Polynomials

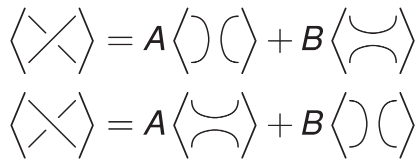
- Read Section 6.1, work on homework.
- History -
  1928 Alexander Polynomial
  1969 Conway - new way to calculate Alexander polynomial
  1984 Jones Polynomial, Kauffman bracket
- **Goal:** Assign a polynomial or Laurent polynomial to (oriented) knots and links so if distinct polynomials are obtained, the links are inequivalent.

Bracket Polynomial

Preliminary Rules for Bracket Polynomial:

Rule 1: $\langle \bigcirc \rangle = 1$

Rule 2:



Rule 3: $\langle L \cup \bigcirc \rangle = C \langle L \rangle$

Type II Move Effect on Bracket

Compute bracket polynomial before and after a Type II move and compare.

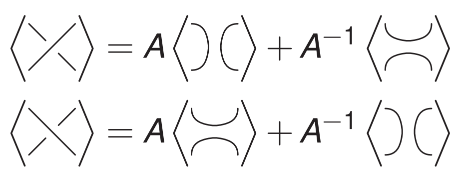
B must equal A^{-1}
C must equal $-A^2 - A^{-2} = (-1)(A^2 + A^{-2})$

New Rules

New Rules for Bracket Polynomial:

Rule 1: $\langle \bigcirc \rangle = 1$

Rule 2:



Rule 3: $\langle L \cup \bigcirc \rangle = (-A^2 - A^{-2}) \langle L \rangle$
$= (-1)(A^2 + A^{-2}) \langle L \rangle$

Effect of Type III Moves?

Check that Type III moves have no effect.

Examples of Computations.

Effect of Type I Moves

Note that Type I moves affect the bracket polynomial.

How to fix?
Definition: The *writhe* of an oriented knot or link projection L. $w(L)$, is the sum of the crossing numbers over all crossings.

X polynomial $X(L) = (-A^3)^{-w(L)} \langle L \rangle$

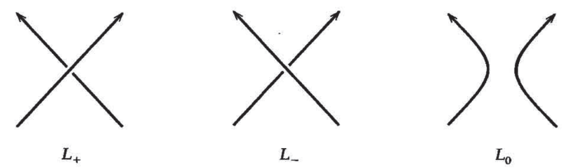
Invariance

Theorem $X(L)$ is an invariant for oriented knots and links.

Definition: The Jones Polynomial $V(L)$ is obtained from $X(L)$ by letting $A = t^{-1/4}$.

Skein Relation

Let L_+, L_-, and L_0 be links that are identical except at one crossing, where they appear as below:



Relation:
$t^{-1} \cdot V(L_+) - t \cdot V(L_-) + (t^{-1/2} - t^{1/2}) \cdot V(L_0) = 0$