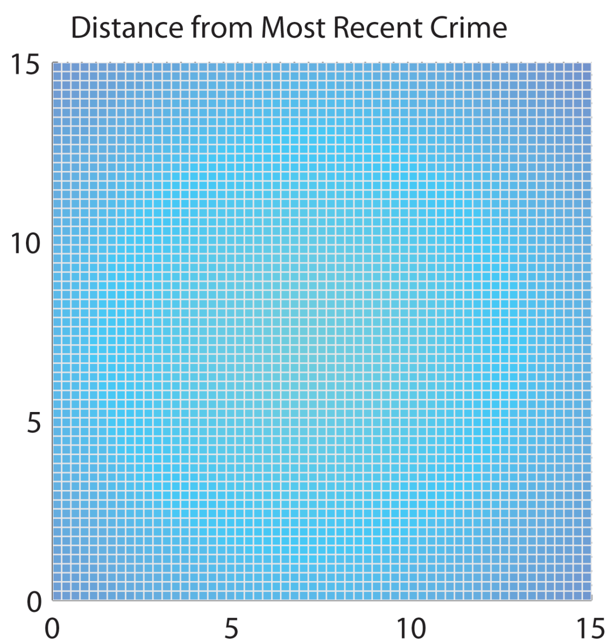


6. Distance from Most Recent Crime Probability Model

Description. This model assumes that the criminal will avoid their most recent crime location in an attempt to simulate random attacks. It assigns crime probabilities based on the distance of each potential site from the most recent site.

This model assumes that the criminal's next location depends solely on their most recent crime, and all other data are ignored. Therefore, while this model may reflect a trend in criminal behavior, it is insufficiently specific for conducting a search. However, this model is able to reduce the search field by disregarding the potentially low probability area around the most recent crime.

Probability Grid. An example probability grid created from six randomly-generated crimes.



7. Distance Decay Applied to Rossmo's Formula

Description. This model uses Rossmo's formula [6] to predict the location of the criminal's anchor point, then, using the idea of a buffer zone, determines where the next crime is most likely to occur.

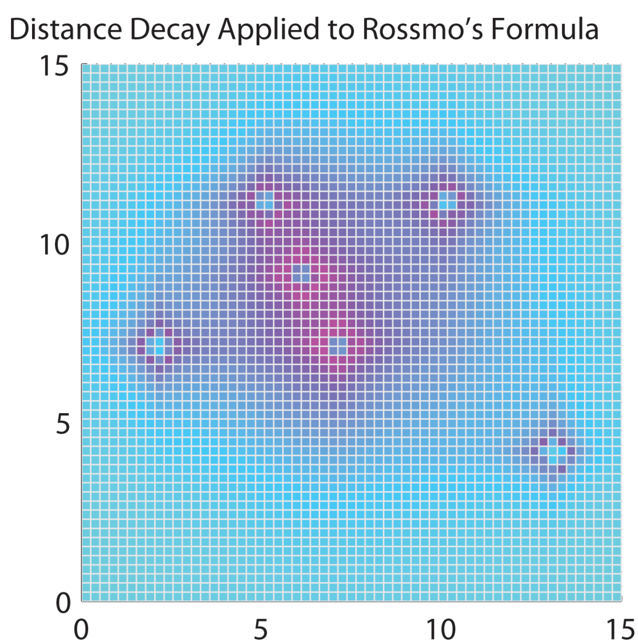
The buffer zone is the region closest to the anchor point that the criminal will avoid. According to the Rossmo model,

> The buffer zone is an area of relative lower probability surrounding an offenders residence. This is a result of two influences: (i) greater risk related to lower anonymity in the offenders own neighbourhood; and (ii) a larger number of criminal opportunities as distance from the home increases.[6]

However, there is also a crime probability decay as distance from the home increases. Based on this behavior, we choose to model this interaction with a gamma probability density function, $x^{5-1}\frac{\exp\left(-x/1\right)}{\Gamma(5)\,1^5}$.

This model requires an empirically determined distance decay function. This function would most likely depend on the type of crime, transportation, and several other variable. Therefore, in practice this model may be difficult to apply.

