Static Fields Homework 6

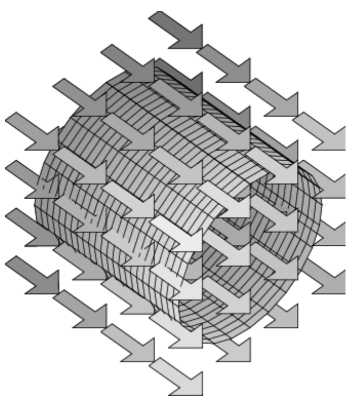
Due 4/20/18 @ 4:00 pm

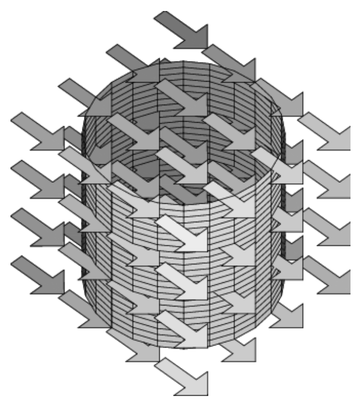
Start your homework early and submit a question about it on Canvas before class on Thursday!

Remember that you should do some sense-making about every problem and result (*e.g.,* describe how you know a result is correct, interpret your answer non-symbolically, or describe new physics insight you gained). Solutions that contain exceptional sense-making will receive bonus points.

PRACTICE:

1. What do you think will be the flux through the cylindrical surface that is placed as shown in the constant vector field in the figure on the left? What if the cylinder is placed upright, as shown in the figure on the right? Explain.





2. Calculate the divergence of each of the following vector fields. You may look up the formulas for divergence in curvilinear coordinates.

   (a) $\vec{\boldsymbol{F}} = z^2\,\hat{x} + x^2\,\hat{y} - y^2\,\hat{z}$

   (b) $\vec{\boldsymbol{G}} = e^{-x}\,\hat{x} + e^{-y}\,\hat{y} + e^{-z}\,\hat{z}$

   (c) $\vec{\boldsymbol{H}} = yz\,\hat{x} + zx\,\hat{y} + xy\,\hat{z}$

   (d) $\vec{\boldsymbol{I}} = x^2\,\hat{x} + z^2\,\hat{y} + y^2\,\hat{z}$

   (e) $\vec{\boldsymbol{J}} = xy\,\hat{x} + xz\,\hat{y} + yz\,\hat{z}$

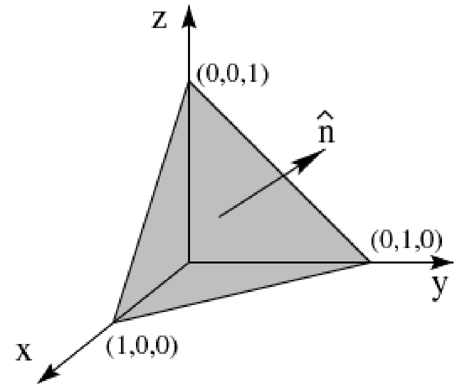
   (f) $\vec{\boldsymbol{K}} = s^2\,\hat{s}$

   (g) $\vec{\boldsymbol{L}} = r^3\,\hat{\phi}$

REQUIRED:

3. A charge q sits at the corner of a cube. Find the flux of $\vec{\boldsymbol{E}}$ through each side of the cube. **Do not do a long calculation (either by hand or by computer)!**

4. Use the cross product to find the components of the unit vector $\hat{n}$ perpendicular to the plane shown in the figure below, i.e. the plane joining the points $\{(1,0,0),(0,1,0),(0,0,1)\}$.

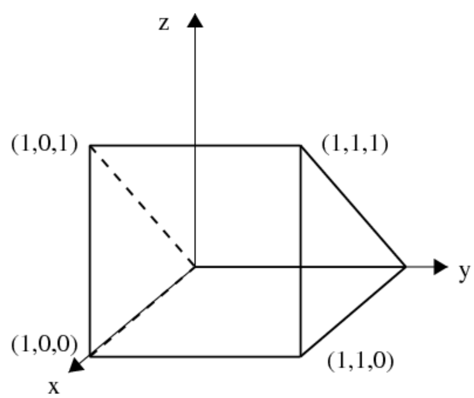


5. Consider the vector field $\vec{F} = (x+2)\hat{x} + (z+2)\hat{z}$.

   (a) Calculate the divergence of $\vec{F}$.

   (b) In which direction does the vector field $\vec{F}$ point on the plane $z = x$? What is the value of $\vec{F} \cdot \hat{n}$ on this plane where $\hat{n}$ is the unit normal to the plane?

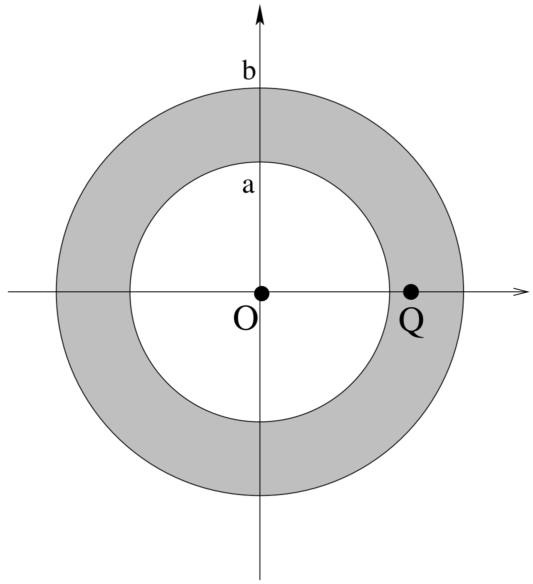
   (c) Verify the divergence theorem for this vector field where the volume involved is drawn below.



6. You can use Gauss's law to find the gravitational field due to a spherical shell of matter (or equivalently, the electric field due to a spherical shell of charge):

$$\vec{g} = \begin{cases} 0 & \text{for } r < a \\ -G\,\frac{M}{b^3-a^3}\left(r - \frac{a^3}{r^2}\right)\hat{r} & \text{for } a < r < b \\ -G\,\frac{M}{r^2}\,\hat{r} & \text{for } r > b \end{cases} \tag{1}$$

(If you want extra practice with Gauss's law, this is a good question to practice on!)



This problem explores the consequences of the divergence theorem for this shell.

(a) Using the given value of the gravitational field, find the divergence of the gravitational field everywhere in space. You will need to divide this question up into three parts: $r < a$, $a < r < b$, and $r > b$.

(b) Discuss the physical meaning of the divergence in this particular example.

(c) For this gravitational field, verify the divergence theorem on a sphere, concentric with the shell, with radius Q, where $a < Q < b$.

(d) Discuss how this example would change if you were discussing the electric field of a uniformly charged spherical shell.