Mth 254, Midterm Examination
October, 2005

1. Find the angle between the normal vectors to the planes $x+y+2z=8$ and $x+y+z=6$. Round your answer (in radians) to 2 decimal places. ll

2. The planes from problem (1) intersect in a line. This line contains the point $(0,4,2)$.
   Find the *parametric form* of the equation of this line.

3. Find the equation of the tangent plane to the surface

$$f(x,y) = 4 + x^2 + y^2 + x^2y^2 \qquad \text{at the point } (1,1,7).$$

4. The point $(0,0)$ is a critical point for the function $f(x,y) = 4 + x^2 + y^2 + x^2y^2$

   Use the second partials test to determine whether this point is a local maximum, local minimum or saddle point.

5. Compute $D_{\mathbf{u}} f\,(1,1)$ for $\mathbf{u} = \left\langle \frac{1}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}} \right\rangle$ and for $f(x,y) = 4 + x^2 + y^2 + x^2y^2$

   Your answer should be a number.

6. In which unit vector direction is $D_{\mathbf{u}} f\,(1,1)$ a maximum, for f as in the previous problem? What is this maximum value of $D_{\mathbf{u}} f\,(1,1)$?

7. f is a function of x and y. Each of x and y are in turn functions of u and v. If you are given the following information, compute $\frac{\partial f}{\partial u}$ at the point where $u=1$, $v=0$.

   $x = 2$ when $u = 1$ and $v = 0$ $\qquad \frac{\partial f}{\partial x} = 2xy \qquad \frac{\partial f}{\partial y} = x^2 + y$

   $y = 1$ when $u = 1$ and $v = 0$ $\qquad \frac{\partial x}{\partial u} = 3u^2 + v^2 \qquad \frac{\partial y}{\partial u} = 2uv$

   $\frac{\partial x}{\partial v} = 2vu \qquad \frac{\partial y}{\partial v} = u^2 + v$

Extra Credit Compute the distance between the parallel planes $x+y+2z=3$ and $x+y+2z=5$.