Mth 254 Project Three – Part One

(1) Find the average value of the functions $f(x,y) = \sqrt{x^2+y^2}$ and $g(x,y) = \sqrt{1-x^2-y^2}$ on the region R consisting of $\{(x,y) \,|\, x^2+y^2 \leq 1\}$.

For Problems (2) and (3), use the following information:
The surface area of a surface $z = f(x,y)$ over a region R in the xy-plane is given by:

$$\text{Surface Area } = \iint_R \sqrt{1+f_x^2+f_y^2}\; dA$$

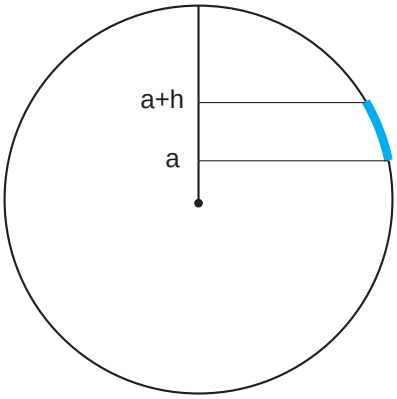
(2) Find the surface area of the upper hemisphere of a sphere of radius A centered at the origin. This surface is given by

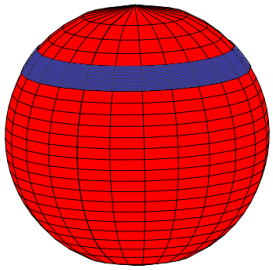
$$f(x,y) = \sqrt{A^2-x^2-y^2}.$$

You should change the double integral to polar coordinates before evaluating.

Use $\iint_R f(x,y)dxdy = \iint_R f(r,\theta)\cdot r\; drd\theta = \int_{\theta_1}^{\theta_2}\left(\int_{r_1}^{r_2} f(r,\theta)\cdot r\, dr\right) d\theta.$
Then try the substitution $u = A^2 - r^2$, $du = -2rdr$.

(3) (a) For the surface in (2), find the surface area of the part of the surface that lies between heights $z = a$ and $z = b$ where $0 \leq a < b \leq A$.
(You will need to find the values of r corresponding to these heights.)
(b) Then let $b = a + h$ and simplify your answer. How does your answer depend on a?





(c) How does the surface area between $z = 0$ and $z = (0.5)A$ compare with the surface area between $z = (0.5)A$ and $z = A$.