Static Fields Homework 5

Due 4/18/18 @ 4:00 pm

Start your homework early and submit a question about it on Canvas before class on Tuesday!

Remember that you should do some sense-making about every problem and result (*e.g.*, describe how you know a result is correct, interpret your answer non-symbolically, or describe new physics insight you gained). Solutions that contain exceptional sense-making will receive bonus points.

PRACTICE:

1. Find the gradient of each of the following functions:

   (a)
   $$f(x, y, z) = e^{(x+y)} + x^2 y^3 \ln \frac{x}{z}$$

   (b)
   $$\sigma(\theta, \phi) = \cos\theta \sin^2 \phi$$

   (c)
   $$\rho(s, \phi, z) = (s + 3z)^2 \cos\phi$$

REQUIRED:

1. (a) Find the electrostatic potential due to an infinite disk, using your results from the finite disk problem.

   (b) Briefly describe in words something you learned from doing this problem that you would like to remember for the future. Make your statement using good scientific writing, as you would in a research paper.

2. A conical surface (an empty ice-cream cone) carries a uniform charge density σ. The height of the cone is a, as is the radius of the top. Find the potential at point P (in the center of the opening of the cone), letting the potential at infinity be zero.

3. The electrostatic potential due to a point charge at the origin is given by:
   $$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

   (a) Find the electric field due to a point charge at the origin as a gradient in rectangular coordinates.

   (b) Find the electric field due to a point charge at the origin as a gradient in spherical coordinates.

(c) Find the electric field due to a point charge at the origin as a gradient in cylindrical coordinates.

4. Consider the fields at a point $\vec{\boldsymbol{r}}$ due to a point charge located at $\vec{\boldsymbol{r}}'$.

   (a) Write down an expression for the electrostatic potential $V(\vec{\boldsymbol{r}})$ at a point $\vec{\boldsymbol{r}}$ due to a point charge located at $\vec{\boldsymbol{r}}'$. (There is nothing to calculate here.)

   (b) Write down an expression for the electric field $\vec{\boldsymbol{E}}(\vec{\boldsymbol{r}})$ at a point $\vec{\boldsymbol{r}}$ due to a point charge located at $\vec{\boldsymbol{r}}'$. (There is nothing to calculate here.)

   (c) Working in rectangular coordinates, compute the gradient of V.

   (d) Write several sentences comparing your answers to the last two questions.

5. Consider the finite line with a uniform charge density from class.

   (a) Write an integral expression for the electric field at any point in space due to the finite line. In addition to your usual physics sense-making, you must include a clearly labeled figure and discuss what happens to the direction of the unit vectors as you integrate.

   (b) Perform the integral to find the z-component of the electric field. In addition to your usual physics sense-making, you must compare your result to the gradient of the electric potential we found in class. (If you want to challenge yourself, do the s-component as well!)