Correcting For Scaling Errors Associated With Gap Based Dendrometer Bands.

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ABSTRACT

Long-term biometric measurements in forests can be used to determine interannual variability in wood volume and aboveground net primary productivity (ANPP), and is essential for estimating net ecosystem production (NEP). The methodology for monitoring tree growth typically includes repeated measurements of stem diameter using fixed dendrometer bands. Dendrometers can provide accurate data over multiple time scales and reduce measurement errors associated with year to year variability of measurement position. However, growth is underestimated if the change in dendrometer gap is measured linearly and assumed to represent actual change in circumference. We show that a solution for a "true" diameter cannot be obtained mathematically when given only a band length and gap width, but diameter can be approximated using a simple model simulation. Results from a simulation of a range of tree sizes and gap widths provided a simple relationship that can be used as a correction factor with minimal error. A scaling exercise using three different forest stands illustrate the magnitude of the errors associated with estimating ANPP from uncorrected dendrometer band data. This error is small on >25 cm diameter stems (2-4%) but can be > 25% on small trees (<10 cm) with a potential error of >60% in certain situations.

KEYWORDS: forest production, dendrometer bands, ANPP, NPP, NEP, forest growth, scaling.

INTRODUCTION

Long-term studies on aboveground production (ANPP), changes in biomass, and net ecosystem production (NEP) in forests require repeated measurements of tree stem diameters on fixed plots. Dendrometers and dendrometer bands have been long used as a simple tool for accurately quantifying changes in tree stem diameter (Liming 1957; Reineke 1932). These fixed dendrometer methods can be superior to both repeated tape/caliper measurements and tree cores, because periodic measurements with a diameter tape may fail to measure the same location on a given stem and cores can be destructive to small trees and impractical to repeat annually.

Many varieties of dendrometers have been developed but most modern studies employ a spring tensioned band fixed around the tree at breast height (1.37 m) that expands with tree stem growth. Data from dendrometer bands are useful at an annual scale for accurate estimates of above ground net primary productivity ANPP (Thomas et al. 2009; Vickers et al. 2012; both use corrected data) but can also be used for finer scale studies of temporal allocation patterns or even daily water storage (Drew and Downes 2009).

The dendrometer device itself can be very complex with log-able units able to record daily or even hourly changes in stem circumference (Link et al. 1998), or the band may be made inexpensively with reference points on the band read by hand with an attached Vernier scale (Cattelino et al. 1986; Liming 1957) or by using calipers. Unfortunately, using the simplest but commonly implemented method of measuring a gap between fixed points with a caliper is inherently biased because a true circumference change is not measured - only approximated by measuring the linear cord distance (Figure 1). This effect may seem minimal but can be quite substantial in certain circumstances, most notably when used on small trees or when used for many consecutive years. Furthermore, there is no exact geometric solution to determine the correct circumference change in subsequent years following the initial instillation; this is because current tree diameter, which is unknown, is needed for a trigonometric solution. The goal of this study is to (1) illustrate the magnitude of this effect and highlight the situations where the errors may be substantial, , (2) to provide an improved method for future work and a correction for past studies that used spring dendrometers and linear measurements to estimate tree diameter change, and (3) to illustrate the effect on plot estimates of ANPP with and without the correction.

MATERIALS AND METHODS

Identification of the Problem:

Dendrometer bands can be fashioned in many different configurations but an increasing use of digital calipers to measure changes in gap width warrants the exploration of associated errors. The errors arise from assuming that a linear distance equates to a semicircular increase in circumference (Figure 1). The errors may seem trivial on large trees or when the gap is very small; however, as the gap increases and becomes large relative to the diameter of the tree, the angle - which is bounded by the triangle created from the gap (cord of circle, c) and the radius - increases greatly (Figure 1). Most investigators periodically reset the bands to prevent the large angles, but diameters are underestimated at any angle and should be corrected for. Furthermore, when the bands are used to estimate annual production, diameter is estimated at each time step so the error is propagated each year and the small errors in the initial diameter estimations become very large.

