Lines and Curves, Calculus on Curves

- read Sections 11.5 and 11.6 in the text
- read Lesson 5 in the study guide
- Try problems in 11.5:
  11, 13, 17-27 odd numbered, 35, 41, 43
  in 11.6:
  9, 15, 17, 23, 25, 27, 33, 37, 43, 47, 51
- View the animations in the online text

Vector Valued Functions

Def. A *vector valued function* F of a real variable t is a function from R to R^2 or R^3 given by:

$$\mathbf{F}(t) = \langle f(t), g(t) \rangle \quad \text{or} \quad \mathbf{F}(t) = \langle f(t), g(t), h(t) \rangle$$

Theorem: Let $\mathbf{F}(t) = \langle f(t), g(t), h(t) \rangle$. Then $\mathbf{F}$ has a limit at c if and only if f, g, and h have limits at c. In this case,

$$\lim_{t \to c} \mathbf{F}(t) = \left\langle \lim_{t \to c} f(t), \lim_{t \to c} g(t), \lim_{t \to c} h(t) \right\rangle$$

Space Curves, Lines

A curve in R^3, or a *space curve* is given by:

$$x = f(t) \quad y = g(t) \quad z = h(t)$$
$$\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$$

A *line* is determined by a point $P_0(x_0, y_0, z_0)$ and a direction vector $\mathbf{v} = \langle a, b, c \rangle$

Vector Form of Equation of a Line: where $(\mathbf{r_0} = \overrightarrow{OP_0})$

$$\mathbf{r}(t) = \mathbf{r_0} + t\mathbf{v}$$

Examples:

Parametric, Symmetric Equations

Parametric Equations of a Line

$$x = x_0 + at \quad y = y_0 + bt \quad z = z_0 + ct$$

Symmetric Equations for the Line

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

Continuity and Differentiability

Def. $\mathbf{F}$ is *continuous* at c if $\lim_{t \to c} \mathbf{F}(t) = \mathbf{F}(c)$

The *derivative* of $\mathbf{F}$ is defined to be

$$\lim_{h \to 0} \frac{\mathbf{F}(t+h) - \mathbf{F}(t)}{h}$$

Note: $\mathbf{F}'(t) = \langle f'(t), g'(t), h'(t) \rangle$

Examples:

Differentiation Rules

Familiar differentiation rules hold for vector valued functions:

- $D_t(\mathbf{F}(t) + \mathbf{G}(t)) = \mathbf{F}'(t) + \mathbf{G}'(t)$
- $D_t(\mathbf{c}) = \mathbf{0}$
- $D_t(h(t)\mathbf{F}(t)) = h(t)\mathbf{F}'(t) + h'(t)\mathbf{F}(t)$
- $D_t(\mathbf{F}(t) \cdot \mathbf{G}(t)) = \mathbf{F}(t) \cdot \mathbf{G}'(t) + \mathbf{F}'(t) \cdot \mathbf{G}(t)$
- $D_t(\mathbf{F}(t) \times \mathbf{G}(t)) = \mathbf{F}(t) \times \mathbf{G}'(t) + \mathbf{F}'(t) \times \mathbf{G}(t)$
- $D_t(\mathbf{F}(h(t)) = \mathbf{F}'(h(t))h'(t)$

Examples:

Velocity and Acceleration

Position: $\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$

Velocity: $\mathbf{v}(t) = \langle f'(t), g'(t), h'(t) \rangle$

Acceleration: $\mathbf{a}(t) = \langle f''(t), g''(t), h''(t) \rangle$

Speed:
$v(t) = |\mathbf{v}(t)| = \sqrt{(f'(t))^2 + (g'(t))^2 + (h'(t))^2}$

Examples:

Tangent Line to a Curve:

Def. The *tangent line* to a curve $\mathbf{r}(t)$ at the point t_0 is the line through $\mathbf{r}(t_0)$ with direction vector $\mathbf{r}'(t_0) = \mathbf{v}(t_0)$

Example:
Circular Motion