Static Fields Homework 8

Due 4/27/18 @ 4:00 pm

Start your homework early and submit a question about it on Canvas before class on Thursday!

Remember that you should do some sense-making about every problem and result (*e.g.,* describe how you know a result is correct, interpret your answer non-symbolically, or describe new physics insight you gained). Solutions that contain exceptional sense-making will receive bonus points.

PRACTICE:

1. Laplace's equation in two dimensions is: $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$. Assume the region if interest is a rectangle of width a and height b.

   (a) Use separation of variables to find the general solution to Laplace's equation in two dimensions.

   (b) Suppose three of the boundaries ($x = 0$, $x = a$, and $y = 0$) are known to have $V = 0$. Find the general solution in this case.

   (c) Suppose only one boundary ($y = 0$) is known to have $V = 0$, and that two boundaries ($x = 0$ and $x = a$) are known to have $\frac{\partial V}{\partial y} = 0$. Find the general solution in this case.

REQUIRED:

2. Consider the electric field $\vec{E} = \alpha\left(\frac{3\cos\theta}{r^4}\hat{r} + \frac{\sin\theta}{r^4}\hat{\theta}\right)$.

   (a) Find the electric potential. In addition to your usual sense-making, include a reasonable graph.

   (b) Find the charge density. In addition to your usual sense-making, include a reasonable graph.

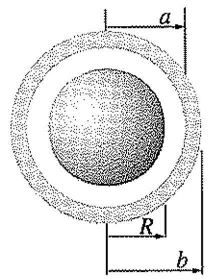
3. Consider the bounded two-dimensional region from class. Three sides are metal and held at $V = 0$ while one is an insulator on which the potential is known to be:
   $V(x, b) = V_0\left(\sin\left(\frac{\pi x}{a}\right) + \sin\left(\frac{2\pi x}{a}\right) - \sin\left(\frac{3\pi x}{a}\right)\right)$

   (a) Starting from the general solution from the practice problem, find a symbolic expression for the potential $V(x, y)$.

   (b) Make several plots of your solution and discuss any interesting features you find. (I particularly recommend both surface plots and plots of x- and y-cross sections at several different values.)

(c) Suppose that the fourth side of the region is also a conductor at constant potential V_0. Find a symbolic expression for $V(x, y)$, graph your solution, and discuss its features.

4. A metal sphere of radius R, carrying charge q is surrounded by a thick concentric metal shell (inner radius a, outer radius b, as shown below). The shell carries no net charge.



   (a) Find the surface charge density σ at R, at a, and at b.
   (b) Find E_r, the radial component of the electric field and plot it as a function of r. Are the discontinuities in the electric field related to the charge density in the way you expect from previous problems?
   (c) Find the potential at the center of the sphere, using infinity as the reference point.
   (d) Now the outer surface is touched to a grounding wire, which lowers its potential to zero (the same as infinity). How do your answers to a), b), and c) change?

5. Consider a square loop with each side length a carrying a uniform linear charge density λ.

   (a) Find the electric field a distance z above the center of the square. (You may start with the electric field due to a single finite line of charge).
   (b) Find the work needed to bring a charge in from infinity along the z-axis.
   (c) Use two different methods to find the value of the electric potential a distance z above the center of the square.

6. Three charges are situated at the corners of a square (side s). Two have charge $-q$ and are located on opposite corners. The third has charge $+q$ and is opposite an empty corner.

   (a) How much work does it take to bring in another charge, $+q$, from far away and place it at the fourth corner?
   (b) How much work does it take to assemble the whole configuration of four charges?