The Method of LaGrange Multipliers

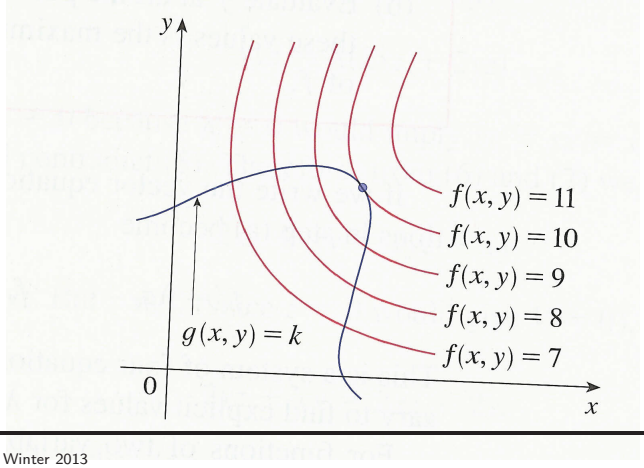
Finding the maximum and minimum value of $f(x, y)$, or $F(x, y, z)$, subject to certain constraints.

Read Lesson 17 and section 12.9

Try:
5, 7, 9, 15, 25, 27

General Problem:

Maximize or minimize $f(x, y)$ subject to the constraint $g(x, y) = k$.



Method of Lagrange Multipliers

To find the max and min values of $f(x, y)$ subject to $g(x, y) = k$,

Find all values of λ, x, and y such that
$\nabla f(x, y) = \lambda \nabla g(x, y) \qquad g(x, y) = k$

Then evaluate f at all points found and choose the largest or smallest of these values.

Method applied to $f(x, y, z)$

To find the max and min values of $f(x, y, z)$ subject to $g(x, y, z) = k$,

Find all values of λ, x, y and z such that
$\nabla f(x, y, z) = \lambda \nabla g(x, y, z) \qquad g(x, y, z) = k$

Then evaluate f at all points found and choose the largest or smallest of these values.

Examples

Find the extreme values of $f(x, y) = x^2 + 4y^2$ on the ellipse $2x^2 + y^2 = 1$.

Find the points on the sphere $x^2 + y^2 + z^2 = 9$ closest to and farthest from $(1, 1, 1)$.

A rectangular box has surface area 1000 in^2 and total edge length 150 in. Find the maximum and minimum volume of such a box.

Two Constraints

The maximum and minimum values of $f(x, y, z)$ subject to two constraints $g(x, y, z) = k$, $h(x, y, z) = c$ can be found by solving

$$\nabla f(x, yz) = \lambda \nabla g(x, y, z) + \mu \nabla g(x, y, z)$$
$$g(x, y, z) = k$$
$$h(x, y, z) = c$$

Example, continued

$V(x, y, z) = xyz$
$2xy + 2yz + 2xz = 1000$
$4x + 4y + 4z = 150$

Other examples

- Largest area rectangle with a given perimeter.
- Largest rectangular box in first quadrant with three faces in coordinate planes and one vertex in plane $x + 2y + 3z = 6$