Model simulation

Unfortunately, no geometric solution exists to determine diameter, circumference, central angle (θ) or arc $length_2$ using solely the gap width (cord) and arc $length_1$. The gap width (c) and arc $length_1$ (L_1) are the only exact measurements available following the first year that the bands were installed, while diameter/radius/circumference are of interest for plot surveys or scaling. When the bands are first installed, an initial diameter and gap width (c) can be measured and used to calculate arc length$_1$ (L_1). In subsequent years it is necessary to mathematically approximate a diameter for each new gap width (c), while arc length$_1$ (L_1) remains constant until the band is reset. The geometry and trigonometry involved in an approximate solution to [Equation 1] can be accomplished using Newton's method for estimating the zero intercept of a function. This solution can be very exact, but requires complex solving software (e.g. MAPLE; Maplesoft, Waterloo, ON, CA), also the zero function needs to be solved for each circle, i.e. each tree, and has multiple solutions at smaller angles.

[Equation 1]

$$\theta r = L_2 = 2\pi r - L_1$$
$$=> \theta = 2\pi - \frac{L_1}{r}$$
$$=> \sin\left(\frac{\theta}{2}\right) = \frac{c}{2r}$$
$$=> \sin\left(\pi - \frac{L_1}{2r}\right) = \frac{c}{2r}$$
$$=> \sin\left(\pi - \frac{L_1}{2r}\right) = \frac{t}{2r}$$
$$=> \sin\left(\pi - \frac{L_1}{2r}\right) - \frac{t}{2r} = 0$$

In Equation 1: L1, L2, r, c and θ are the arc lengths, radius, cord length and internal angle (in radians) for a given circle (Figure 1). To overcome the complexity of the purely mathematical method, we employed a modeling simulation to approximate the relationship among diameter, cord length and arc length. This relationship allowed us (1) to estimate the magnitude of the errors across a range of conditions and (2) through the life span of an individual band, and finally (3) to correct for this error.

Simulated data was created using a matrix of 90,500 circles with a range of diameters from 1-500 cm in 1 cm increments and angles between 0° and 180° in 1 degree increments assigned to each diameter; thus creating a corresponding cord length (Figure 1). This cord length was substituted for the arc length [Arc $Length_2$] which is the arc length bounded within the angle [θ] and added to the remaining arc length [Arc $Lengh_2$] to calculate an estimated circumference. This value was used to calculate an estimated diameter (D_{est}) which could be compared to the actual diameter (Figure 2, top panel). The ratio of the actual diameter (D) to the estimated diameter ($D:D_{est}$) was calculated, as this quantity could be used as a simple multiplicative correction factor for all diameters estimated from gap based dendrometers bands. The values for $D:D_{est}$ were plotted against the ratio of cord length: arc $lenght_1$ (Figure 2, top panel).

Tree and stand level errors

To examine the propagation of diameter errors through time, as might occur during long term monitoring of stem diameter in studies of forest production, we simulated a 10 cm Douglas fir tree [*Pseudotsuga menziesii* (Mirb.) Franco] that grows at 0.5 cm per year for 20 years. The diameter increment data was then used to estimate annual above ground biomass from species specific allometric equations for bole, branch, foliage and bark (Hudiburg et al. 2009; Ter-Mikaelian and Korzukhin 1997), of which the difference between successive years is equal to above ground net primary productivity (ANPP). This exercise reflects the “worst case scenario” where a band is left to expand to 180° so that the cord length equals the tree diameter. In practice, most investigators would reset the band and new reference points would be marked on the band immediately following the last cord length measurement. This reset band would then be used to calculate a new band length (arc $lenght_1$) and the process would be repeated every few years.

