**Static Fields Homework 10**

*Due 5/4/18 @ 4:00 pm*

Start your homework early and submit a question about it on Canvas before class on Thursday!

Remember that you should do some sense-making about every problem and result (*e.g.,* describe how you know a result is correct, interpret your answer non-symbolically, or describe new physics insight you gained). Solutions that contain exceptional sense-making will receive bonus points.

**REQUIRED:**

1. (a) Find the magnetic field for a finite segment of straight wire, carrying a uniform current $I$. Put the wire on the $z$ axis, from $z_1$ to $z_2$.

   (b) Show that your answer to part (a) is the curl of the magnetic vector potential.

2. Consider a point a distance $z$ above the center of an infinitesimally thin, square sheet of current. The current is parallel to one of the square sides. (Obviously, since the current cannot just begin and end in the middle of nowhere, this current is just the building block for some larger current.)

   (a) Use the Biot-Savart Law to find the magnetic field at the point $z$. You may use any symmetry arguments you like, but do **not** use Ampere's Law.

   *Note: if you choose to use Mathematica or Maple to evaluate the integral, it may take you into complex number land, even though the integral is clearly real. To address this issue, you should be explicit about what assumptions you want the program to make ("Assume" in Maple and "Assumptions" in Mathematica)*

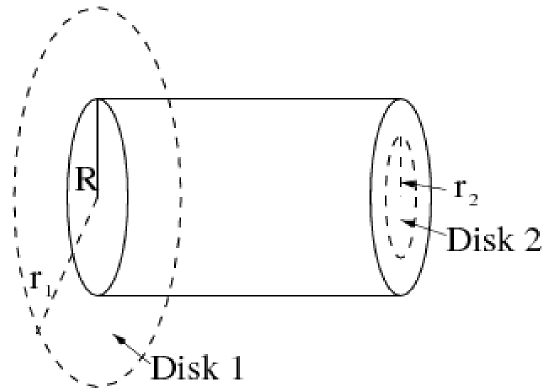
   (b) Consider your previous answer in the limit that the square becomes infinitely large.

   (c) Discuss your answer in the light of the magnetic field above an infinite sheet of current as found using Ampere's Law.

3. In this problem, you will be investigating, from several different points of view, a cylindrical wire of finite thickness $R$, carrying a non-uniform current density $J = \kappa s$, where $\kappa$ is a constant and $s$ is the distance from the axis of the cylinder.

   (a) Find the total current flowing through the wire.

   (b) Find the current flowing through Disk 2, a central (circular cross-section) portion of the wire out to a radius $r_2 < R$.




(c) Use Ampère's law in integral form to find the magnetic field at a distance $r_1$ outside the wire.

(d) Use Ampère's law in integral form to find the magnetic field at a distance $r_2$ inside the wire.

(e) Use theta functions to write the magnetic field everywhere (both inside and outside of the wire) as a single function.

(f) Evaluate

$$\int \left(\vec{\nabla} \times \vec{\boldsymbol{B}}\right) \cdot d\vec{\boldsymbol{A}}$$

for Disk 2, a circular disk of radius $r_2 < R$. Use this result and part (d) to verify Stokes' theorem on this surface.

(g) Evaluate

$$\int \left(\vec{\nabla} \times \vec{\boldsymbol{B}}\right) \cdot d\vec{\boldsymbol{A}}$$

for Disk 1, a circular disk of radius $r_1 > R$. Use this result and part c) to verify Stokes' theorem on this surface.

4. Find the volume current density that produces the following magnetic field (expressed in cylindrical coordinates):

$$\vec{B}(\vec{r}) = \begin{cases} \frac{\mu_0\, I\, s}{2\pi a^2}\hat{\phi} & s \leq a \\ \frac{\mu_0\, I}{2\pi s}\hat{\phi} & a < s < b \\ 0 & s > b \end{cases}$$

What is a physical situation that corresponds to this current density?