## Mth 254H Winter 2013 Project One

You are standing on the surface  $z = 25 - x^2 - y^2$  at the point (1, 1, 23). Depending on the direction that you head in, your height may increase, decrease, or remain the same. You want to determine the rate of increase or decrease of your height (with respect to horizontal distance traveled) when you head in various directions.



1. Let  $\theta$  in polar coordinates represent your direction of travel. Find a unit vector  $\mathbf{U}_{\theta}$  in the xy plane in the direction  $\theta$ .

The line in the plane through the point P = (1, 1) in the direction  $\theta$  can be parameterized as

$$r(t) = P + t \cdot \mathbf{U}_{\theta}$$

Show that the distance from r(t) to r(0) is t.

- 2. Find a function h(t) that gives the height of the surface above the point r(t).
- 3. Find  $\frac{dh}{dt}$  when t = 0. This gives the rate of change of height with respect to horizontal distance travelled in the direction  $\theta$ .
- 4. Let  $c(\theta)$  be the derivative you found in the previous problem. Find the maximum and minimum values of  $c(\theta)$  for  $\theta \in [0, 2\pi]$ . Which angles  $\theta$  correspond to the maximum and minimum values?
- 5. For which values of  $\theta$  is  $c(\theta) = 0$ ?
- 6. What is the relation between the angles you found in problem (4) and the angle(s) you found in problem (5) ?