Actual plot level data and dendrometer bands from 3 differing forested stands were used to illustrate errors of production estimates from uncorrected dendrometer band data. Two central Oregon, USA ponderosa pine stands(*Pinus ponderosa* Lawson & C. Lawson), a young (YP) and mature (MP) stand aged 25 and 66 years respectively (see Thomas et al. 2009; Vickers et al. 2012 for complete site descriptions), and a 47 year old mature Douglas fir stand [*Pseudotsuga menziesii* (Mirb.) Franco] in the Coast Range of western Oregon, USA (MF) have been monitored repeatedly as part of the Ameriflux Network (http://ameriflux.ornl.gov/, Ameriflux site codes: USME-3, USME-2 and US-MRf, respectively; site descriptions, detailed site data, locations and histories are available online). These sites differ considerably in structure and density (Figure 3) and have large differences in the central angle (θ) at the last dendrometer band measurement. Measurements at the YP site ceased in 2007 and both the MP and MF site have had the bands reset in 2010. ANPP was scaled at these site similarly to the exercise described above but production was calculated for each tree and summed over the total plot area.

RESULTS AND DISCUSSION

From the modeling simulations, the true diameter: estimated diameter ratio ($D{:}D_{est}$) increases exponentially as the central angle (θ) or the *cord:arc* $length_1$ ratio increases (Figure 2, top panel, solid line). The ratio of true diameter to estimated diameter is 1.00 when θ is 0º and reaches a maximum of 1.22 when the angle is 180º. The fit of the line that equates *cord:arc* $length_1$ ratio to the ratio of the two diameters ($D{:}D_{est}$) can be best described with a 6th order polynomial function [fit with Eureqa, (Schmidt and Lipson 2009)].

[Equation 2]

$$Correction\ Factor = \frac{D}{D_{est}} = b_0 + b_1\left(\frac{Cord}{Arc\ Lengh_1}\right) + b_2\left(\frac{Cord}{Arc\ Lengh_1}\right)^2 + b_3\left(\frac{Cord}{Arc\ Lengh_1}\right)^3 + b_4\left(\frac{Cord}{Arc\ Lengh_1}\right)^4 + b_5\left(\frac{Cord}{Arc\ Lengh_1}\right)^5 + b_6\left(\frac{Cord}{Arc\ Lengh_1}\right)^6$$

In Equation 2, D refers to the true diameter, D_{est} is the erroneously estimated diameter calculated from ($Cord + Arc\ length_1$)/π, and *Cord* and $Arc\ length_1$ are circle components (Figure 1). This fit of Equation 2 (Figure 2, top panel) is predictably significant at the p=0.0001 level with a R^2 of effectively 1.000 (Table 1). The fit of Equation 2 is not without error (Figure 2, top panel, shaded region) although the errors are very small with a maximum error of approximately 0.005% at very large angles.

Errors from production estimates scaled from allometric equations can be much larger than the errors from estimating diameter alone (Figure 2, bottom panel). In our simulated tree, the annual increment change erroneously declines (Figure 2, bottom panel, solid line) when compared to the actual fixed rate of change of 0.5 cm yr^{-1} (Figure 2, bottom panel). These errors are further compounded when ANPP is scaled from allometric equations that predict 3-dimensional values (volume or mass) from 1-dimensional data. Furthermore, in time series of dendrometer data, any small error in the diameter estimated at the first time step is propagated for each additional year in an additive manner (Figure 2, bottom panel, long dashed line). The errors associated with uncorrected ANPP estimates can be very large, i.e. 40% when the internal angle (θ) of the band passes 90º and greater than 60% when the ban approaches 180º. These are extreme cases and can be avoided with periodic resetting of the band gap; however, errors of 20% or more are possible on smaller trees where gaps are commonly 40 - 60º. Angles of this size are not uncommon in real world situations as shown by plot level data from existing long term research sites (Figure 3).

