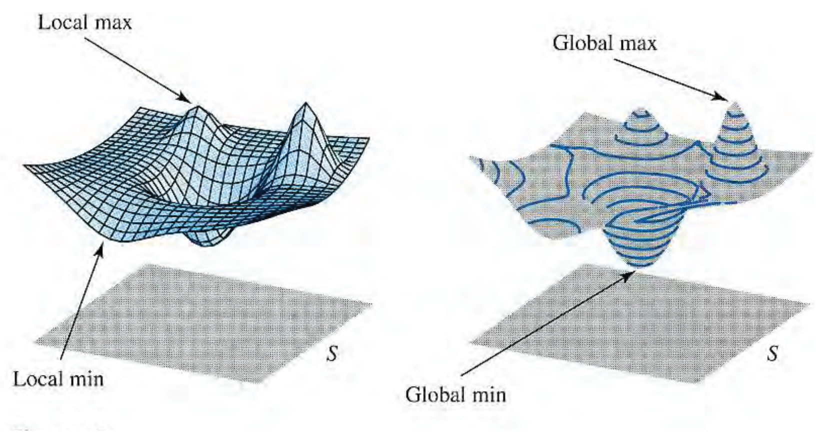
Maxima and Minima

Review the definitions of *global maximum value, global minimum value, global extreme value* and *local maximum value, local minimum value, local extreme value* from single variable Calculus.

Read Lesson 16 and section 12.8

Try: 9-19 odd numbered, 29, 33, 37, 39, 47, 51

Max Min Existence and Critical Points



Existence

- **Max Min Existence Theorem:** If f is continuous on a closed and bounded set S, then f achieves both a global maximum value and a global mimimum value.

  Proof: See Mth 311 (Advanced Calculus) :

Where Extreme Values Occur

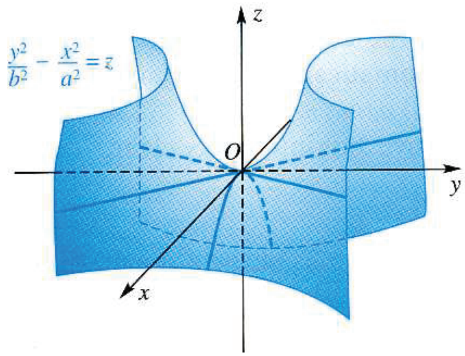
Definition: The following three types of points are *critical points*:

- **Boundary Points**
- **Stationary Points**
  (interior points p where f is differentiable and $\nabla f(p) = \mathbf{0}$).
  This corresponds to a horizontal tangent plane.
- **Singular Points**
  (interior points p where f is not differentiable.)

Critical Point Theorem:
Let f be defined on a set S containing p. If $f(p)$ is an extreme value, then p *must be* a critical point.

Saddle Points

Definition: A *saddle point* for a function p is a critical point p where $\nabla f(p) = \mathbf{0}$ such that $f(p)$ is *not* a local extreme value.



Second Partials Test

Theorem: Suppose $f(x, y)$ has continuous second partials in a neighborhood of (a, b) and that $\nabla f(a, b) = 0$.
Let $D = D(a, b) = f_{xx}(a, b) f_{yy}(a, b) - f_{xy}^2(a, b)$.
Then

1. If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local maximum value
2. If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local minimum value
3. If $D < 0$, (a, b) is a saddle point
4. If $D = 0$, this test is inconclusive

Examples: