Assignment

Read: Section 11.3 – Dot Products
Lesson 3 in Study Guide

Try:
§11.3 #9, 11, 15, 21, 23, 25, 27, 29, 33
Begin work on assignment 5 in My Mathlab

Dot Product of Vectors

Def: If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, the dot product of **a** and **b**,$\mathbf{a} \cdot \mathbf{b}$ is

$$a_1 b_1 + a_2 b_2 + a_3 b_3$$

Note: Result is a scalar **!!**

Similar definition for n-dimensional vectors.

Examples

Properties of Dot Products

Read in text

1. $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$
2. $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$
3. $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$
4. $(c\mathbf{a}) \cdot \mathbf{b} = c(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \cdot (c\mathbf{b})$
5. $\mathbf{0} \cdot \mathbf{a} = \mathbf{0}$

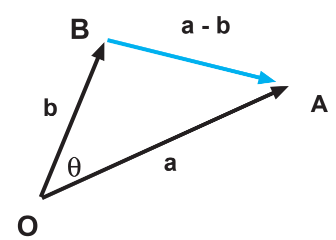
Why these hold:

Angle between vectors:

Def: The angle θ between **a** and **b** is the angle between the representations of these vectors starting at the origin, and is restricted: $0 \leq \theta \leq \pi$

Theorem: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}|\ |\mathbf{b}| \cos\theta$

Proof:



Angle from Dot Product

Corollary: The angle θ between non zero vectors $\mathbf{a}$ and $\mathbf{b}$ is given by $\cos\theta = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|}$

Note: $\mathbf{a}$ and $\mathbf{b}$ are orthogonal *if and only if* $\mathbf{a}\cdot\mathbf{b} = 0$.

Direction Angles

Def. The *direction angles* of a nonzero $\mathbf{a}$ are the angles α, β, and γ in $[0,\pi]$ that $\mathbf{a}$ makes with the positive x, y, and z axes.

The cosines of these angles are called the *direction cosines of* $\mathbf{a} = \langle a_1, a_2, a_3\rangle$.

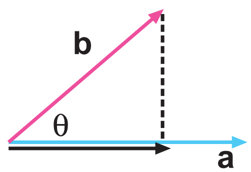
Note:

$$\cos\alpha = \frac{a_1}{|\mathbf{a}|} \qquad \cos\beta = \frac{a_2}{|\mathbf{a}|} \qquad \cos\gamma = \frac{a_3}{|\mathbf{a}|}$$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ and

$\mathbf{a} = |\mathbf{a}|\,\langle\cos\alpha, \cos\beta, \cos\gamma\rangle$

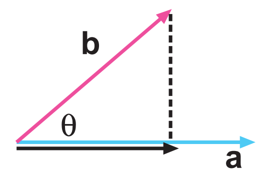
Projections



The *vector projection of* $\mathbf{b}$ onto $\mathbf{a}$ is the following **vector** in the direction of $\mathbf{a}$

$$proj_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}\frac{\mathbf{a}}{|\mathbf{a}|} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\mathbf{a} = \frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{a}\cdot\mathbf{a}}\mathbf{a}$$

Scalar Projection



The *scalar projection of* $\mathbf{b}$ onto $\mathbf{a}$ is the following **scalar**:

$$\text{scal}_{\mathbf{a}}\mathbf{b} = |\mathbf{b}|\cos\theta = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$$

Example: (work done by force)