Errors at the plot level scale representing a range of real world situations are shown in figure 3 and illustrate both how easily this error could be ignored and also how large the error can be when trees are small and central angles are large. At the quickly growing mature Douglas fir site (MF) the errors increased sharply following band installation but only resulted in an underestimation of ANPP of 4% after 4 years. The slower growing mature ponderosa pine site (MP) had error of similar magnitude but increased much slower than MF. Dendrometer bands at both of these sites were reset in 2010, hence the reduction in error for the last year. Although ANPP errors can be small on the >25 cm diameter stems (2-4%, Figure 3) even after multiple years, the errors can increase rapidly and result in substantial underestimation of ANPP at the slowly growing young ponderosa pine site (YP). This error exceeded 25% on these small trees (<10 cm) and could have the potential to reach >60% if not corrected (Figure 2).

The data presented here highlight a potential negative bias in forest production data scaled from gap based dendrometer bands measured with a linear caliper over multiple years. From a simple modeling exercise, the errors can be approximated and accounted for using a simple correction factor. Future work should incorporate this information and past analyses that used linearly measured dendrometer bands should be examined for a potential underestimation of diameter, diameter change and production estimates.

ACKNOWLEDGEMENTS

This work was supported by the Office of Science (BER), U.S. Department of Energy (DOE) (grant DE-FG02-06ER64318), for the AmeriFlux project ‘‘On the effects of disturbance and climate on carbon storage and the exchanges of carbon dioxide, water vapor and energy exchange of evergreen coniferous forests in the Pacific Northwest: integration of eddy flux, plant and soil measurements at a cluster of supersites.’’ We wish to thank L. Hopp, M. Goeckede, C. Hanson, J. Campbell and A. Schmidt for the many midday discussions that lead to this work.

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TABLES

Table 1: Summary Statistics of Equation 2

Summary of Fit

RSquare	1
RSquare Adj	1
Root Mean Square Error	0.000016
Mean of Response	1.053412
Observations (or Sum Wgts)	90500

Analysis of Variance

Source	**DF**	**Sum of Squares**	**Mean Square**	**F Ratio**
Model	6	348.27812	58.0464	2.25e+11
Error	90493	2.33484e-5	2.58e-10	**Prob > F**
C. Total	90499	348.27814		0.0000

Parameter Estimates

Term	**Estimate**	**Std Error**	**t Ratio**	**Prob>|t|**
b_0	1.0000459	3.216e-7	3.1e+6	0.0000
b_1	-0.004476	1.461e-5	-306.4	0.0000
b_2	0.1013471	0.000206	493.00	0.0000
b_3	0.6733074	0.001233	545.96	0.0000
b_4	0.1869086	0.00356	52.50	0.0000
b_5	-1.373648	0.004889	-281.0	0.0000
b_6	1.846399	0.002561	720.85	0.0000

FIGURE LEGENDS

Figure 1: A diagram of cross sectional geometry when linear gap dendrometer bands are used. Typically, studies use Cord length to estimate a portion of the stem circumference (Arc Length) resulting in increasing errors as the central angle [θ] increases. Two time periods are shown, t1 and t2, where the diameter of the tree increases from $Radius_{t1}$ to $Radius_{t2}$. Arc angle [θ], Cord length and Arc $Length_2$ change accordingly while Arc $Length_1$ (dendrometer band length) remains the same.

Figure 2, TOP: The ratio of true diameter to the diameter estimated using linear gap length as a proxy for Arc $Length_2$ increases as arc angle [θ] or the ratio of Cord length to Arc $Length_1$ increases (solid line). The results are from the simulation of 90,500 circles with random diameters and random cord lengths which were used to develop Equation 2. Equation 2 describes the relationship between Cord:Arc Length ratio (x) where y is the correction factor that is multiplied to estimated diameter to convert to true diameter; errors from the model deviating from true diameters are small (shaded region).

BOTTOM: The estimated annual diameter change decreases relative to actual diameter change (solid lines) for a simulated scenario where a Douglas fir with 10 cm DBH grew 0.5 cm annually which changed the arc angle of the dendrometer band from 0-180º. The annual increment growth errors are compounded and resulted in an increasing error of estimated ANPP (dashed line).