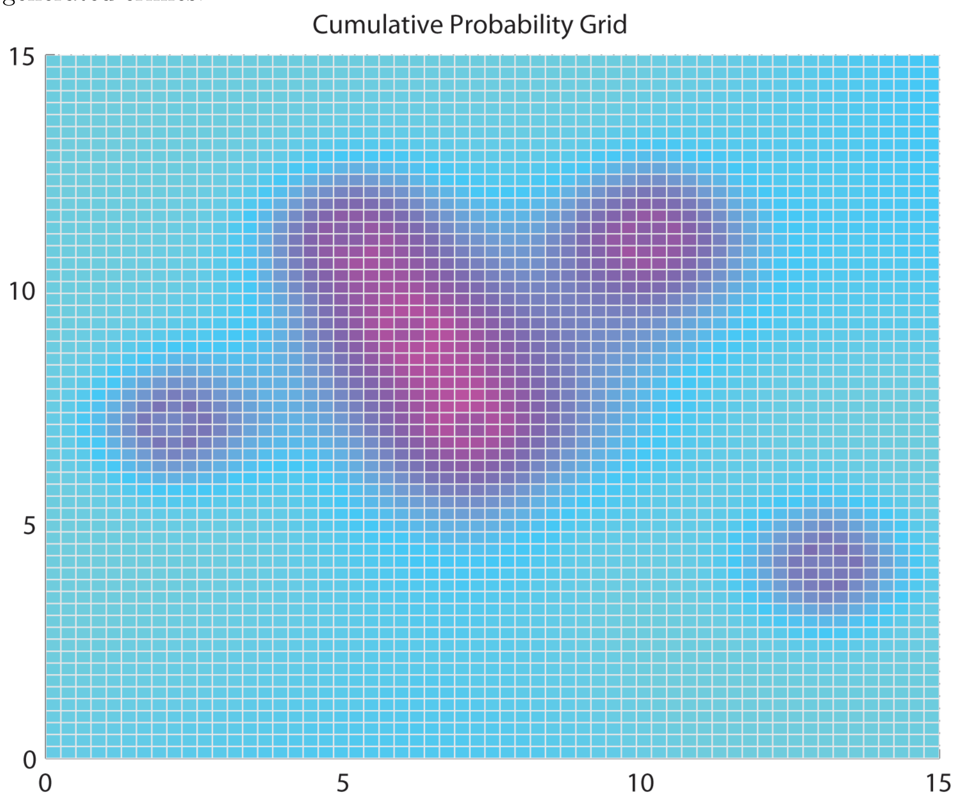
Probability Grid. An example probability grid created from six randomly-generated crimes.



8. Cumulative Probability Grid

Description. Because each of these models have strengths and weaknesses, our final model weights and stacks each of the probability grids in order to narrow down the search field for the next crime location. The weights will depend on the type of crime, due to the variability in observed spatial reasoning of offenders committing different types of crimes.

Probability Grid. An example probability grid created from six randomly-generated crimes.



Part 4. Results

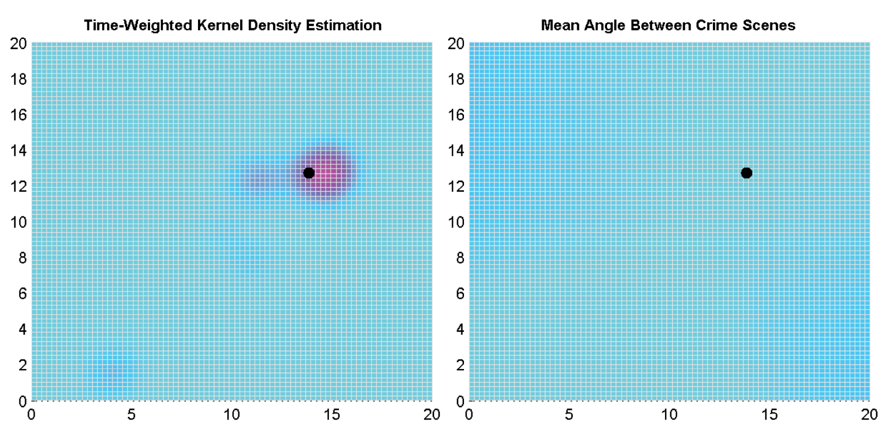
In order to test the effectiveness of our final model, we obtained the data for Peter Sutcliffe's 13 confirmed murders. We input the first 12 data points into the model to predict the location of Sutcliffe's final crime.

