§ 12.7–Tangent Planes

This lesson covers the material in Section 12.7

Read Lesson 15 in the Study Guide and Section 12.7 in the text.

Continue working on online homework.

Try: 9-25 odd numbered, 29, 35, 39

Tangent Plane

For a differentiable function $z = f(x, y)$:

$$\mathbf{z - z_0 = f_x(a,b)(x-a) + f_y(a,b)(y-b) + \epsilon_1 \Delta x + \epsilon_2 \Delta y}$$

and the *tangent plane* to the surface $z = f(x, y)$ at the point $(a, b, f(a, b))$ is

$$\mathbf{z - z_0 = C_1(x-a) + C_2(y-b)}$$

where $z_0 = f(a, b)$, $C_1 = f_x(a, b)$ and $C_2 = f_y(a, b)$

or:

$$\boxed{\mathbf{z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)}}$$

Tangent Plane

So, for a differentiable function $z = f(x, y)$, the *tangent plane* to the surface at (a, b) is given by

$$\boxed{z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)}$$

Def. For a level surface $F(x, y, z) = k$ where F is differentiable at $P(a, b, c)$, and with $\nabla F(a, b, c) \neq \mathbf{0}$, the *tangent plane* to the surface at P is the plane through P with normal vector $\nabla F(a, b, c)$.

$$\boxed{\nabla F(a,b,c) \cdot \langle x-a, y-b, z-c \rangle = 0}$$

Tangent Plane, continued

The first formula,

$$\boxed{z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)}$$

is a special case of the second formula

$$\boxed{\nabla F(a,b,c) \cdot \langle x-a, y-b, z-c \rangle = 0}$$

when $F(x, y, z) = f(x, y) - z = 0$

Similarity to Tangent Line Formulas:

For a differentiable function $y = f(x)$, the *tangent line* to the function at $x = a$ is given by

$$\boxed{z = f(a) + f'(a)(x - a)}$$

Def. For a level curve $F(x, y) = k$ where F is differentiable at $P(a, b)$, and with $\nabla F(a, b) \neq \mathbf{0}$, the *tangent line* to the level curve at P is the line through P with normal vector $\nabla F(a, b)$.

$$\boxed{\nabla F(a, b) \cdot \langle x - a, y - b \rangle = 0}$$

Differentials

Let $z = f(x, y)$ be differentiable and let dx and dy be variables, thought of as small changes in x and y. *The differential* of the dependent variable z, written dz, is

$$dz = df(x, y) = f_x dx + f_y dy$$

dz can be used as an approximation to the actual change in z:

$$\Delta z = f(x + dx, y + dy) - f(x, y)$$