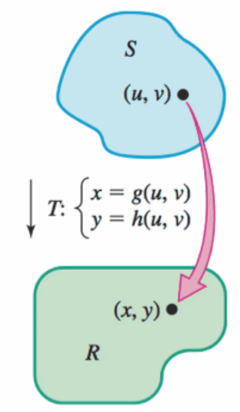
Change of Variable

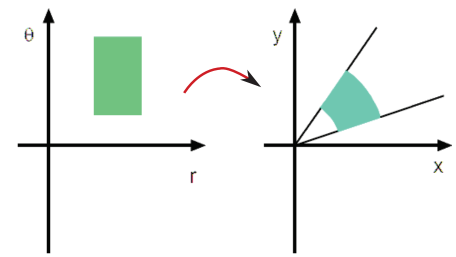
There are two and three dimensional change of variable techniques for integration.
Read 13.7 (Lesson 24)

Suggested Homework:

13.7 - 7, 11, 15, 17, 19, 23, 27, 33, 37, 41

Transformations from R^2 to R^2





Jacobian Determinant

DEFINITION Jacobian Determinant of a Transformation of Two Variables

Given a transformation $T: x = g(u, v)$, $u = h(u, v)$, where g and h are differentiable on a region of the uv-plane, the **Jacobian determinant** (or **Jacobian**) of T is

$$J(u, v) = \frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}.$$

Change of Variable -Double Integrals

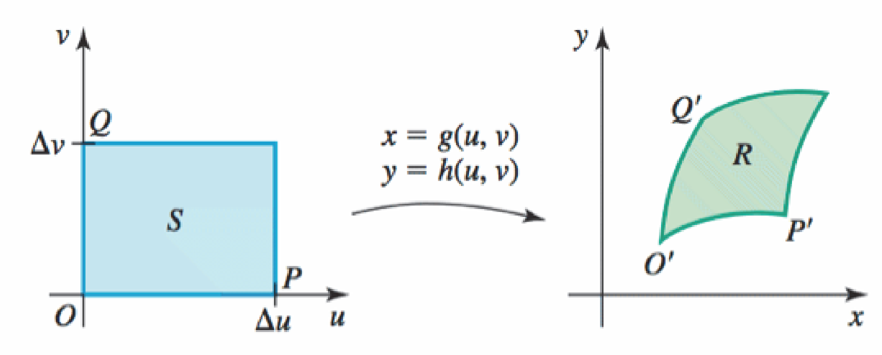
THEOREM 13.8 Change of Variables for Double Integrals

Let $T: x = g(u, v)$, $y = h(u, v)$ be a transformation that maps a closed bounded region S in the uv-plane onto a region R in the xy-plane. Assume that T is one-to-one on the interior of S and that g and h have continuous first partial derivatives there. If f is continuous on R, then

$$\iint_R f(x, y)\, dA = \iint_S f(g(u, v), h(u, v))\, |J(u, v)|\, dA.$$

Examples:

Change of Variable - explanation



3D - Jacobians

DEFINITION **Jacobian Determinant of a Transformation of Three Variables**

Given a transformation $T: x = g(u, v, w)$, $y = h(u, v, w)$, and $z = p(u, v, w)$, where g, h, and p are differentiable on a region of uvw-space, the **Jacobian determinant** (or **Jacobian**) of T is

$$J(u, v, w) = \frac{\partial(x, y, z)}{\partial(u, v, w)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix}.$$

3D - Change of Variable

THEOREM 13.9 **Change of Variables for Triple Integrals**

Let $T: x = g(u, v, w)$, $y = h(u, v, w)$, and $z = p(u, v, w)$ be a transformation that maps a closed bounded region S in uvw-space to a region $D = T(S)$ in xyz-space. Assume that T is one-to-one on the interior of S and that g, h, and p have continuous first partial derivatives there. If f is continuous on D, then

$$\iiint_D f(x, y, z)\, dV = \iiint_S f(g(u, v, w), h(u, v, w), p(u, v, w))\, |J(u, v, w)|\, dV.$$

Examples: