13.5 – Cylindrical and Spherical Integrals

Read Lesson 22 in the Study Guide and Section 13.5 in the text.
- triple integrals in cylindrical coordinates - triple integrals in spherical coordinates

Suggested Homework:

Try 11, 13, 17, 19, 21, 23, 29, 31, 33
35, 37, 39, 41, 43, 45, 47, 49, 51 63

Cylindrical coordinates:

$$\iiint_B f(x,y,z)dV = \int_\alpha^\beta \int_{h_1(\theta)}^{h_1(\theta)} \int_{u_1(r,\theta)}^{u_2(r,\theta)} f(x,y,z)r\,dz\,dr\,d\theta$$

Reason:

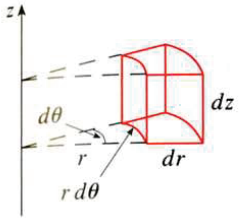
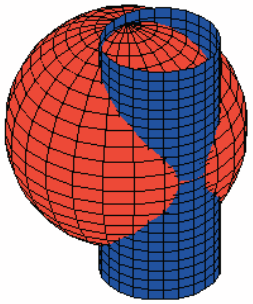


FIGURE 3
Volume element in cylindrical coordinates: $dV = r\,dz\,dr\,d\theta$

Example:

- Find the volume cut out of the sphere of radius a centered at the origin by the cylinder $r = a\cos\theta$

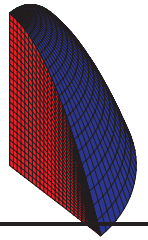


Example:

- Set up in cylindrical coordinates the integral

$$\iiint_E yx^2 + xy^2 dV$$

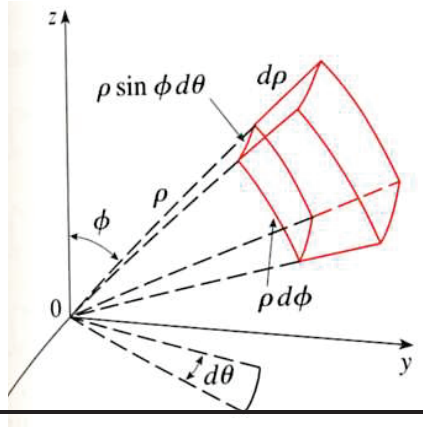
where E is the solid in the first octant that lies under the paraboloid $z = 4 - x^2 - y^2$



Spherical Coordinates:

$$\iiint_B f(x,y,z)dV = \int_c^d \int_\alpha^\beta \int_a^b f(x,y,z)\rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

Reason:



Examples:

- Find the volume of a sphere of radius a by using a triple integral in spherical coordinates.
- Set up an integral in spherical coordinates for

$$\iiint_B xe^{(x^2+y^2+z^2)}dV$$

where B is the region in the first octant between the spheres of radius 1 and 4 centered at the origin.