Figure 3: Mean and standard deviation of diameter, height, stand density and most recent central angle (θ) varies across three different forest stands, a mature Douglas fir stand (MF) and a mature and young ponderosa pine stands (YP and MP). For a range of corrected ANPP (gray bars), the errors associated with using uncorrected diameter growth (solid line) accumulates over time and is largest for a young stand with trees <10 cm DBH.

FIGURE 1

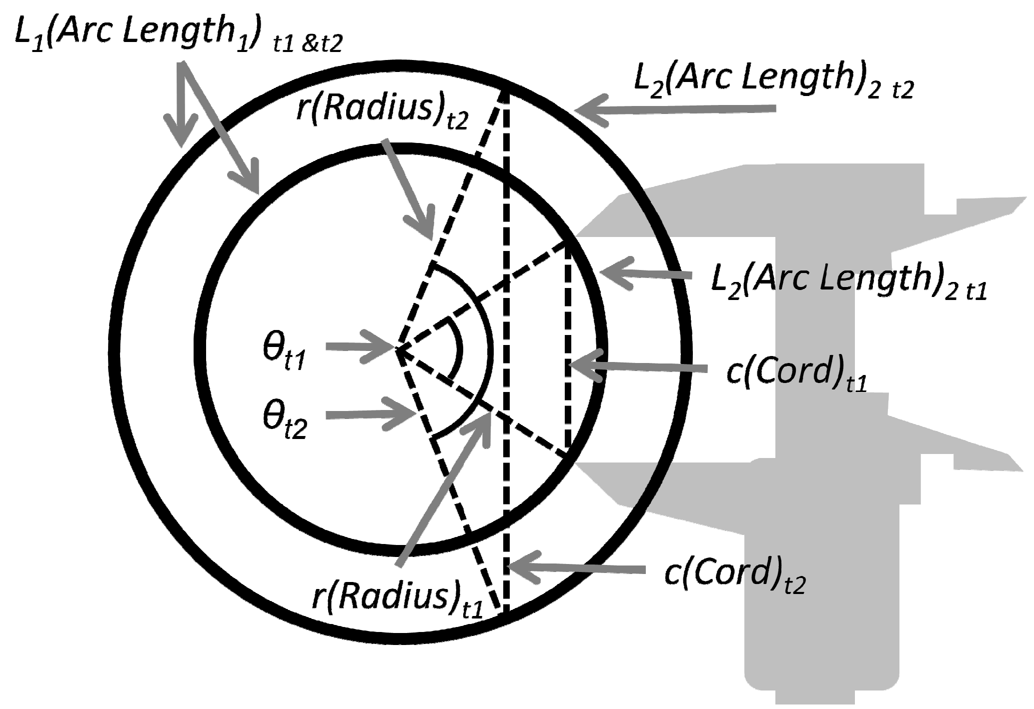

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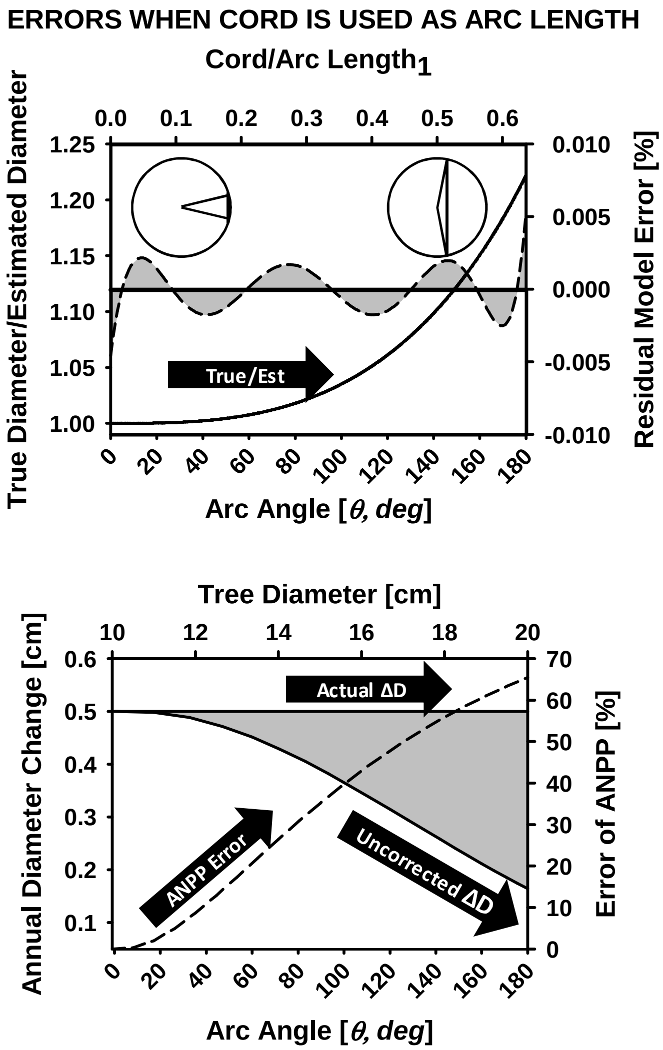
FIGURE 2



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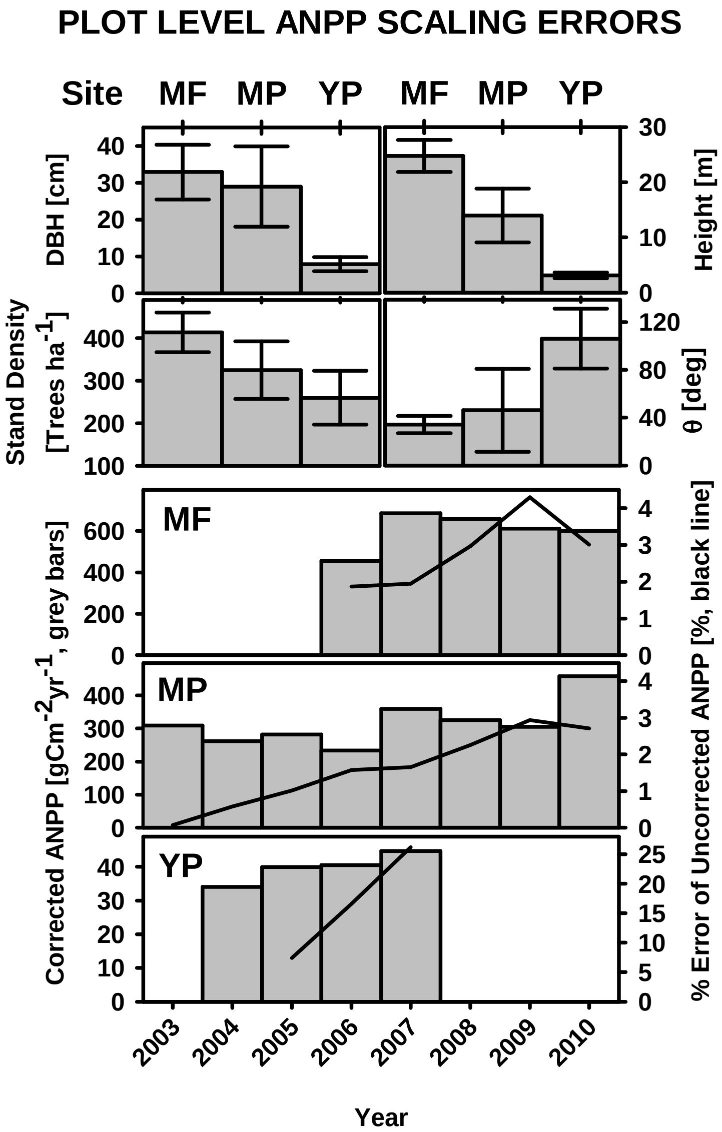

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