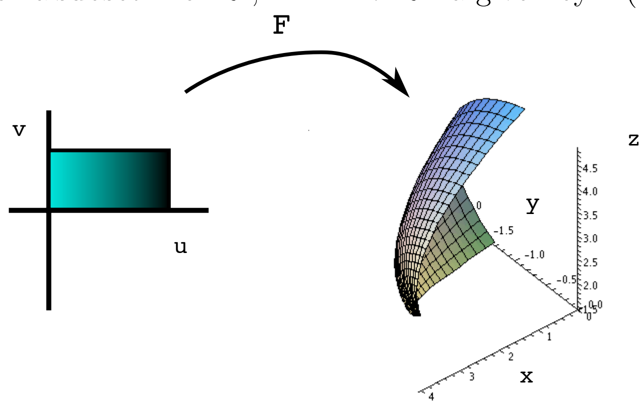
Mth 254 Project Three – Part Two

For the next two problems, use:
If a function defined on a subset D of R^2, $F : D \to R^3$ ia given by $F(u,v) = (x(u,v), y(u,v), z(u,v))$,



the surface area of the image of F is always $\iint_D \left| \left(\frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u} \right) \times \left(\frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v} \right) \right| dA$

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(4) If $D = [0, 2\pi] \times [0, 2\pi]$, there is a function from D onto the surface of the torus obtained by rotating the circle of radius 1 in the $x - z$ plane centered at $(4, 0)$ about the z-axis.
This function is given by (in cylindrical coordinates)

$$r(u,v) = 4 + \cos v, \quad \theta(u,v) = u, \quad z(u,v) = \sin v$$

(a) Compute what x and y are as functions of u and v.

(b) Use the chain rule to compute $\left(\frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u} \right)$ and $\left(\frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v} \right)$

(5) Compute the surface area of the torus described in the previous problem. How does your answer compare with the circumference of the circle you rotate to get the torus times the circumference of a circle of radius 4?