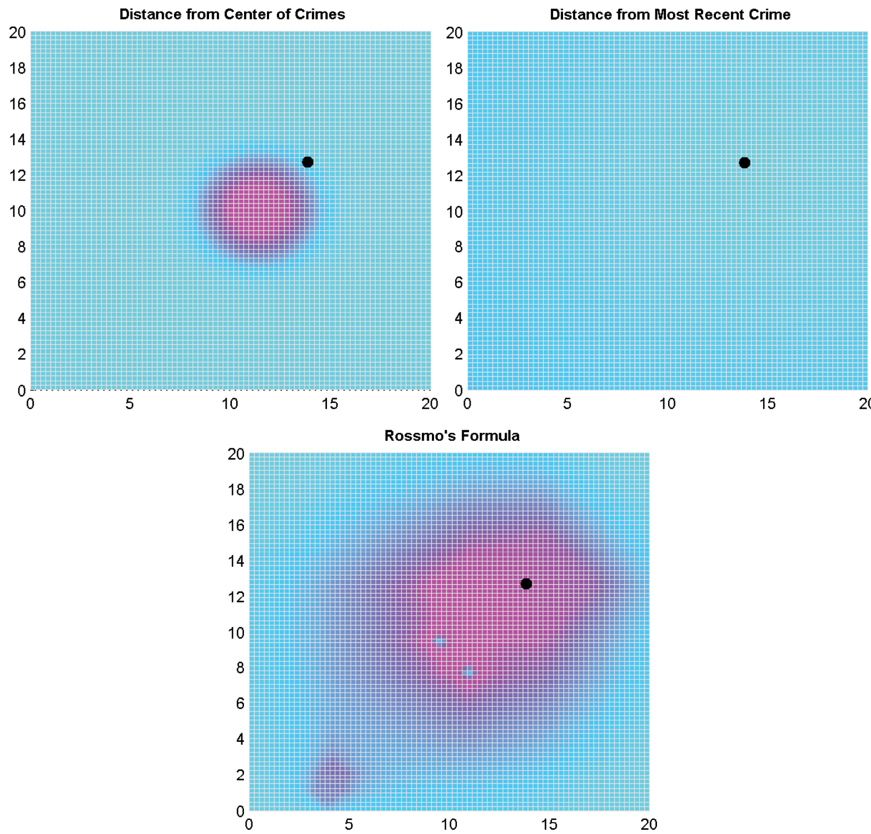
Table 3. Sutcliffe Data

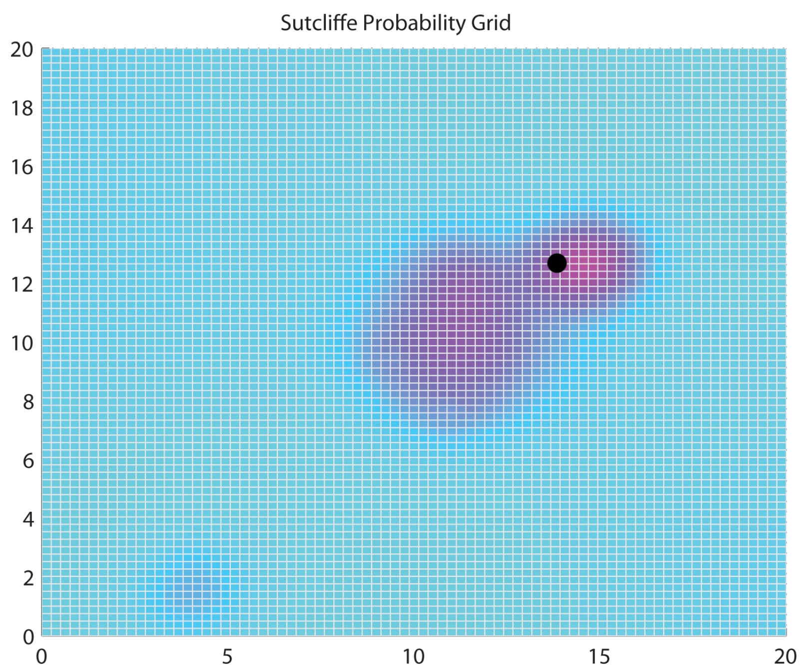
Murder	Latitude	Longitude	Days Since First Crime
1	-1.54	53.81	0
2	-1.53	53.80	82
3	-1.50	53.83	464
4	-1.76	53.81	523
5	-1.53	53.81	605
6	-2.25	53.43	702
7	-1.76	53.79	814
8	-1.77	53.65	824
9	-2.22	53.46	929
10	-1.87	53.71	1252
11	-1.56	53.81	1403
12	-1.67	53.80	1756
13	*(-1.57)*	*(53.82)*	*(1854)*

(Source: http://bbs.keyhole.com/ubb/ubbthreads.php?ubb=download&Number=408713)

In the following graphs, the location of last crime is highlighted:







The model was able to successfully predict the location of Sutcliffe's last murder.

8.1. **Error Analysis.** Since the model ranks all sites by their probability of hosting the next crime, rather than defining a specific region containing it, its error must be defined in terms of the smallest region that would contain the next crime. However, the error might be estimated by obtaining data on the coordinates of past serial crimes, using the model to predict the last crime in each series, and finding the average search area required to reach the actual crime site. (Since it is assumed in the model that the last crime falls inside the probability grid, the search area can be no larger than the entire grid.)

Each of the five individual models has scenarios which it would model poorly. By weighting their probability grids before the final grid is generated, the best-suited models' outputs can be favoured above the worst.

The intention of our model is not to provide a guaranteed region in which the next crime will occur, but rather to be used in combination with other resources (e.g. behavioural profiling) to help police further narrow the field.

Part 5. Conclusion

Due to the limited time and data we were only able to test the Sutcliffe case. However, due to the success of the model in accurately predicting a region for the final Sutcliffe murder, it is reasonable to assume that this model could hold under further scrutiny. This suggests that it could be beneficial to further test this model with other cases, and possibly refine the model by taking into account other aspects of serial criminal spatial reasoning. It would be possible to further refine and test the model if further data were available.

Part 6. Strengths and Weaknesses

Our model is very flexible in that it utilizes several different possible spatial reasoning methods. It expands on previously determined geographic profiling methods in order to narrow down the search region for the next crime location. It combines several different observed offender spatial reasoning behaviors which allows for flexibility in adjusting the model to fit different types of serial crimes.

This model does not take into account non-stationary anchor point, behavioral effects, possible specific target profile, population and neighborhood differences, and potentially several other aspects of an offender's spatial reasoning. This model also would fail to produce a reasonable search region if the crime location spread was very large (i.e. interstate).

Part 7. References

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