Recent Results, Definitions

Def: If $z = f(x, y)$, then f is *differentiable* or *locally linear* at (a, b) if

$$\Delta z = f(a + \Delta x, b + \Delta y) - f(a, b)$$

can be expressed as

$$\Delta z = f_x(a, b)\Delta x + f_y(a, b)\Delta y + \epsilon_1 \Delta x + \epsilon_2 \Delta y$$

where ϵ_1 & $\epsilon_2 \to 0$ as $(\Delta x, \Delta y) \to (0, 0)$

i.e

$$\mathbf{z - z_0 = C_1(x - a) + C_2(y - b) + \epsilon_1 \Delta x + \epsilon_2 \Delta y}$$

Differentiability

Theorem: Suppose f has partial derivatives f_x and f_y defined on an open region containing (a, b), with f_x and f_y continuous at (a, b). Then f is differentiable at (a, b).

Theorem: Suppose f is differentiable at (a, b), then f is continuous at (a, b).

Chain Rule

The Chain Rule, Case 1:
If $f(x, y)$ is a function of two variables x and y, and if $x = x(t)$ and $y = y(t)$ are functions of a third variable t, then

$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle \cdot \left\langle \frac{dx}{dt}, \frac{dy}{dt} \right\rangle$$