§ 13.2–Double Integrals over General Regions

This lesson covers the material in Section 13.2

Read Lesson 19 in the Study Guide and Section 13.2 in the text.

Continue working on online homework.

Try: 7, 13, 19, 23, 25, 29, 35, 39, 43, 49, 51, 53, 57, 63

Bounded Regions

Definition: If D is a *bounded region,* contained in a rectangle R, and $f(x, y)$ is defined on D, we can define a new function on R by

$$F(x,y) = \begin{cases} f(x,y) \text{ if } (x,y) \in \mathbf{D} \\ 0 \text{ if } (x,y) \notin \mathbf{D} \end{cases}$$

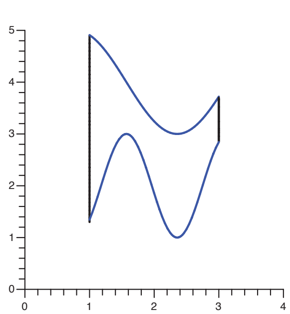
Then

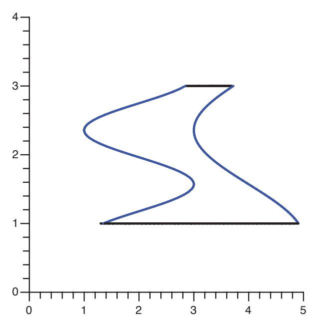
$$\iint_D f(x,y)dA \text{ is defined to be } \iint_R f(x,y)dA$$

Types of Regions:

Def: A *y-simple* region is a region $D = \{(x,y) \,|\, a \le x \le b,\ g_1(x) \le y \le g_2(x)\}$

An *x-simple* region is a region $D = \{(x,y) \,|\, c \le y \le d,\ h_1(y) \le x \le h_2(y)\}$





How to Compute

Note: some regions are both x-simple and y-simple, some regions are neither.

If f is continuous on a y-simple region,

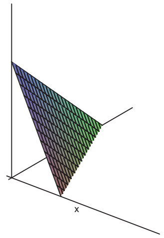
$$\iint_D f(x,y)dA = \int_a^b \left(\int_{g_1(x)}^{g_2(x)} f(x,y)dy \right) dx$$

If f is continuous on an x-simple region,

$$\iint_D f(x,y)dA = \int_c^d \left(\int_{h_1(x)}^{h_2(x)} f(x,y)dx \right) dy$$

Examples:

- Find the volume of the solid bounded by $y = 0, z = 0, y = x$, and $6x + 2y + 3z = 6$



- $\iint_D x\sqrt{y^2 - x^2}\,dA$ for $0 \le y \le 1$, $0 \le x \le y$

Examples:

- Sketch the region and change the order of integration for $\int_0^1 \left(\int_{y^2}^{2-y} f(x,y)dx \right) dy$
- $\iint_D (x+y)dA$ for the region bounded by $y = \sqrt{x}$ and $y = x^2$