Partial Derivatives

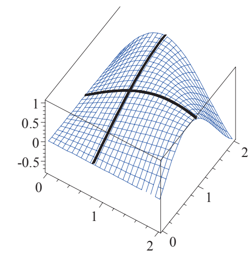
Read Lesson 12 in the study guide

Read Section 12.4 in the text

Continue work on online homework
Also Try 7 - 15 odd numbered, 19, 21, 25, 27, 29, 35, 37, 41, 43, 45

Motivation for Partial Derivatives

Consider $f(x, y) = \sin(xy)$ near the point $(\frac{\pi}{4}, 1)$.



We get different slopes if we approach this point along the two different curves on the surface obtained by fixing x and y.

Partial Derivatives

Def. If f is a function of two variables, its partial derivatives f_x and f_y are defined by

$$f_x(x, y) = \lim_{h\to 0} \frac{f(x+h, y) - f(x, y)}{h}$$

$$f_y(x, y) = \lim_{h\to 0} \frac{f(x, y+h) - f(x, y)}{h}$$

Notation:

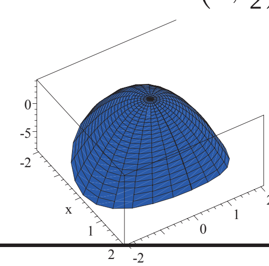
$f_x(x, y) = f_x = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} f(x, y) = \frac{\partial z}{\partial x} = f_1 =$
$D_1 f = D_x f$

How to find partial derivatives

- To find f_x regard y as a constant and differentiate
- To find f_y regard x as a constant and differentiate.

Example:

$f(x, y) = 4 - 2x^2 - 3y^2$
Find partial derivatives at $\left(1, \frac{1}{2}\right)$



Higher order derivatives

$$(f_x)_x = f_{xx} = f_{11} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 z}{\partial x^2}$$

$$(f_x)_y = f_{xy} = f_{12} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 z}{\partial y \partial x}$$

Examples:

$z = x^4 - 2x^2y$

$z = \frac{x}{x+2y}$

Preview of Result on Mixed Partials

Clairaut's Theorem: If $f(x, y)$ is defined on an open set D containing (a, b) and if both f_{xy} and f_{yx} are continuous on D, then $f_{xy}(a, b) = f_{yx}(a, b)$

Functions of more than 2 variables

$f(x, y, z)$ $\quad \frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial y} \quad \frac{\partial f}{\partial z}$

$f(x_1, x_2, \cdots, x_n)$ $\quad \frac{\partial f}{\partial x_1} \quad \frac{\partial f}{\partial x_2} \quad \cdots \quad \frac{\partial f}{\partial x_n}$

Differentiability

Def: If $z = f(x, y)$, then f is *differentiable* or *locally linear* at (a, b) if

$$\Delta z = f(a + \Delta x, b + \Delta y) - f(a, b)$$

can be expressed as

$$\Delta z = f_x(a, b)\Delta x + f_y(a, b)\Delta y + \epsilon_1 \Delta x + \epsilon_2 \Delta y$$

where ϵ_1 & $\epsilon_2 \to 0$ as $(\Delta x, \Delta y) \to (0, 0)$

f is differentiable on an open set D if it is differentiable at each point of D.

Conditions for Differentiability

Theorem: Suppose f has partial derivatives f_x and f_y defined on an open region containing (a, b), with f_x and f_y continuous at (a, b). Then f is differentiable at (a, b).

Theorem: Suppose f is differentiable at (a, b), then f is continuous at (a, b).