## Mth 254, Final Examination

1. The equation  $xy^2 + y \cos x - 1 = 0$  implicitly defines y as a function of x near the point (0, 1). Use the technique covered in class to find  $\frac{dy}{dx}$  at this point. Show your work to receive credit.

**Hint:** The equation is of the form F(x, y) = 0.

- 2. Set up an iterated double integral for  $\iint_R f \, dA$  where  $f(x, y) = y \sin(x)$  and where R is the region in the first quadrant between the curves  $y = 4x^2$  and  $y = x^3$
- 3. Set up a triple integral in spherical coordinates for  $\iiint_D f \, dV$  where  $f(x, y, z) = x^2 + y^2 + z^2$  and where D is the region between spheres of radius 2 and 4 centered at the origin.
- 4. Set up a double integral in polar coordinates that gives the area of the region in the first quadrant bounded by the x axis and by the polar curve  $r = 2\cos 2\theta$ .
- 5. Set up an integral that gives the surface area of the part of the surface  $z = 9 x^2 y^2$  that lies above the circle of radius 2 centered at the origin in the xy plane.
- 6. Set up an iterated triple integral for  $\iiint_D f \, dV$  where  $f(x, y, z) = x^2 \cos(yz)$  and where D is the solid region bounded by the planes x = 0, y = 0, z = 0, z = 4 y, and y = 4 x.
- 7. The reduced row echelon form of the augmented matrix associated with a linear system in variables x, y, and z is

$$\left(\begin{array}{ccc|c}
1 & 2 & 0 & 2\\
0 & 0 & 1 & -3\\
0 & 0 & 0 & 0
\end{array}\right)$$

Write down the **solution set** to this system.

8. You are given that the first matrix below has reduced row echelon form equal to the second matrix.

 $\begin{pmatrix} 1 & 1 & 3 \\ 3 & 1 & 7 \\ 2 & -1 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ . Use this information to determine if the set of column vectors below is linearly

independent.

$$\left\{ \begin{pmatrix} 1\\3\\2 \end{pmatrix}, \begin{pmatrix} 1\\1\\-1 \end{pmatrix}, \begin{pmatrix} 3\\7\\3 \end{pmatrix} \right\}.$$
 Explain your answer.

- 9.  $\begin{pmatrix} 1\\ 1\\ -1 \end{pmatrix}$  is an eigenvector of the matrix  $\begin{pmatrix} -7 & 8 & -3\\ -13 & 14 & -3\\ -11 & 8 & 1 \end{pmatrix}$ . What is the associated **eigenvalue**? Note: You do not need to compute all the eigenvalues to answer this. Show all your work.
- 10. The number 2 is an eigenvalue of the matrix  $\begin{pmatrix} 1 & -1 \\ -2 & 0 \end{pmatrix}$ . Find **all eigenvectors** associated with this eigenvalue using the method covered in class. Show all your work.