§ 13.1–Double Integrals

This lesson covers the material in Section 13.1

Read Lesson 18 in the Study Guide and Section 13.1 in the text.

Continue working on online homework.

Try: 5-23 odd numbered, 27

Double Integral Definition

Def: The *norm* or *mesh* of a partition P of a rectangle $R = [a, b] \times [c, d]$ into subrectangles is the length of the longest diagonal of the subrectangles and is denoted $|P|$.

The double integral of $f(x, y)$ over a rectangle $R = [a, b] \times [c, d]$ is

$$\iint_R f(x, y)dA = \lim_{|P|\to 0} \sum_{i=1}^{n} \sum_{j=1}^{m} f(x_{ij}^* y_{ij}^*)\Delta A_{ij}$$

provided the limit exists.

Volume

$\iint_R f(x, y)dA = \lim_{|P|\to 0} \sum_{i=1}^{n} \sum_{j=1}^{m} f(x_{ij}^* y_{ij}^*)\Delta A_{ij}$

Note:

- If $f(x, y) > 0$ on R, this represents the volume under the graph of f and above the xy plane.
- The sum in the definition is a double Riemann sum.
- If f is continuous, the double integral exists and does not depend on the choice of (x_{ij}^*, y_{ij}^*).

Existence of Double Integrals

Integrability Theorem: If f is bounded on the closed rectangle R and is continuous except on a finite number of smooth curves, then f is integrable on R. In particular, if f is continuous, then it is integrable.

Example:

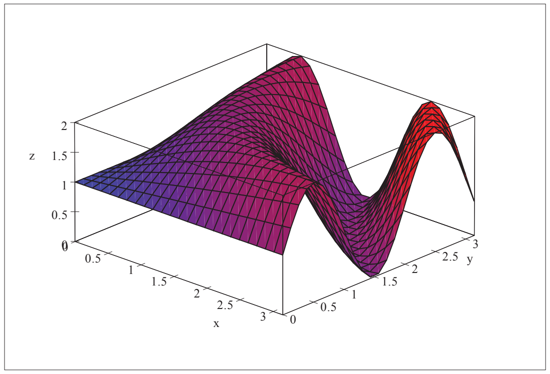
$\iint_R 2x + 2dA$ where $R = [0, 2] \times [0, 2]$

Properties

- $\iint_R 1dA =$ area of R
- $\iint\limits_R (f(x,y) + g(x,y))\, dA$
  $= \iint\limits_R f(x,y)dA + \iint\limits_R g(x,y)dA$
- $\iint\limits_R cf(x,y)dA = c \iint\limits_R f(x,y)dA$
- If $f(x,y) \geq g(x,y)$ on R, then
  $\iint\limits_R f(x,y)dA \geq \iint\limits_R g(x,y)dA$

Other Examples:

$\iint\limits_R \sin(xy) + 1dA$ where $R = [0,\pi] \times [0,\pi]$



How to evaluate this?

Iterated Integrals

Main Results:

$$\iint_R f(x,y)dA = \int_c^d \left[\int_a^b f(x,y)dx\right] dy$$

$$\iint_R f(x,y)dA = \int_a^b \left[\int_c^d f(x,y)dy\right] dx$$

Reason:

Examples

- $\displaystyle\iint_R x^2y\, dA$ over $[-2,2] \times [0,3]$
- $\displaystyle\iint_R yx\sin(x^2)\, dA$ over $[0,1] \times [0,1]$
- $\displaystyle\iint_R f(x)g(y)\, dA$ over any rectangle $[a,b] \times [c,d]$

Average Value

DEFINITION **Average Value of a Function over a Plane Region**

The **average value** of an integrable function f over a region R is

$$\bar{f} = \frac{1}{\text{area of } R} \iint_R f(x, y)\, dA.$$