### 13.4 - Triple Integrals

Read Lesson 21 in the Study Guide and Section 13.4 in the text.

- triple integrals over rectangular regions
- triple integrals over general regions

#### **Suggested Homework:**

Try 7, 11, 15-37 odd numbered, 41

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### Existence and Properties

#### **Notes:**

- The sum in the definition is a triple Riemann sum.
- If f is continuous, the triple integral exists and does not depend on the choice of  $(x_{iik}^*, y_{iik}^*, y_{iik}^*)$
- Same properties as double integrals

#### Definition of Triple Integral

**Def:** The triple integral of f(x, y, z) over a rectangular box  $B = [a, b] \times [c, d] \times [r, s]$  is

$$\iiint\limits_B f(x,y,z)dz =$$

$$\lim_{\ell,m,n\to\infty} \sum_{i=1}^{\ell} \sum_{j=1}^{m} \sum_{k=1}^{n} f(x_{ijk}^{*}, y_{ijk}^{*}, z_{ijk}^{*}) \Delta V_{ijk}$$

provided the limit exists.

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#### **Evaluating**

#### **Fubini's Theorem:**

If f(x, y, z) is continuous on  $B = [a, b] \times [c, d] \times [r, s]$ , then

$$\iiint\limits_R f(x,y,z)dV = \int_r^s \left( \int_c^d \left( \int_a^b f(x,y,z) dx \right) dy \right) dz$$

**Note:** Also equal to iterated integrals in the other 5 orders.

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#### **Examples**

$$\iiint\limits_R f(x,y,z)dV \text{ where } f(x,y,z) = x\sin(x+y+z) \text{ and }$$
 where

$$R = [0,1] \times [0,2] \times [0,3]$$

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## Special 3-dimensional regions

**Def:** A z-simple region is a region  $\mathbf{E} =$ 

$$\{(x,y,z)|(x,y)\in D, u_1(x,y)\leq z\leq u_2(x,y)\}$$

Other simple regions are similarly defined.

#### General Regions

If E is a bounded region, contained in a box B, and f(x, y, z) is defined on E, Then

$$\iiint\limits_E f(x,y,z)dV = \iiint\limits_B F(x,y,z)dV$$

where F(x, y, z) = f(x, y, z) if  $(x, y, z) \in E$  and = 0 otherwise.

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### **Evaluating on Special Regions**

If f is continuous on a z-simple region,

$$\iiint\limits_E f(x,y,z)dV = \iint\limits_D \left( \int_{u_1(x,y)}^{u_2(x,y)} f(x,y,z)dz \right) dA$$

Similarly for other simple regions.

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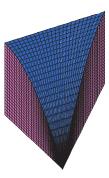
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## Examples

• Evaluate  $\iiint_E f(x, y, z) dV$  where f(x, y, z) = xyz and where E is the solid region bounded by the coordinate planes and 2x + y + 3z = 6

# More Examples

• Set up iterated integrals for the volume of the region bounded by the planes  $z=0,\ x=0,\ y=0,\ y=1-x$  and the surface  $z=1-x^2$ 



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