Limits and Continuity

Read Lesson 11 in the study guide

Read Section 12.3 in the text

Continue work on online homework
Also try 11-45, odd numbered

Limits and Continuity

Def: Suppose $z = f(x, y)$ has domain D with points (x, y) arbitrarily close to (a, b).

$$\lim_{(x,y)\to(a,b)} f(x, y) = L$$

provided $f(x, y)$ can be made arbitrarily close to L by requiring (x, y) to be sufficiently close to (a, b).

That is, for each $\epsilon > 0$, there is a $\delta > 0$ so that if

$$\text{if } 0 < d((x, y), (a, b)) < \delta$$
$$\text{then } |f(x, y) - L| < \epsilon$$

Examples:

$$f(x, y) = \frac{3x^2y}{2x^2 + y^2} \qquad \lim_{(x,y)\to(0,0)} \frac{3x^2y}{2x^2 + y^2}$$

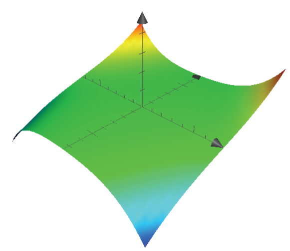
What happens as (x, y) approaches $(0, 0)$ along the axes?

$$\frac{3x^2y}{2x^2 + y^2} = 3\left(\frac{x^2}{2x^2 + y^2}\right) y \to 0 \text{ along the axes.}$$

In fact, $\to 0$ no matter how you come in to origin. Why?

Example, continued

$$f(x, y) = \frac{3x^2y}{2x^2 + y^2}$$



More Examples

Result: If $f(x,y) \to L_1$ as $(x,y) \to (a,b)$ along C_1 and if $f(x,y) \to L_2 \neq L_1$ as $(x,y) \to (a,b)$ along C_2 then

$$\lim_{(x,y)\to(a,b)} f(x,y) \text{ does not exist.}$$

Example:

$$f(x,y) = \frac{xy}{x^2+y^2}$$

What happens if you approach along y=0 or $y = x$?

Limit Rules

Note: Same limit rules apply as in Single Var Calc.

Assume c is constant.

THEOREM 12.1 Limits of Constants and Linear Functions

Let a, b, and c be real numbers.

1. Constant functions $f(x, y) = c$: $\lim_{(x,y)\to(a,b)} c = c$
2. Linear function $f(x, y) = x$: $\lim_{(x,y)\to(a,b)} x = a$
3. Linear function $f(x, y) = y$: $\lim_{(x,y)\to(a,b)} y = b$

Rules, Continued

Suppose $\lim_{(x,y)\to(a,b)} f(x,y) = L$ and $\lim_{(x,y)\to(a,b)} g(x,y) = M$.

1. **Sum** $\lim_{(x,y)\to(a,b)} [f(x,y) + g(x,y)] = L + M$
2. **Difference** $\lim_{(x,y)\to(a,b)} [f(x,y) - g(x,y)] = L - M$
3. **Constant multiple** $\lim_{(x,y)\to(a,b)} [c\,f(x,y)] = c\,L$
4. **Product** $\lim_{(x,y)\to(a,b)} f(x,y)\,g(x,y) = L\,M$
5. **Quotient** $\lim_{(x,y)\to(a,b)} \left[\frac{f(x,y)}{g(x,y)}\right] = \frac{L}{M}$, provided $M \neq 0$
6. **Power** $\lim_{(x,y)\to(a,b)} [f(x,y)]^n = L^n$
7. ***m*/*n* power** If m and n have no common factors and $n \neq 0$,

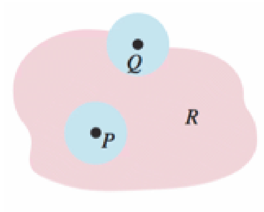
then $\lim_{(x,y)\to(a,b)} (f(x,y))^{m/n} = L^{m/n}$, ($L > 0$ if n even)

Interior and Boundary Points

DEFINITION Interior and Boundary Points

Let R be a region in $\mathbb{R}^2$. An **interior point** P of R lies entirely within R, which means it is possible to find a disk centered at P that contains only points of R (Figure 12.40).

A **boundary point** Q of R lies on the edge of R in the sense that *every* disk centered at Q contains at least one point in R and at least one point not in R.



Open and Closed Sets

DEFINITION **Open and Closed Sets**

A region is **open** if it consists entirely of interior points. A region is **closed** if it contains all its boundary points.

Continuity

Def: $f(x, y)$ is *continuous* at (a, b) if

$$\lim_{(x,y)\to(a,b)} f(x, y) = f(a, b)$$

Note: Polynomial functions and rational functions are continuous at all points in their domains. *Compositions* of continuous functions are continuous.

More Variables

Functions of three or more variables

-Similar definitions for limits, continuity