Motion in Space

- read Sections 11.7 in the text
- read Lesson 6 in the study guide
- Try problems in 11.7:
  7, 9, 11, 13, 21, 23, 27, 29, 35, 39
- View the animations in the online text

Space Curves

Recall, a curve in R^3, or a *space curve* is given by:

$$x = f(t) \quad y = g(t) \quad z = h(t)$$
$$\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$$

Position: $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$

Velocity and Acceleration

Position: $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$

Velocity: $\mathbf{v}(t) = \mathbf{r}\,'(t) = \langle f'(t), g'(t), h'(t) \rangle$

Acceleration:
$\mathbf{a}(t) = \mathbf{r}\,''(t) = \langle f''(t), g''(t), h''(t) \rangle$

Speed:
$|\mathbf{v}(t)| = |\mathbf{r}\,'(t)| = \sqrt{(f'(t))^2 + (g'(t))^2 + (h'(t))^2}$

Unit Tangent Vector: $\mathbf{T}(t) = \dfrac{\mathbf{v}(t)}{|\mathbf{v}(t)|}$

Circular Motion

Theorem: If $|r(t)|$ is constant, then position and velocity are perpendicular.

2 dimensional motion:

An object moving in the plane, subject only to a vertical force of gravity, with initial velocity $\mathbf{v}(0) =< u_0, v_0 >$ and initial position $\mathbf{r}(t) =< x_0, y_0 >$ satisfies:

$$\mathbf{v}(t) =< u_0, v_0 - g \cdot t >, \text{and}$$

$$\mathbf{r}(t) =< u_0 \cdot t + x_0, -\frac{1}{2} g \cdot t^2 + v_0 \cdot t + y_0 >$$

2 dimensional motion continued:

Given an object moving in the plane subject only to a vertical force of gravity, be able to find:

- the time until the object hits the ground
- the range, i.e. the horizontal distance traveled before hitting the ground, and
- the maximum height achieved by the object.

Generalization to 3 dimensional motion

Examples