Static Fields Homework 7

Due 4/25/18 @ 4:00 pm

Start your homework early and submit a question about it on Canvas before class on Tuesday!

Remember that you should do some sense-making about every problem and result (*e.g.,* describe how you know a result is correct, interpret your answer non-symbolically, or describe new physics insight you gained). Solutions that contain exceptional sense-making will receive bonus points.

PRACTICE:

1. You have a charge distribution composed of two point charges: one with charge $+3q$ located at $x = -d$ and the other with charge $-q$ located at $x = +d$.
   (a) Sketch the charge distribution.
   (b) Write an expression for the *volume* charge density $\rho(\vec{r})$ everywhere in space.

2. One way to write volume charge densities without using piecewise functions is to use step (Θ) or δ functions. If you need to review these, see the following links in the math-physics book:

   http://physics.oregonstate.edu/BridgeBook/book/math/step

   http://physics.oregonstate.edu/BridgeBook/book/math/deltaintro

   http://physics.oregonstate.edu/BridgeBook/book/files/math/deltadensity

REQUIRED:

1. A positively charged dielectric cylindrical shell of inner radius a and outer radius b has a cylindrically symmetric internal charge density

$$\rho = 3\,\alpha\ \sin\left(\frac{\pi(s-a)}{b-a}\right)$$

   where α is a constant with appropriate dimensions.

   (a) Sketch the charge density and find the total charge on the shell.
   (b) Write the volume charge density everywhere in space as a single function.
   (c) Use Gauss's Law and symmetry arguments to find the electric field in each of the regions given below:
   (i) $s < a$
   (ii) $a < s < b$
   (iii) $s > b$

(d) Sketch the s-component of the electric field as a function of s.

(e) Briefly describe in words something you learned from doing this problem that you would like to remember for the future. Make your statement using good scientific writing, as you would in a research paper.

2. Referring to the charge distribution in the Gauss's Law problem which you have solved above, take the limit as $a \to b$ so that the shell becomes infinitely thin, but keeping the total charge on a unit length of the cylinder constant. Redo each part of the previous problem for this situation.

   (a) Find the surface charge density on the shell.

   (b) Write the volume charge density everywhere in space as a single function.
   Be careful: Integrate your charge density to get the total charge as a check.

   (c) Use Gauss's Law and symmetry arguments to find the electric field at each region given below:
   (i) $s < b$
   (ii) $s > b$

   (d) Sketch the s-component of the electric field as a function of s.

   (e) Compare the surface charge density on the shell to the discontinuity in the s-component of the electric field.

3. For an infinitesimally thin cylindrical shell of radius b with uniform surface charge density σ, the electric field is zero for $s < b$ and $\vec{E} = \frac{\sigma b}{\epsilon_0 s}\,\hat{s}$ for $s > b$. Use Gauss' Law to find the charge density everywhere in space.

4. Consider the vector field in rectangular coordinates (assume the coordinates are scaled to be dimensionless):

$$\vec{E} = \frac{q}{4\pi\epsilon_0}[(2xy^3z + z)\hat{x} + (3x^2y^2z)\hat{y} + (x^2y^3 + x)\hat{z}]$$

   (a) Using only the x-component of $\vec{E}$, find as much information as possible about the potential from which this electric field might have come.

   (b) Repeat this exercise for the y- and z-components of $\vec{E}$. Does this field come from a potential?

   (c) Consider the different vector field:

$$\vec{E} = \frac{q}{4\pi\epsilon_0}(-y\hat{x} + x\hat{y})$$

   Does this field come from a potential?

   (d) Consider the different vector field:

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\left(s\hat{\phi}\right)$$

   Does this field come from a potential?